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Tini a Tangaroa

Modelling the length frequency of red rock lobsters (*Jasus edwardsii*) in New Zealand

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EXECUTIVE SUMMARY

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In the past, commercial catch length frequency (LF) data for New Zealand red rock lobster stocks were processed as empirical LFs and an equation based on the catch, the number of fish measured, and the number of days sampled was used to derive a measure of uncertainty for each LF. However, there are two main issues with deriving empirical LFs in this way: (1) if LF data are missing within a stratum (e.g., an area wasn't sampled in a year/month), but there was some catch taken from that stratum, then the aggregated LF scaled by the catch may not be representative of the entire catch; and (2) the measure of uncertainty is somewhat arbitrary and may not adequately characterise the true uncertainty.

This document describes the development of a methodology for predicting commercial catch LFs and associated uncertainty for New Zealand red rock lobster stocks using data collected by the observer catch sampling and voluntary logbook programmes. This methodology is applied to two Quota Management Areas (CRA 2 and CRA 3).

¹ Quantifish Ltd, New Zealand.

1. INTRODUCTION

Commercial catch length frequencies (LFs) of fish or crustaceans are usually derived empirically from measurements of length (or an alternative measure of size) where the measurements are done at sea or in processing facilities on land. Deriving LFs empirically generally involves:

1. summarising individual measurements of length into counts by length bin for each sex category (i.e., rounded to the nearest cm or categories that cover a range of lengths) for different strata (e.g., by month and/or statistical area);
2. rescaling (i.e., multiplying) the counts within each stratum by the commercial catch that was taken from that stratum;
3. aggregating the scaled data into fewer strata; and
4. converting the counts to proportions, if required.

However, there are two main issues with using empirical LFs:

1. if LF data are missing within a stratum (e.g., a statistical area wasn't sampled in a year/month), but there was some catch taken from that stratum, then the aggregated scaled LF may not be representative of the entire catch; and
2. there is no measure of uncertainty for the derived LF within each stratum.

Often the first point above is ignored which may not matter if only small amounts of catch are taken from strata that were not sampled for lengths, or if the LFs in the strata that were not sampled are similar to the LFs in the strata that were sampled. However, ignoring the first point could result in biases in derived LFs, particularly if a large proportion of the catch was taken from a stratum that was not sampled for lengths and the LFs in that stratum were significantly different from the LFs in the strata that were sampled. The second point above is usually addressed by defining proxies for the uncertainty (e.g., using a measure of effort such as the number of trawl tows or an *ad hoc* formula for deriving a relative data set weight), which can be subjective and may not characterise the uncertainty well.

Alternatively, Bull & Dunn (2002) developed software to derive LFs and associated coefficients of variation (CVs) using bootstrapping of stratified length data (where the strata are usually the spatial strata defined in trawl surveys, the trip, and the trawl tow). However, the approach used in this software does not address the first problem identified above. Furthermore, bootstrapping is only asymptotically consistent under some conditions and, therefore, will not always yield valid results (Hinkley 1994). For instance, the result may depend on the representativeness of the samples, the independence of samples, or the sample size.

A better approach would be to:

1. summarise individual measurements of length into counts by length bin for different strata;
2. model the binned measurements of length using a categorical (e.g., multinomial or Dirichlet-multinomial, see Appendix I) distribution;
3. rescale the predicted counts within each stratum by the commercial catch that was taken from each stratum;
4. aggregate the scaled predictions into fewer strata; and
5. convert the counts to proportions, if required.

Modelling the LFs in this way can allow predictions to be made for missing strata and, naturally, provides a measure of uncertainty for each derived LF. This type of model-based approach has already been explored for stock assessment. Thorson (2014) suggested that LF data should be standardised in the same way that abundance index data (e.g., catch per unit effort) are treated before being used in stock assessment. He also proposed that future research should develop methods to impute compositional data in strata with missing data. Thorson et al. (2016) focussed on using such models to

estimate uncertainty (i.e., the effective sample size) for use in stock assessments. A similar approach has also been applied to pāua (*Haliotis iris*) stocks in New Zealand (Neubauer & Tremblay-Boyer 2019, Neubauer 2020) using a custom Stan (Stan Development Team 2018) model and assuming a Dirichlet-multinomial distribution. While this custom model is capable of addressing both issues identified above for pāua, the custom modelling framework is not easily portable to other fish stocks.

This document describes the development of a methodology for predicting commercial catch LFs and associated uncertainty for New Zealand red rock lobster (*Jasus edwardsii*, CRA) stocks, using data collected by the observer catch sampling (CS) programme (Mackay & George 2002), the voluntary logbook (LB) data collected by commercial fishers (Walker & Mackay 2002), and commercial catch data. This methodology was developed to address both of the issues identified above and is easily portable to other fish stocks. This methodology is applied to two Quota Management Areas (QMAs): CRA 2 and CRA 3 (Figure 1). These two QMAs were selected for their contrasting data collection regimes, where the LB programme is the primary source of LF data in CRA 2 while the CS programme has predominated in CRA 3. LF data are a key input for New Zealand red rock lobster stock assessments, where their purpose is to inform the model of the range of sizes that are removed by the commercial catch and, also, to indirectly provide information about growth and recruitment.

1.1 Design of the observer catch sampling programme

The observer CS programme places independent observers on a vessel to record information from a full day of fishing, measuring and recording the entire lobster catch from every pot lifted. For each day of fishing, observers recorded statistical area, weather information, skipper, vessel key, and for each pot lift they recorded depth, pot type, the number of predators, soak time, and the total number of lobsters caught. Observers measured every lobster in a pot to avoid any selection process. They were allowed to skip entire pots when they were already in the process of measuring from a pot and were at risk of slowing down the operation of the vessel. Skipped pots were enumerated with a total count, but were not separated out in terms of legal/sub-legal counts. For the pots with measurements, there was a tail width (TW) taken for every lobster (rounded down to the nearest 0.1 mm), with the sex category and the maturity status (if female) recorded. A count of the number of damaged limbs was also recorded for each measured lobster.

1.2 Design of the voluntary logbook programme

The voluntary LB programme began in 1993. In areas where the programme operates, participation by fishers is voluntary and is consequently non-random. All fishers in a QMA, regardless of the scale or type of their fishing activities, were encouraged to participate in the programme, with the intent of gaining the participation of as many operators as possible. Each participant was asked to sample the entire catch from four selected pots each time they were hauled. These four pots, selected from the hundred or so pots that each fisherman uses and marked with a tag on the pot, were assumed to be representative of their remaining pots².

Each time a designated pot was lifted, the depth, soak time, the number of octopus, and the number of dead lobsters was recorded, as well as the TW, sex category, and maturity (of female lobsters) of each live lobster. Using specially modified callipers, TW was measured across segment two of the abdomen, rounded down to the nearest millimetre.

The design capped the number of lobsters that were measured in any single potlift to reduce the commitment on the part of the voluntary operator. Initially this number was 31 (regardless of the size or maturity status of the measured lobster) because of the configuration of the paper form with the number of rows which could be easily fitted on a single page. When the form was redesigned in 1998, this

² Ideally the pots would be selected randomly, but it was felt that this would have been too difficult to implement reliably. Consequently, LB participants were asked to select pots that represented their average fishing effort. They were instructed to fish these pots in exactly the same manner as the remainder of their pots.

number was reduced to 25. Additional lobsters beyond the cap limit were recorded in two categories, those above the minimum legal size (MLS) limit and those below. This categorisation was meant to be independent of the legal status of the individual. For example, a female that was alive when the pot was lifted and above the female size limit for the area, but which could not be landed because it was carrying eggs (berried), was to be included in the ‘LiveAbove’ category.

2. METHODS

The subscripts used throughout this document are summarised in Table 1.

2.1 Previous method for generating length frequencies of lobsters

For a given stock, the lobster stock dynamics (LSD) model (Webber et al. 2018) tracks the numbers of lobsters in size classes or TW bins (l) in three different sex/maturity categories (s), each fishing year (y) (also referred to as ‘year’), season (t), and region (r). The number of individuals in each category of the model is denoted mathematically as

$$N_{y,t,r,s,l}$$

with support $N_{y,t,r,s,l} \in [0, \infty]^3$. Henceforth, the year, season, and region subscripts are dropped for clarity. Typically, 2 mm TW size bins are used, where the first bin includes lobsters that are ≥ 30 to < 32 mm TW, and the final bin includes lobsters that are ≥ 90 mm TW. This can be expressed mathematically as

$$[30,32), [32,34), \dots, [88,90), [90, \infty)$$

Occasionally, a larger accumulator bin (such as 100 mm TW) may be used if many of the measured lobsters are greater than 90 mm TW.

For the LSD stock assessment model to predict the numbers at length in the population each year, there must be data to inform the relative numbers at length each year, either in the population or in the catch. For New Zealand rock lobster stock assessments, commercial catch LFs provide this information to the stock assessment. These commercial catch LFs were processed from the sampling data collected by the CS programme (Section 1.1) and the LB programme (Section 1.2) as follows (e.g., see Starr et al. 2021).

For a given fishing year (y), month (m), statistical area (a), data source (CS or LB) (d), trip (j), and potlift (i) counts of the number of lobsters measured by sex category (s) and length bin (l) are collected by the CS and LB programmes. These counts can be expressed mathematically as

$$n_{y,m,a,d,j,i,s,l} \in [0, \infty]$$

It is usually better to look at aggregated versions of these data by summing the counts of lobsters within pots. For example, aggregated to fishing year, month, statistical area, data source, and sex category for each length

Equation 1:

$$n_{y,m,a,d,s,l} = \sum_{j,i} n_{y,m,a,d,j,i,s,l}$$

The sampled proportion-at-length is then calculated as

³ The notation $x \in [0,1]$ means that x can take any value between 0 and 1, including 0 and 1; $y \in (0,1)$ means that y can take any value between 0 and 1, excluding 0 and 1; and $z \in [0,1)$ means that z can take any value between 0 and 1, including 0 but excluding 1.

Equation 2:

$$p_{y,m,a,d,s,l} = \mathbf{p}_{y,m,a,d,s} = n_{y,m,a,d,s,l} / \sum_l n_{m,a,s,l}$$

where $\mathbf{p}_{y,m,a,d,s}$ is a vector that sums to one.

The catch (tonnes) in each fishing year, month, and statistical area is calculated from the catch and effort database and denoted $C_{y,m,a}$. Proportions were scaled by the catch and aggregated by fishing year, season, data source, and sex category:

Equation 3:

$$\mathbf{P}_{y,t,d,s} = \sum_{m \in t,a} C_{y,m,a} \mathbf{p}_{y,m,a,d,s} / \sum_{m \in t,a,s} C_{y,m,a} \mathbf{p}_{y,m,a,d,s}$$

This catch scaling is done at the fishing year, month, and statistical area level, because the CS and LB data cannot be linked to the catch effort data in any finer detail (i.e., there is no way to link the CS or LB data with the associated catch data for the same trip).

A relative weight for each fishing year, season, data source, and sex category ($\omega_{y,t,d,s}$) was assigned to each LF. This weight combined the representativeness of the catch for each fishing year/month/statistical area, the cube root of the number of fish measured ($n_{y,m,a,d,s,l}$), and the cube root of the number of days sampled ($D_{y,m,a,d}$). These weights were arbitrarily selected to represent the difference between two sampling methods, with the CS programme measuring a large number of lobsters over relatively fewer pots lifted and days sampled while the LB programme samples a much larger selection of days but fewer pots and lobsters measured per day.

Equation 4:

$$\omega_{y,t,d,s} = \sum_{m,a} \left(C_{y,m,a}^3 \sqrt[3]{\sum_{s,l} n_{y,m,a,d,s,l}} \sqrt[3]{D_{y,m,a,d}} / \sum_{m,a} C_{m,a,d} \right)$$

In summary, the LFs are scaled by the catch to ensure that they are representative of the catch and a data set weight is assigned to each LF. However, this approach ignores catch in statistical areas that were not sampled for lengths. Furthermore, the calculation of the data set weights is *ad hoc* and somewhat subjective. Both of these issues can be solved by modelling the LF data rather than using the data to derive empirical LFs.

2.2 New method for predicting length frequencies of lobsters

Before developing a new method for predicting LFs from CS and LB data, it is worth considering the generative process for these data:

1. a fisher/vessel goes out on a trip on a particular day/month/fishing year;
2. during that trip the fisher lifts one or more pots within a region/statistical area; and
3. the fisher or observer measures and records the sex category and TW of lobsters within those pots.

Therefore, there are three important components to the sampling:

1. the number of days or trips sampled and the temporal distribution of these trips (i.e., did trips occur on all months within a year);
2. the number of pots lifted during each trip and the spatial distribution of these pots; and
3. the number of lobsters measured.

Any new method should adequately capture the variability in these components and provide reasonable predictions of the LFs and the uncertainty associated with these predictions, ideally at the level that the data are collected (i.e., by pot). However, in the method that follows, the data were not fitted at the pot (*i*) level because initial exploratory work that fitted models to pot-by-pot data took too long to run. Instead, the data were aggregated to the trip (*j*) level and the models were fitted to trip-by-trip data. It is also important to note that the catch should not be considered during model fitting to the LF data and, thus, should not be used in deriving the uncertainty/data set weighting for each LF. However, the catch should be used to rescale the model predicted LFs.

For a given fishing year (*y*), month (*m*), statistical area (*a*), data source (LB or CS) (*d*), and trip (*j*), counts of the number of lobsters measured by sex category (*s*) and length bin (*l*) are generated and arranged as vectors by length bin (*l*)

$$n_{y,m,a,d,j,l} = \mathbf{n}_{y,m,a,d,j} \in [0, \infty]$$

The sex category subscript is dropped for clarity and because models are fitted separately for each sex category. Often there are few lobsters below a certain size or above a certain size, so each end of the LF is aggregated into the first or final bin, respectively (Table 2).

The predictor term (η) for each length bin (*l*) during fishing year (*y*), month (*m*), and statistical area (*a*) was

Equation 5: $\eta_{y,m,a,l} = \boldsymbol{\eta}_{y,m,a} = \mathbf{X}\boldsymbol{\beta} + \mathbf{Z}\boldsymbol{\mu} = \beta_l^0 + X^y\beta_l^y + X^m\beta_l^m + X^a\beta_l^a + Z^{y,a}\mu_l^{y,a}$

In this equation, $\boldsymbol{\beta}$ and $\boldsymbol{\mu}$ are the coefficients at the population level (analogous to fixed effects) and group level (analogous to random effects), respectively, $\mathbf{X}$ and $\mathbf{Z}$ are the corresponding design matrices, β_l^0 is the intercept for each length, β_l^y are coefficients accounting for each fishing year, β_l^m are coefficients accounting for each month, β_l^a are coefficients for each statistical area, $\mu_l^{y,a}$ are coefficients for an interaction term by fishing year and statistical area. The fishing year, month, and statistical area terms are treated as population-level effects and the fishing year by statistical area interaction term is treated as a group-level effect.

The model predicts the proportion ($\mathbf{p}$) within each TW bin (*l*) by fishing year, month, and statistical area

Equation 6: $\mathbf{p}_{y,m,a} = \text{logit}(\boldsymbol{\eta}_{y,m,a})$ where $\mathbf{p}_{y,m,a} \in (0,1)$ and $\sum \mathbf{p}_{y,m,a} = 1 \forall y, m, a$

Counts of individuals within each TW bin at the trip level were fitted to using a multinomial distribution

Equation 7: $\mathbf{n}_{y,m,a,d,j} \sim \text{multinomial}(n, \mathbf{p}_{y,m,a})$

and the following priors were used

Equation 8:

$$\begin{aligned} \beta_l^0 &\sim N(0, 10^2) \\ \beta_l^y &\sim N(0, 1^2) \\ \beta_l^m &\sim N(0, 1^2) \\ \beta_l^a &\sim N(0, 1^2) \\ \mu_l^{y,a} &\sim N\left(0, (\sigma_l^{y,a})^2\right) \\ \sigma_l^{y,a} &\sim \text{student}(3, 0, 2.5) \end{aligned}$$

Bayesian inference was done using the R package *brms* (Bürkner 2017). The *brms* package is a wrapper for doing mixed effects models using Stan (Stan Development Team 2018) for Markov chain Monte Carlo (MCMC). Using *brms* should enable this modelling approach to be implemented easily for other fish stocks. Initial model fits were done using variational inference, which provides an approximation of the posterior distribution. The approximate posterior distributions were used to explore different model structures and ensure that the model was not severely ill-conditioned. Full Bayesian inference was then done using the Hamiltonian Monte Carlo (HMC) algorithm. Initially, MCMC mixing was poor, but adequate MCMC mixing within a reasonable timeframe was achieved by:

- removing early model years with too few data;
- using QR⁴ decomposition of the data;
- the application of priors (Equation 8); and
- specifying initial values for all model parameters (using the median obtained using variational inference).

Model fits to these data are not presented at the level to which they were fitted (i.e., by trip). Instead, for each QMA the data were aggregated to the two levels described below with model predictions made at these aggregated levels:

1. By fishing year, month, data source, and length bin separately for each statistical area and sex category

Equation 9:

$$n_{y,m,a,d,s} = \sum_j n_{y,m,a,d,j,s}$$

2. By fishing year, season, and length bin separately for each region and sex category

Equation 10:

$$n_{r,y,t,s} = \sum_{a \in r, m \in t, d, j} n_{y,m,a,d,j,s}$$

Predictions at any of the two levels above (done using the fitted function in the *brms* package) also produces a standard deviation (SD, σ) for each bin. The total SD across bins, associated with Equation 9 can, for example, be calculated using

Equation 11:

$$\sigma_{y,m,a,s} = \sqrt{\sum_l \sigma_{y,m,a,s,l}^2}$$

The catch (tonnes) in each fishing year, month, and statistical area is calculated from the catch and effort database and denoted $C_{y,m,a}$. The fitted models were used to simulate the predicted LF by region, year, month, statistical area, and sex category from the posterior distribution (using the *posterior_epred*⁵ function in the *brms* package) or posterior predictive distribution (using the *posterior_predict* function in the *brms* package). The simulated LFs were scaled by the catch and aggregated by posterior sample (x), region, fishing year, season (t), sex category, and length bin then converted to proportions using

⁴ In linear algebra, a QR decomposition, also known as a QR factorisation or QU factorisation, is a decomposition of a matrix A into a product $A = QR$ of an orthogonal matrix Q and an upper triangular matrix R .

⁵ The *posterior_epred* function computes posterior draws of the expected value/mean of the posterior predictive distribution. This can be done for the data used to fit the model (i.e., posterior predictive checks) or for new data. By definition, these predictions have smaller variance than the posterior predictions performed by the *posterior_predict* method. This is because only the uncertainty in the mean is incorporated in the draws computed by *posterior_epred* while any residual error is ignored. However, the estimated means of both methods averaged across draws should be very similar.

Equation 12:

$$P_{r,y,t,s,x} = \sum_{a \in r, m \in t} C_{y,m,a} p_{y,m,a,s,x} / \sum_{a \in r, m \in t, s} C_{y,m,a} p_{y,m,a,s,x}$$

From Equation 12, the mean and SD by region, fishing year, season, sex category, and length bin were calculated from either the posterior or posterior predictive distribution

Equation 13:

$$P_{r,y,t,s,l} = P_{r,y,t,s} \text{ and } \sigma_{r,y,t,s,l} = \sigma_{r,y,t,s}$$

Finally, the data set weight for each region, fishing year, season, and sex category was calculated from the posterior or posterior predictive SD as

Equation 14:

$$\omega_{r,y,m,a,s} = \frac{1}{\sqrt{\sum_l \sigma_{r,y,t,s,l}^2}}$$

2.3 Using bootstrapping to generate length frequencies of lobsters

Comparisons between the data sets weights produced in Equation 14 and the effective sample size derived using bootstrapping of the lengths (Bull & Dunn 2002) were done for CRA 2. The Bull & Dunn (2002) software was used, but this software was designed for generating LFs for fish stocks sampled using trawl surveys and, hence, required information for each trawl survey stratum, trip, and tow. For lobsters these fields were substituted by setting:

- stratum to period (i.e., fishing year/season);
- trip to month;
- tow to statistical area;
- length to TW (rounded down to the nearest mm);
- weight to the catch (kg) for each stratum; and
- no.m, no.f, no.u, and no.t to the number of males, the number of immature females, the number of mature females, and the total number measured, respectively.

For each LF produced using the Bull & Dunn (2002) software, the effective sample size ($N_{r,y,t,s}^b$) for each region, fishing year, season, and sex category was solved numerically using

Equation 15:

$$\log(CV_{r,y,t,s}^b) = \log \left(\frac{\sqrt{N_{r,y,t,s}^b P_{r,y,t,s}^b (1 - P_{r,y,t,s}^b)}}{N_{r,y,t,s}^b P_{r,y,t,s}^b} \right)$$

where $P_{r,y,t,s}^b$ is the proportion within each TW bin for each region, fishing year, season, and sex category derived from bootstrapping and $CV_{r,y,t,s}^b$ is the coefficient of variation for each TW bin for each region, fishing year, season, and sex category derived from bootstrapping.

For CRA 2 and CRA 3, an alternative comparison to the data sets weights produced in Equation 14 was done. This involved using $P_{r,y,t,s}$ and $\sigma_{r,y,t,s}$ derived in Equation 13 and calculating a CV (from the posterior or posterior predictive distribution) as

Equation 16:

$$CV_{r,y,t,s} = \frac{\sigma_{r,y,t,s}}{P_{r,y,t,s}}$$

then using Equation 15 to derive an effective sample size ($N_{r,y,t,s}$) for each region, fishing year, season, and sex category.

3. RESULTS

3.1 Length frequency data

The raw proportions at length (Equation 2) derived in this analysis were compared with the raw proportions at length derived independently (P. Starr, pers. comm) for CRA 2 (Figure 2, Figure 3) and CRA 3 (Figure 4, Figure 5) and were found to be equivalent based on visual inspection. This cross validation ensured that the proportions at length were being calculated correctly and that the data obtained for this analysis were the same as used in previous data requests.

The first three fishing years (1986, 1990, and 1991) were dropped from the CRA 2 model because there were too few vessels, days, trips, pots, and lobsters sampled by the CS programme and no LB samples (Table 3) and, during these years, there were no samples in statistical areas 905 and 906 (Table 4) and only the SS season was sampled. The choice of years to drop was less clear for CRA 3 where only the first two years (1986, 1989) were dropped, while the following three years (1990, 1991, 1992) were also considered for omission. Although there was a reasonable number of vessels, days, trips, pots, and lobsters sampled by the CS programme during these years (Table 5), there was no sampling in statistical area 911 (Table 6).

In CRA 2, there were few male and immature female lobsters below 36 mm TW measured and, therefore, all lobsters from 30 to 36 mm TW were added to the [36, 38) bin (Table 2). There were few mature females below 42 mm TW, so all mature females from 30 to 42 mm were added to the [42, 44) bin. Likewise, there were few immature females above 70 mm TW, so all immature females greater than 70 mm TW were added to the [68, 70) bin.

In the past, dropping fishing years and aggregating TW bins is usually done after the empirical LFs are produced but before they are input into the stock assessment (e.g., see Starr et al. 2021). However, initial model runs that included all years and attempted to fit to all TW bins took a long time to run and resulted in inadequate MCMC mixing (not shown). When these years and TW bins were dropped, MCMC mixing was adequate for all model runs and the run time was reasonable (Appendix II).

3.2 Length frequency models

For all three sex categories and both QMAs, the model intercept parameters increased from low values for the smallest TW bins, peaked near the MLS, then began to decline (Figure 6, Figure 7). There are five intercepts from the CRA 3 models with low means with high uncertainty (Figure 7), which suggested that these intercepts were poorly defined and the associated TW bins should probably be dropped in future model runs (i.e., the three largest bins for males, the largest immature female bin, and the smallest mature female bin). The fishing year population level effects (i.e., fixed effects) for each TW bin varied by year, with some obvious correlation among TW bins (Figure 8, Figure 9). For example, the fishing year coefficients were very similar for the 75 and 77 mm TW bins for males in CRA 2 (Figure 8) and a similar pattern was observed for the 65 and 69 mm TW bins for mature females in CRA 3 (Figure 9). The population level effects for month suggested some strong seasonal differences in CRA 2 (Figure 10), but were more consistent among months in CRA 3 (Figure 12). The population level effects for statistical area were different in both CRA 2 and CRA 3 (Figure 12, Figure 13) which wasn't surprising, given that these statistical areas are spatially distinct.

The conditional effect of a particular explanatory variable within a model is the effect of that variable conditioned on the remaining explanatory variables fixed at a reference value (the reference value used for all models was the parameter associated with the smallest TW bin, the first fishing year, April, and statistical area 905 or 909). Conditional effects for fishing year illustrate how the underlying LFs (i.e.,

the mean across all months and statistical areas) have changed through time (Figure 14, Figure 15). There does not appear to be any persistent changes in the conditional effects for fishing year in CRA 2, but, in CRA 3, the peak TW bin for males and mature females seems to have increased to larger TW bins from the early- to mid-1990 and then stabilised (Figure 15). The LFs for immature females in CRA 3 are much more uncertain (saw-toothed, Figure 15) compared with CRA 2 (Figure 14), reflecting the small number of observations for this sex category in CRA 3. The conditional effects for month look quite different when comparing CRA 2 with CRA 3, with CRA 2 having a much longer tail (i.e., more larger lobsters) and CRA 3 having much more peaked LFs (Figure 16, Figure 17), possibly reflecting a higher exploitation rate in CRA 3. The conditional effects for statistical area in CRA 2 are unusual in that non-adjacent areas were more similar (i.e., statistical areas 906 and 908 were similar to each other, and statistical areas 905 and 907 were similar to each other; Figure 18). In CRA 3, statistical area 911 is clearly different to statistical areas 909 and 910 with a higher proportion of larger lobsters (Figure 19).

3.3 Predicted length frequency

Predictions of the LFs for CRA 2 and CRA 3 are presented in Appendix III. When the CS and/or LB sample sizes are adequate for a stratum, the fits to the LF data generally look good. Furthermore, when both CS and LB data are available for a stratum, and both have adequate sample sizes, they generally exhibit the same LF pattern. Using the previous method for generating LFs, the CS and LB LFs were sometimes at odds with each other. These differences were thought to be due to either low sample sizes for at least one of the programmes, or because the CS programme was often temporally limited (i.e., CS samples for a particular QMA tended to all be done over just a few days, compared with the LF data, which were collected throughout the year across a wider range of participants).

3.4 Scaled predicted length frequency

Scaled predicted LFs for CRA 2 and CRA 3 are presented in Appendix IV. When the sample sizes were high, the LF standardisation generally didn't alter the raw LFs very much, but at low sample sizes or when sampling was non-representative the procedure did correct the LFs substantially. For example, the predicted LF is very different to the raw LF for mature females in CRA 3 region 1 in SS 1995 (see Section 12.6), because all of the data for that stratum was CS data and came from a single month (November) and a single statistical area (910) (see Section 11.6). Scaling the LFs by the catch generally did very little to alter the LFs. This is because the CS programme is adjusted each year so that the samples are approximately proportional to the catch by statistical area, and the LB data set is self-weighting. Scaled LFs for CRA 2 and CRA 3 in Appendix IV show the posterior distribution of the mean, which, by definition, is a much tighter distribution than the posterior predictive distribution which includes the full uncertainty (e.g., Figure 20).

Comparisons of the scaled LFs derived using the previous method, the new method, and bootstrapping for CRA 2 showed that all three methods yielded similar LFs (Figure 21 to Figure 26). Sometimes the bootstrapping method produced spurious results (e.g., CRA 2 males during SS of 1997, Figure 21) and often the previous method applied to the CS data resulted in highly variable 'saw toothed' LFs (e.g., immature females during SS of 2003, Figure 22). Comparison between the previous method and the new method for CRA 3 also showed that all three methods generally yielded similar LFs (Figure 27 to Figure 38).

3.5 Length frequency data set weights

The relationship between the previous data set weights (Equation 4) and the new weights (Equation 14) was more or less linear for the LB data and the CS data in CRA 3, but was different when compared with the CS data in CRA 2 (Figure 39, Figure 40). In CRA 3, the new data set weights tended to plateau when compared with the old weights (Figure 40).

The CRA 2 data set weights derived from the new methodology (Equation 14) were also compared with the corresponding effective sample sizes, produced using bootstrapping (Equation 15), and were found to be approximately linearly related (Figure 41).

Finally, the data set weights derived from the new methodology (Equation 14) and the effective sample sizes derived from these data set weights (using Equation 16) were also approximately linear for both CRA 2 and CRA 3 (Figure 42, Figure 43). This was the case when the effective sample sizes were derived from the posterior or the posterior predictive distribution.

4. DISCUSSION

This methodology built on work done by Thorson (2014), Thorson et al (2016), Neubauer & Tremblay-Boyer (2019), and Neubauer (2020). This methodology addresses two key issues that arise from using empirical LF data:

1. if LF data are missing within a stratum (e.g., a statistical area wasn't sampled in a year/month) but there was some catch taken from that stratum, then the aggregated scaled LF may not be representative of the entire catch; and
2. there is no measure of uncertainty for the LF within each stratum.

Furthermore, this implementation uses the *R* package *brms*, making this method more easily accessible to other fisheries scientists.

There are several issues with the data used in this analysis. For example, there is no way of telling if a measured lobster in the logbook database is a red rock lobster or a packhorse rock lobster (*Sagmariasus verreauxi*), and in most QMAs fishers only record up to 25 lobsters per pot. These issues are more fully documented and discussed in Webber & Starr (in prep.).

The data in this analysis were fitted using a multinomial distribution, although a Dirichlet-multinomial model would also be a good choice (e.g., Neubauer & Tremblay-Boyer 2019, Neubauer 2020; also see Table 7 in Appendix I). The Dirichlet distribution would not be a good choice when fitting to these data, since they are count data with many zero counts for some TW bins—the Dirichlet distribution is for modelling simplex data (i.e., a vector of proportions that sums to one) and does not allow any of the proportions to be zero. This issue can be avoided by adding a small number to the zero proportions and renormalising the LF to sum to one; however, the choice of the small number is subjective and can influence model fits to LF data (Francis 2014). The logistic-normal distribution would also not be suitable because it does not allow for zeros. The Dirichlet-multinomial was not explored here because this distribution is not an option in *brms*. Although *brms* does allow for custom families to be defined, this does not yet apply to categorical distributions. It was desirable to maintain the analysis in *brms* so that this methodology would be accessible to other researchers.

The model structure included population-level effects for fishing year, month, and statistical area and group-level effects for a fishing year/statistical area interaction term. This structure models each of the TW bins independently; however, there is likely to be some correlation structure among different TW bins. It is possible to impose such a correlation structure in *brms* (i.e., smoothing over TW bins). However, this raises a theoretical question: should the smoothing be applied during the LF standardisation or should this be left to the stock assessment? Future research might further consider this question.

The catch scaling (Equation 12) was done using catch stratified by fishing year, month, and statistical area ($C_{y,m,a}$). However, male and female lobsters have different weights for a given TW and the ratio of males:females in the catch can differ by fishing year, month, and statistical area. In the future, this difference should be taken into account and the catch should be further stratified by sex before catch

scaling is applied. This would require the development of a model for predicting sex ratios which can then be used to further stratify the catch by sex (and the sex ratios can also be used within stock assessments).

It is reassuring that the data set weights derived from this new methodology were approximately linear with the previous data set weights (e.g., Starr et al. 2021) and with the effective sample sizes produced using bootstrap resampling (Bull & Dunn 2002). These similarities imply that the empirical approach implemented in previous rock lobster stock assessments was roughly approximating the uncertainty associated with each LF as estimated by this model. The new data set weights tended to plateau when compared with the old weights in CRA 3. Very high LF data set weights were becoming an issue in some previous stock assessments (e.g., CRA 4, Rudd et al. 2021, Starr et al. 2021), which at the time was solved by simply capping the LF weights arbitrarily. The new methodology may provide a less subjective solution to this issue.

4.1 Using these length frequencies in stock assessments

In New Zealand red rock lobster stock assessments, a data set weight for an LF applies to the full vector of TW bins (i.e., all TW bins get a single effective sample size for a given stratum). However, for rock lobsters in CRA 2 and CRA 3 there is sometimes greater uncertainty for smaller or larger individuals, or for a range of TW bins that were not observed for whatever reason (see Appendix IV). For example, there is much higher uncertainty for immature females in CRA 3 region 1 at smaller sizes (i.e., before they are fully selected by the fishery, see Section 12.5). This is likely to be a common feature among other fish and crustacean LFs. A data set weight for an LF that applies to the full vector (e.g., when using a multinomial or Dirichlet distribution) could result in a stock assessment model that ignores the higher uncertainty associated with these TW bins. However, if the logistic-normal distribution were used to fit to these LFs within a stock assessment, then the SDs for each TW bin as estimated by this methodology could be used, rather than summing over TW bins (i.e., the input would then be a vector of LF proportions and a vector of SDs). This approach may better reflect LF uncertainty in stock assessment.

This analysis will be repeated before the stock assessment for each QMA and reported in the relevant data Fisheries Assessment Reports for those assessments. When each stock assessment is done and this analysis is repeated, the following points should also be considered:

- drop those TW bins in CRA 3 with low intercept values and high uncertainty (see Figure 7);
- identify a consistent approach for selecting which standardised LFs to include or exclude from the stock assessment (e.g., in CRA 3, the years 1990, 1991, and 1992 should be dropped for statistical area 911 because there are no data for that statistical area during these years; see Section 3.1).

4.2 Future research

Future research should consider:

- loosening up the priors for the population-level effects (Equation 8) to see how the model responds;
- exploring correlation structures/smoothing among TW bins (as described in the discussion above);
- exploring model runs that include depth (categorical) as an explanatory variable;
- using the Dirichlet-multinomial distribution;
- developing a similar but separate model for predicting sex ratios (e.g., fitting to sex ratio proportions across all three sex categories using a multinomial distribution, or fitting to the proportion male versus female separately to the proportion immature versus mature for females using two separate binomial distributions); and
- further stratifying the catch by sex (using the sex-ratio model in the point above) before the catch scaling (Equation 12) is done.

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7. TABLES

Table 1: Summary of subscripts used throughout this document.

Subscript	Variable	Support
<i>y</i>	Fishing year	1986–2020
<i>m</i>	Month	4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15
<i>t</i>	Season	AW = autumn/winter SS = spring/summer
<i>a</i>	Statistical Area	CRA 2: 905, 906, 907, 908 CRA 3: 909, 910, 911
<i>r</i>	Region	CRA 3 region 1: 909, 910 CRA 3 region 2: 911
<i>s</i>	Sex category	Males, immature females, mature females
<i>d</i>	Data source	CS = catch sampling LB = logbook
<i>j</i>	Trip	–
<i>i</i>	Pot	–
<i>l</i>	Length bin	[30,32), [32,34), ..., [88,90), [90, ∞)
<i>x</i>	Posterior sample / iteration	–

Table 2: Bin limits for CRA 1 and CRA 2 indicated in grey.

Length bin	Midpoint	Bin	CRA 2			CRA 3		
			MM	IF	MF	MM	IF	MF
[30, 32)	31	1						
[32, 34)	33	2						
[34, 36)	35	3						
[36, 38)	37	4						
[38, 40)	39	5						
[40, 42)	41	6						
[42, 44)	43	7						
[44, 46)	45	8						
[46, 48)	47	9						
[48, 50)	49	10						
[50, 52)	51	11						
[52, 54)	53	12						
[54, 56)	55	13						
[56, 58)	57	14						
[58, 60)	59	15						
[60, 62)	61	16						
[62, 64)	63	17						
[64, 66)	65	18						
[66, 68)	67	19						
[68, 70)	69	20						
[70, 72)	71	21						
[72, 74)	73	22						
[74, 76)	75	23						
[76, 78)	77	24						
[78, 80)	79	25						
[80, 82)	81	26						
[82, 84)	83	27						
[84, 86)	85	28						
[86, 88)	87	29						
[88, 90)	89	30						
[90, ∞)	91	31						

Table 3: CRA 2: number of vessels, skippers, trips, days sampled, pots lifted, and lobsters measured in the catch sampling (CS) and logbook (LB) programmes each fishing year. The fishing years indicated in grey were not included in the LF models.

Fishing year	Catch sampling (CS)					Logbook (LB)					
	vessels	days	trips	pots	lobsters	vessels	skippers	days	trips	pots	lobsters
1986	2	3	3	159	562	—	—	—	—	—	—
1990	2	3	3	127	364	—	—	—	—	—	—
1991	1	2	2	115	918	—	—	—	—	—	—
1993	—	—	—	—	—	26	23	253	1 915	4 494	18 092
1994	—	—	—	—	—	30	25	254	1 613	4 307	18 133
1995	—	—	—	—	—	26	21	176	1 048	2 809	11 657
1996	—	—	—	—	—	21	19	187	902	2 508	12 522
1997	—	—	—	—	—	19	16	169	766	2 111	9 224
1998	—	—	—	—	—	21	17	191	781	2 199	9 172
1999	12	12	12	917	3 073	20	17	153	949	2 642	10 323
2000	13	13	13	1 002	3 022	16	16	198	950	2 568	9 186
2001	10	11	11	859	3 014	13	13	268	743	1 787	5 893
2002	12	13	14	724	2 274	18	18	267	1 197	2 738	8 269
2003	12	12	12	737	1 898	13	13	289	979	2 440	7 445
2004	12	12	12	530	1 353	14	14	319	1 144	2 730	8 532
2005	10	12	12	752	2 303	18	18	301	1 442	3 563	10 748
2006	9	12	12	772	1 964	17	17	251	1 057	2 765	8 836
2007	11	12	12	828	2 237	14	13	245	1 088	2 678	9 008
2008	7	12	12	874	2 568	15	15	257	1 281	3 302	10 415
2009	8	11	14	881	2 769	15	14	240	1 195	2 861	8 662
2010	12	13	13	783	1 926	16	16	274	1 471	3 518	12 003
2011	12	14	17	857	2 175	17	19	252	1 357	3 177	8 831
2012	11	13	13	835	2 465	17	17	256	1 502	3 696	11 636
2013	9	10	12	864	2 868	19	18	284	1 585	3 853	11 976
2014	8	12	12	882	2 666	15	16	266	1 373	3 147	8 016
2015	8	13	13	768	1 450	16	17	274	1 342	2 921	6 315
2016	9	13	13	763	2 005	15	16	223	859	1 829	3 658
2017	9	14	14	873	2 652	14	14	235	928	1 971	4 234
2018	9	11	12	965	2 785	11	11	176	453	1 230	4 374
2019	—	—	—	—	—	13	14	196	470	1 318	4 273
2020	10	16	19	1 551	5 174	15	15	174	526	1 490	5 072

Table 4: CRA 2: number of vessels and trips by statistical area in either the catch sampling (CS) or logbook (LB) programmes each fishing year. The fishing years indicated in grey were not included in the LF models.

Fishing year	905		906		907		908	
	vessels	trips	vessels	trips	vessels	trips	vessels	trips
1986	—	—	—	—	1	2	1	1
1990	—	—	—	—	—	—	2	3
1991	—	—	—	—	—	—	1	2
1993	3	80	15	1 187	3	191	5	457
1994	8	271	15	805	6	262	6	275
1995	1	43	12	635	6	194	7	176
1996	2	43	9	537	7	154	3	168
1997	1	21	10	431	6	181	3	133
1998	3	102	11	454	4	67	3	158
1999	4	89	11	499	6	231	3	142
2000	3	63	11	534	5	189	3	177
2001	2	71	8	382	4	167	4	134
2002	3	90	10	704	5	195	5	222
2003	2	91	8	582	3	159	4	159
2004	2	97	10	811	3	34	5	214
2005	1	70	11	872	4	236	5	276
2006	1	30	9	606	3	186	6	247
2007	2	3	8	504	2	183	8	410
2008	1	22	9	628	2	144	5	499
2009	1	20	8	577	3	200	6	412
2010	3	96	9	717	5	185	7	486
2011	2	132	11	722	3	185	5	335
2012	2	91	8	699	4	208	5	517
2013	1	118	9	680	4	304	6	495
2014	1	93	9	728	7	361	3	203
2015	2	162	8	565	5	332	4	296
2016	2	119	6	294	6	301	4	158
2017	2	143	7	309	4	220	6	270
2018	2	30	5	206	4	141	4	88
2019	1	66	4	136	3	127	5	141
2020	1	59	6	189	4	146	6	151

Table 5: CRA 3: number of vessels, skippers, trips, days sampled, pots lifted, and lobsters measured in the catch sampling (CS) and logbook (LB) programmes each fishing year. The fishing years indicated in grey were not included in the LF models.

Fishing year	Catch sampling (CS)					Logbook (LB)					
	vessels	days	trips	pots	lobsters	vessels	skippers	days	trips	pots	lobsters
1986	2	2	2	87	483	—	—	—	—	—	—
1989	7	10	10	626	5 176	—	—	—	—	—	—
1990	4	17	17	1 299	13 493	—	—	—	—	—	—
1991	5	21	21	1 404	14 386	—	—	—	—	—	—
1992	6	23	23	1 522	13 157	—	—	—	—	—	—
1993	17	54	56	3 539	28 636	14	13	43	207	701	6 174
1994	18	51	51	4 633	43 470	16	14	112	360	1 125	8 557
1995	14	37	37	2 906	32 311	12	10	66	211	674	6 808
1996	12	32	32	2 612	30 646	5	8	60	164	557	6 710
1997	8	29	29	2 379	31 013	3	2	36	41	141	1 471
1998	11	20	32	2 007	22 720	4	2	38	51	170	894
1999	8	16	21	1 390	15 001	1	1	29	30	101	517
2000	7	13	20	1 463	13 615	1	1	39	39	132	769
2001	9	22	29	2 395	21 849	1	1	24	24	69	348
2002	12	19	29	2 165	18 666	—	—	—	—	—	—
2003	10	22	28	1 980	13 611	—	—	—	—	—	—
2004	11	25	29	2 019	13 387	—	—	—	—	—	—
2005	11	28	29	2 296	13 177	—	—	—	—	—	—
2006	10	26	28	2 312	13 259	—	—	—	—	—	—
2007	12	24	28	1 881	11 536	—	—	—	—	—	—
2008	12	26	28	2 221	14 490	—	—	—	—	—	—
2009	12	24	29	2 504	18 439	—	—	—	—	—	—
2010	9	21	27	2 093	14 862	1	1	11	11	34	126
2011	8	26	29	2 000	15 481	5	5	70	118	387	2 615
2012	7	24	28	1 282	13 750	2	2	45	45	141	1 400
2013	11	24	28	1 816	18 735	5	5	55	93	286	3 299
2014	9	25	25	1 847	18 659	7	7	215	364	1 126	7 937
2015	7	23	23	1 518	14 854	5	5	163	283	911	9 089
2016	9	22	22	1 339	13 639	7	7	136	244	824	7 967
2017	8	23	24	1 389	12 870	10	10	152	346	1 107	12 257
2018	10	25	29	1 766	16 438	6	6	143	267	894	8 755
2019	8	18	18	943	9 438	7	7	183	327	1 167	10 431
2020	8	26	34	1 957	17 563	7	7	178	323	1 135	10 725

Table 6: CRA 3: number of vessels and trips by statistical area in either the catch sampling (CS) or logbook (LB) programmes each fishing year. The fishing years indicated in grey were not included in the LF models.

Fishing year	909		910		911	
	vessels	trips	vessels	trips	vessels	trips
1986	1	1	–	–	1	1
1989	1	4	6	6	–	–
1990	1	6	3	11	–	–
1991	1	9	4	12	–	–
1992	1	8	5	15	–	–
1993	3	33	11	140	10	90
1994	3	33	12	199	12	179
1995	2	48	13	109	6	91
1996	3	44	8	92	5	60
1997	1	21	5	20	3	29
1998	1	8	7	36	6	39
1999	2	5	2	12	4	34
2000	1	4	3	12	3	43
2001	2	5	6	20	2	28
2002	2	4	5	15	5	10
2003	1	2	5	11	4	15
2004	2	4	4	10	5	15
2005	2	5	5	11	4	13
2006	1	2	6	13	3	13
2007	1	1	8	13	3	14
2008	2	6	7	8	3	14
2009	3	7	6	12	3	10
2010	2	5	5	15	2	18
2011	2	65	5	34	2	48
2012	1	36	4	16	2	21
2013	1	44	7	68	4	9
2014	3	232	6	47	3	110
2015	2	104	4	55	2	147
2016	2	108	6	46	3	112
2017	2	134	7	125	3	111
2018	3	135	5	65	3	96
2019	3	88	5	177	3	80

8. FIGURES

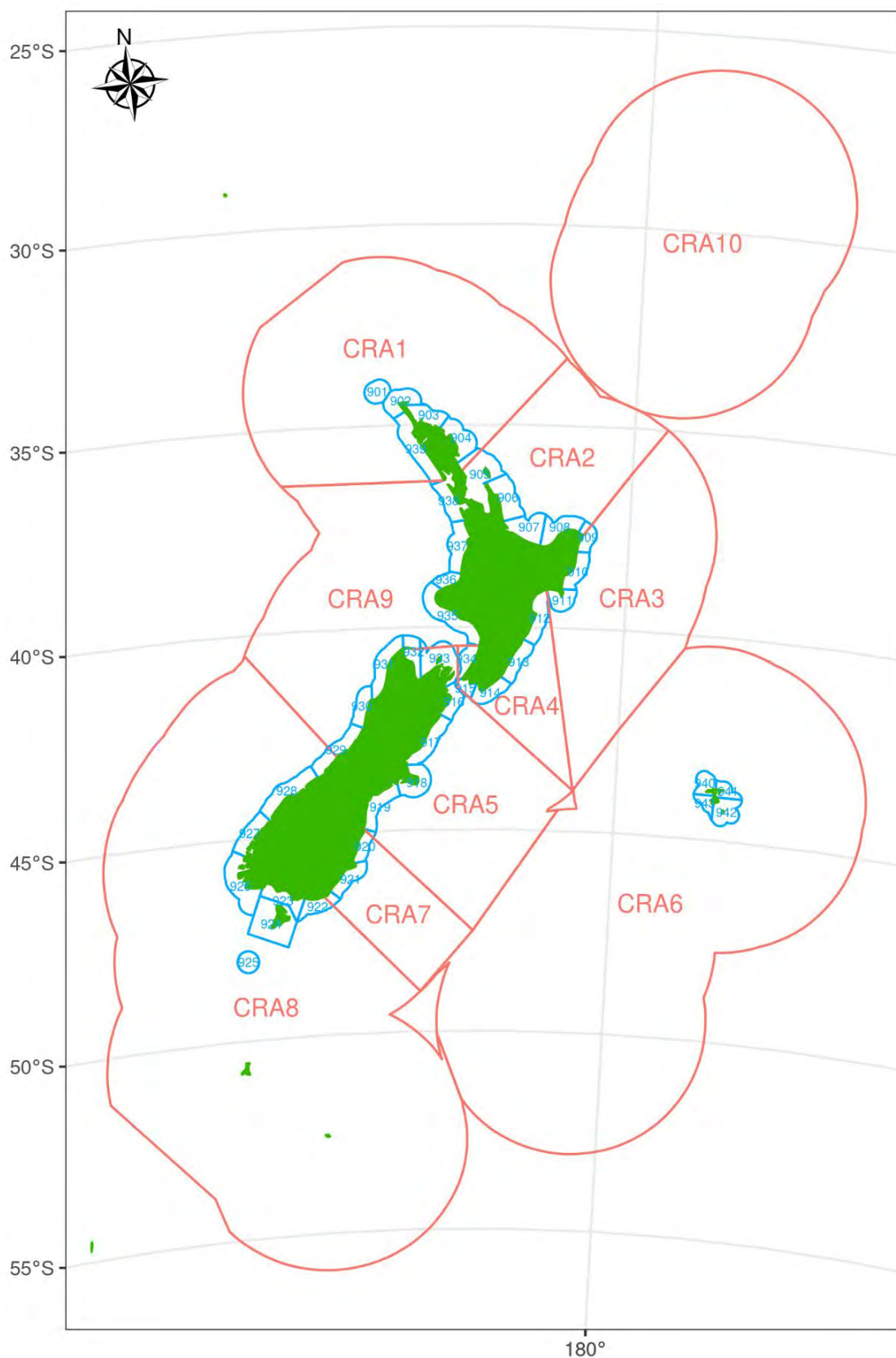


Figure 1: New Zealand red rock lobster Quota Management Area (QMA) boundaries (red lines) and statistical area boundaries (blue lines).

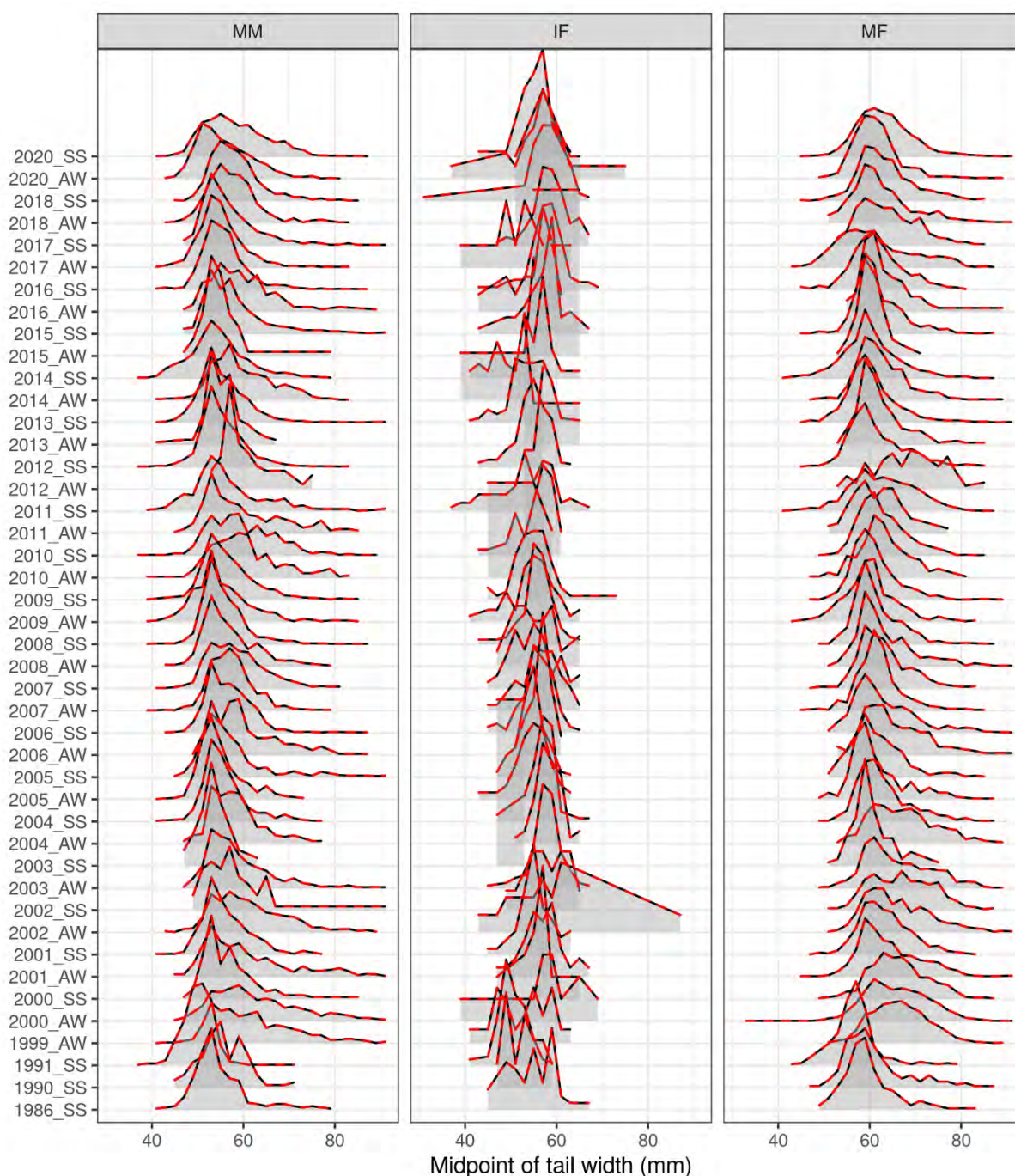


Figure 2: CRA 2: LF data from the catch sampling (CS) programme showing raw LFs (Equation 2) generated independently (Starr & Webber 2018, Webber et al. 2021) (grey) and in this analysis (dashed red line).

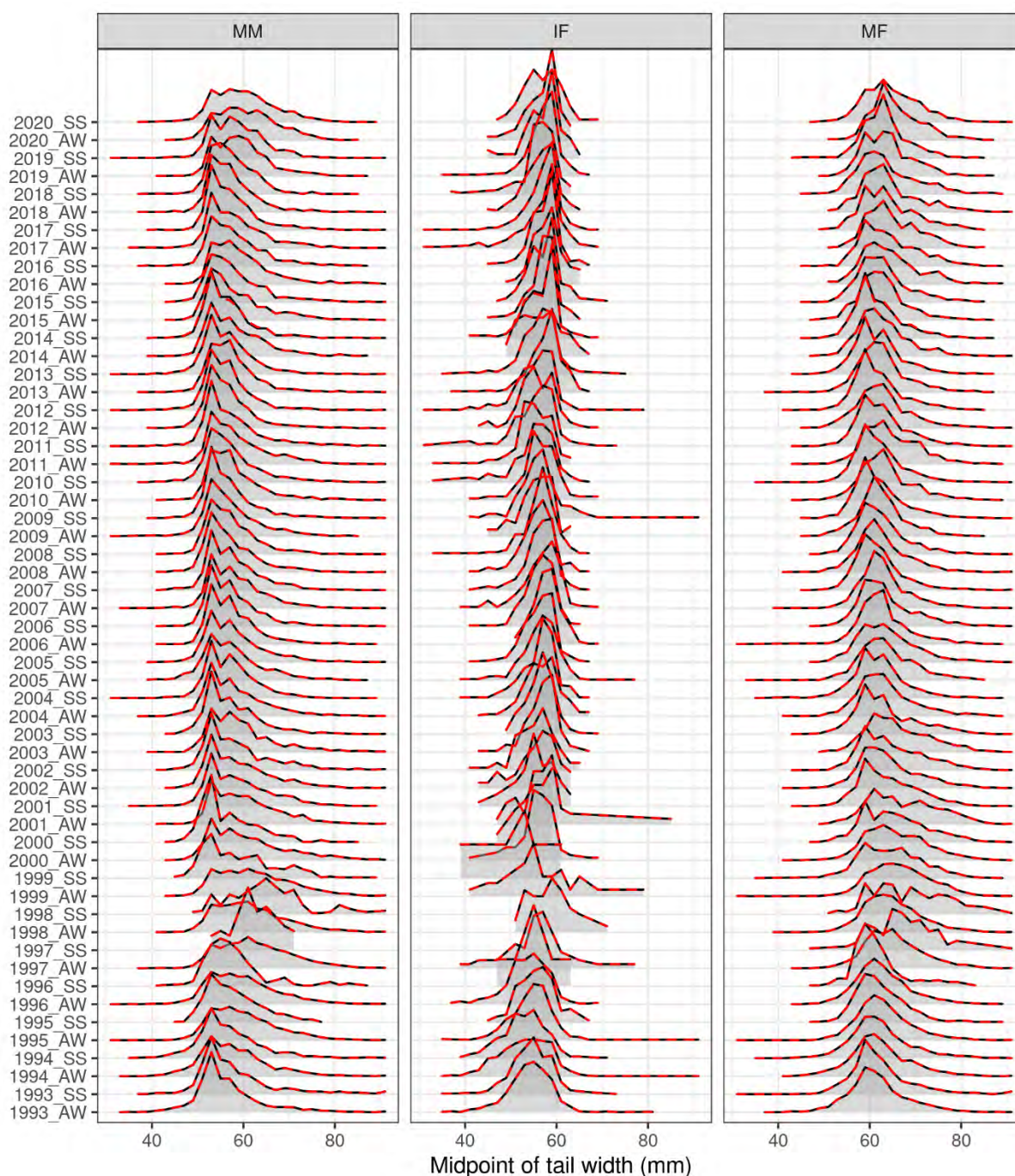


Figure 3: CRA 2: LF data from the logbook (LB) programme showing raw LFs (Equation 2) independently (Starr & Webber 2018, Webber et al. 2021) (grey) and in this analysis (dashed red line).

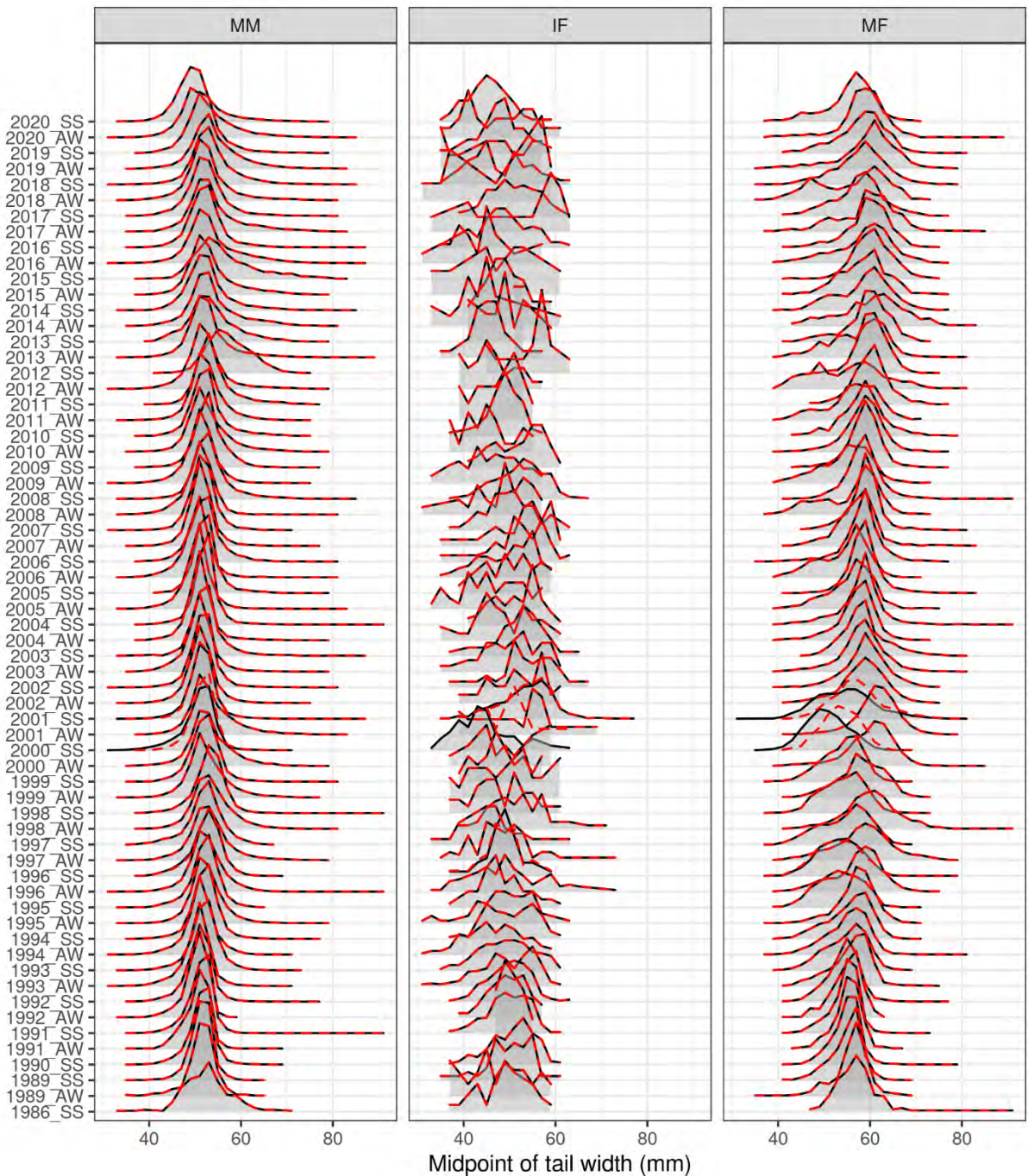


Figure 4: CRA 3: LF data from the catch sampling (CS) programme showing raw LFs (Equation 2) generated independently (Starr et al. 2020, Webber et al. 2021) (grey) and in this analysis (dashed red line).

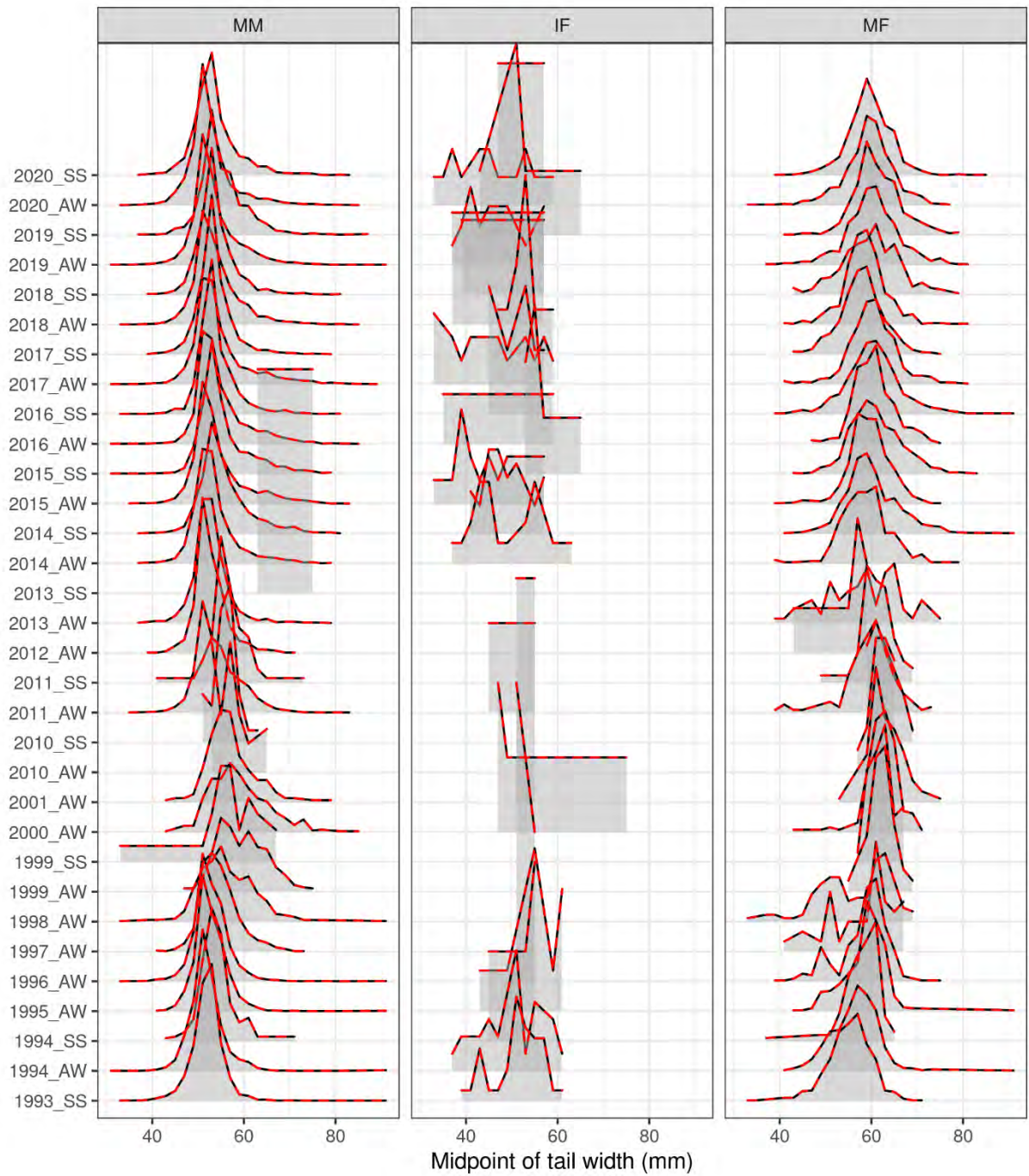


Figure 5: CRA 3: LF data from the logbook (LB) programme showing raw LFs (Equation 2) generated independently (Starr et al. 2020, Webber et al. 2021) (grey) and in this analysis (dashed red line).

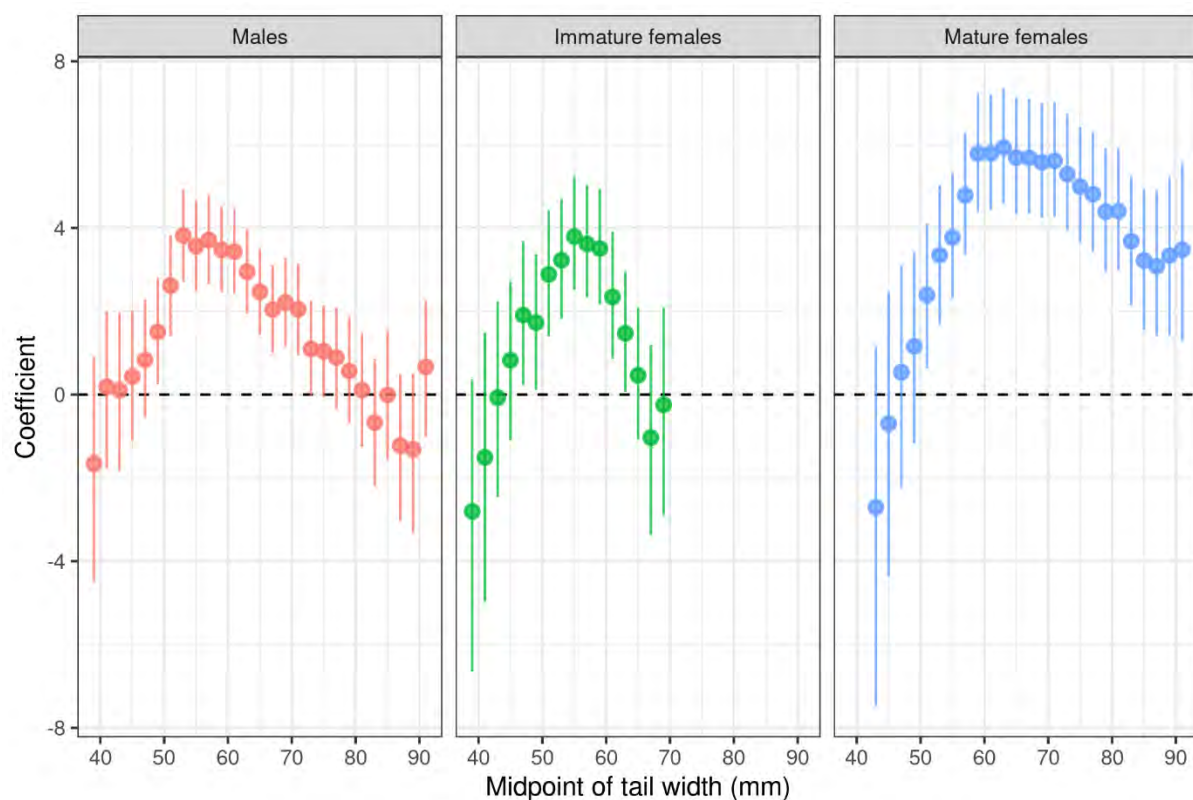


Figure 6: CRA 2: the population level effect of intercept by TW bin for each sex category, showing the mean and 95% credible intervals.

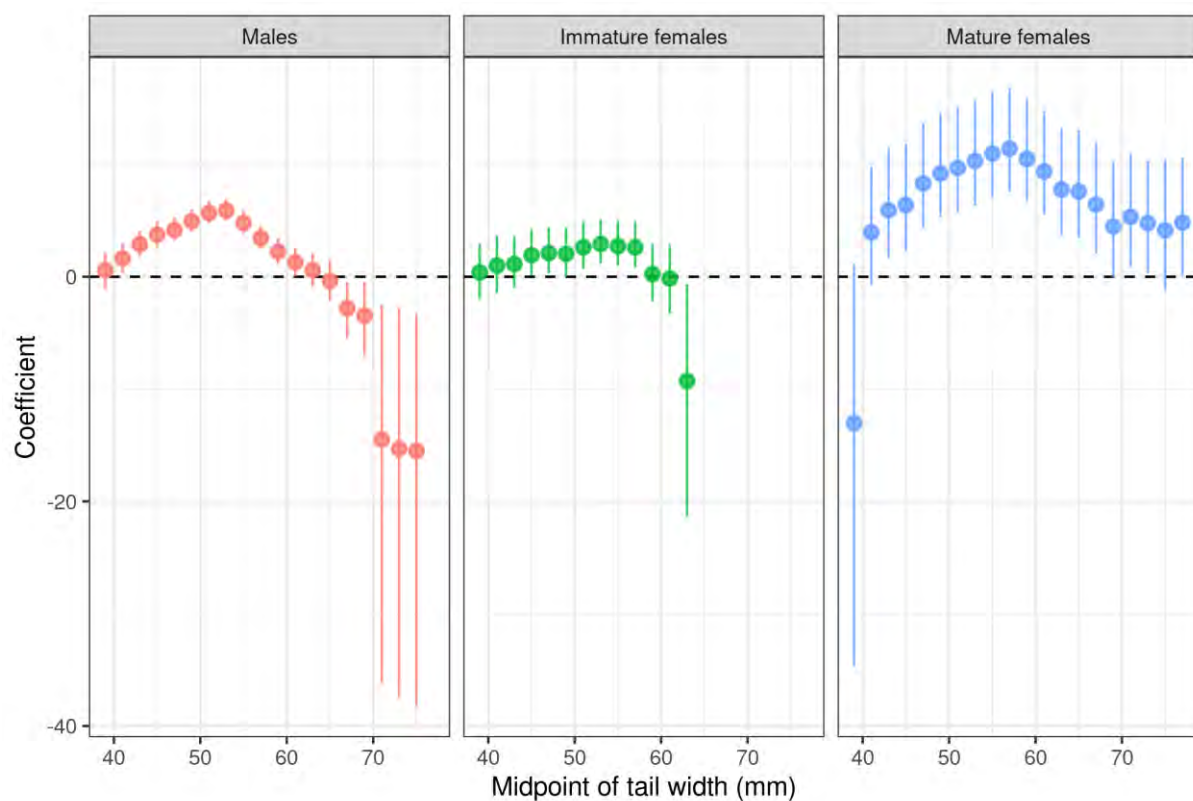


Figure 7: CRA 3: the population level effect of intercept by TW bin for each sex category, showing the mean and 95% credible intervals.

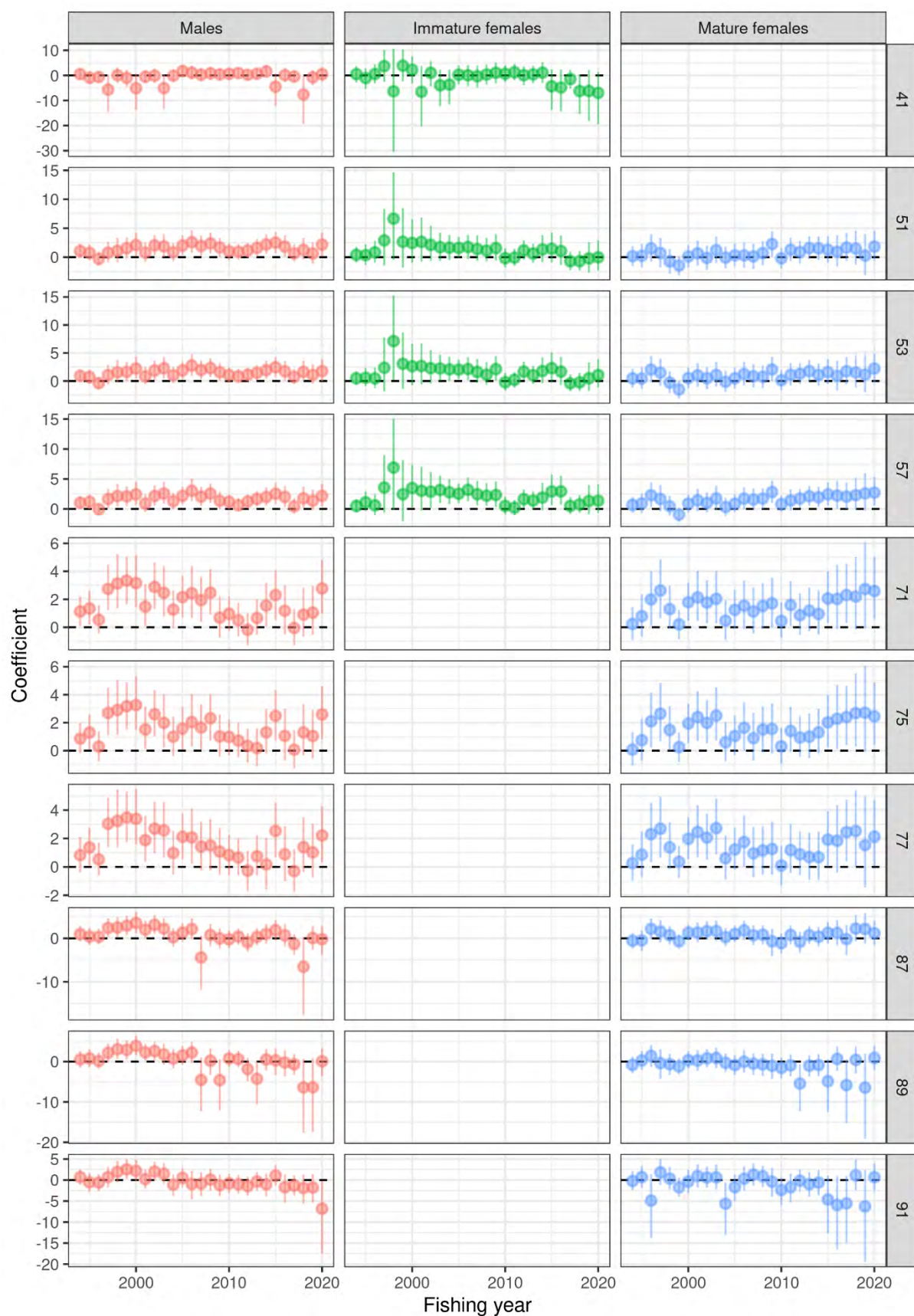


Figure 8: CRA 2: the population level effect of fishing year by TW bin for each sex category, showing the mean and 95% credible intervals.

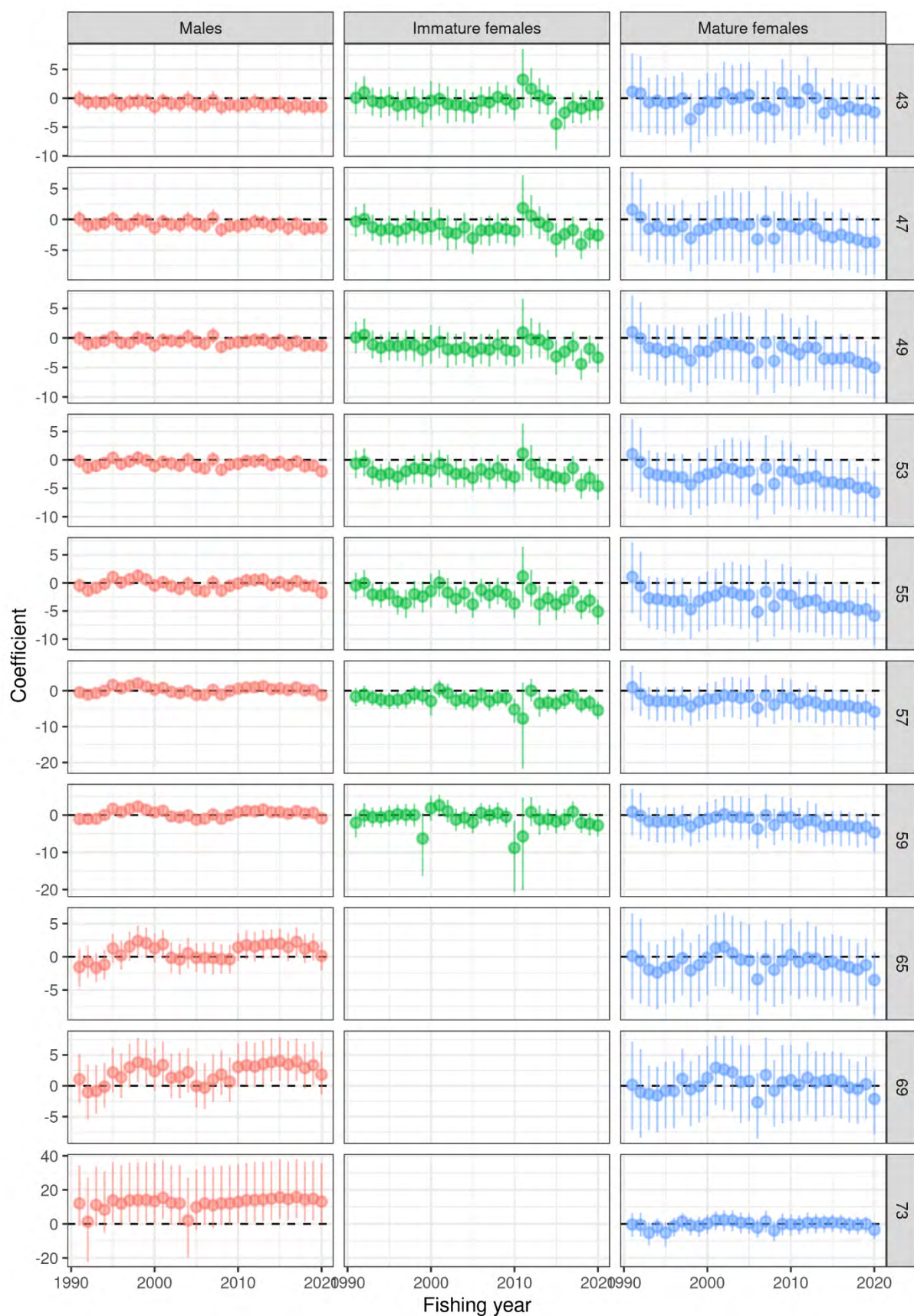


Figure 9: CRA 3: the population level effect of fishing year by TW bin for each sex category, showing the mean and 95% credible intervals.

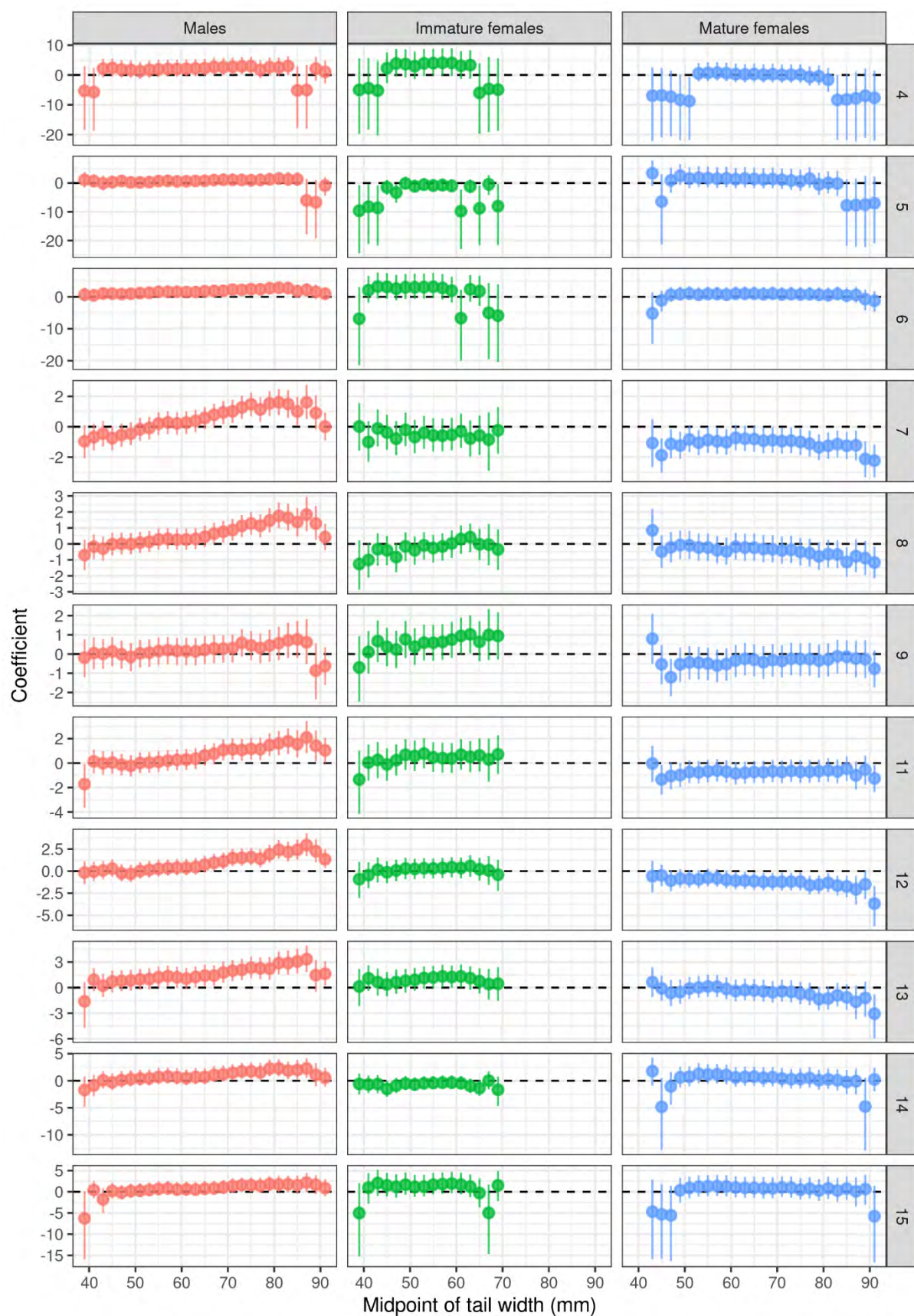


Figure 10: CRA 2: the population level effect of fishing month (4 = April, 15 = March) by TW bin for each sex category, showing the mean and 95% credible intervals.

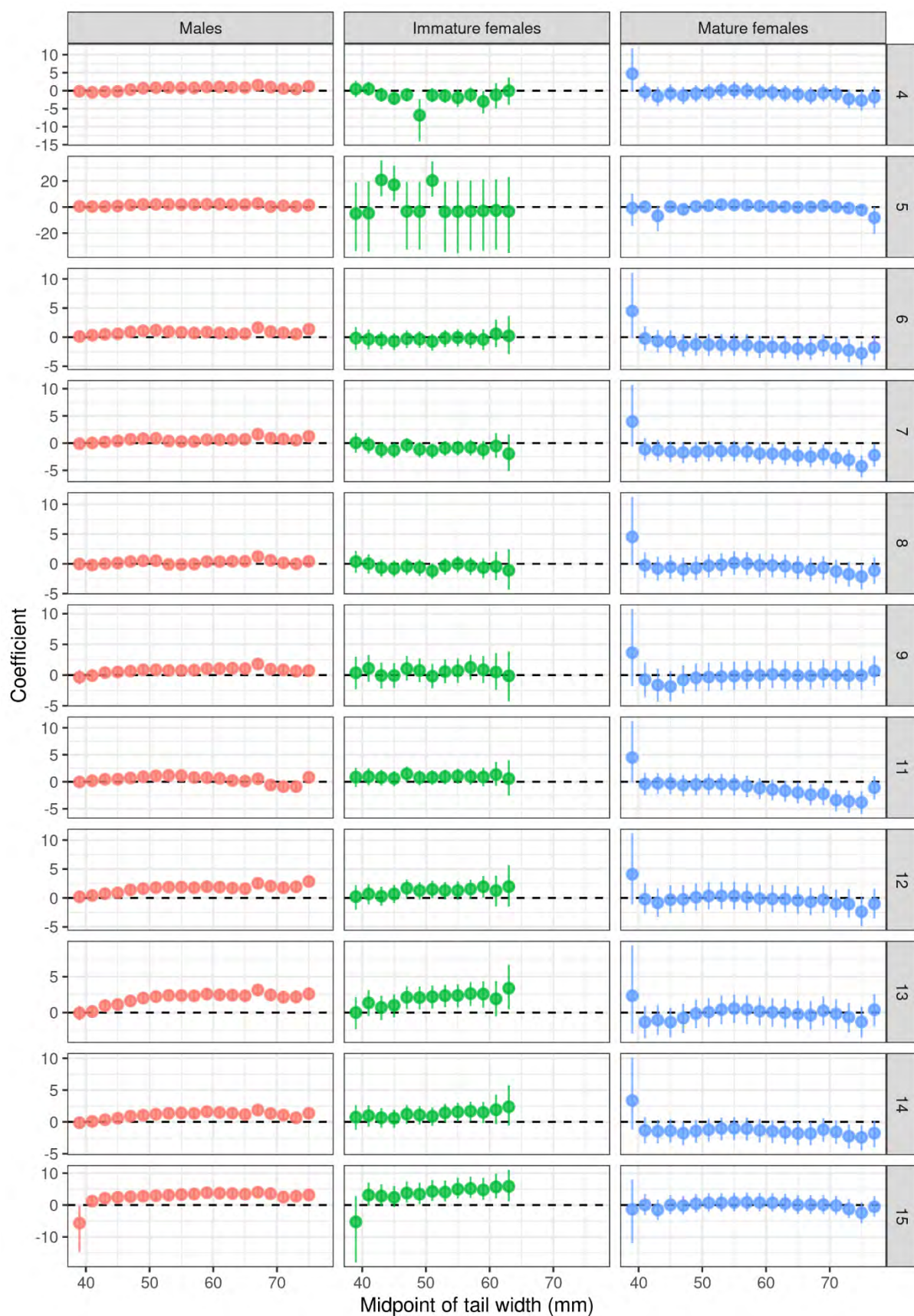


Figure 11: CRA 3: the population level effect of fishing month (4 = April, 15 = March) by TW bin for each sex category, showing the mean and 95% credible intervals.

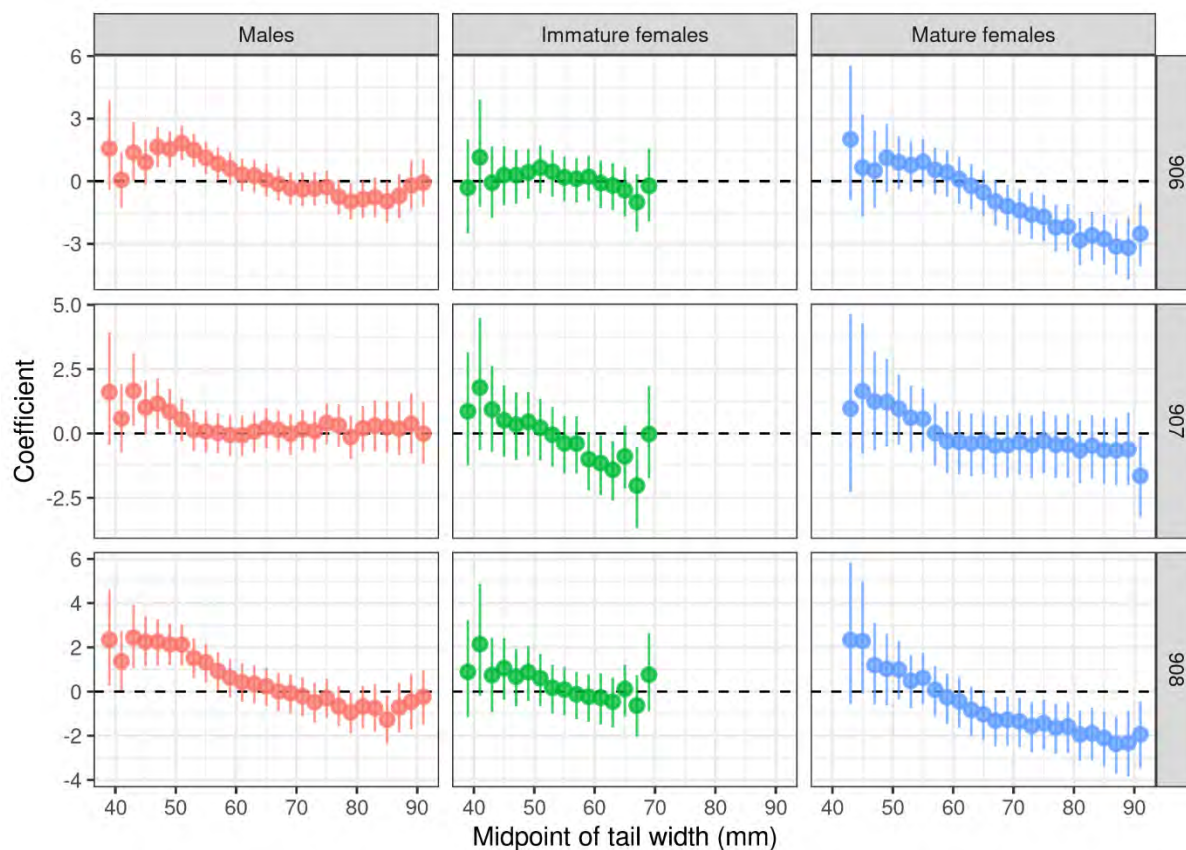


Figure 12: CRA 2: the population level effect of statistical area by TW bin for each sex category, showing the mean and 95% credible intervals. Statistical area 905 is not shown as it is the reference level.

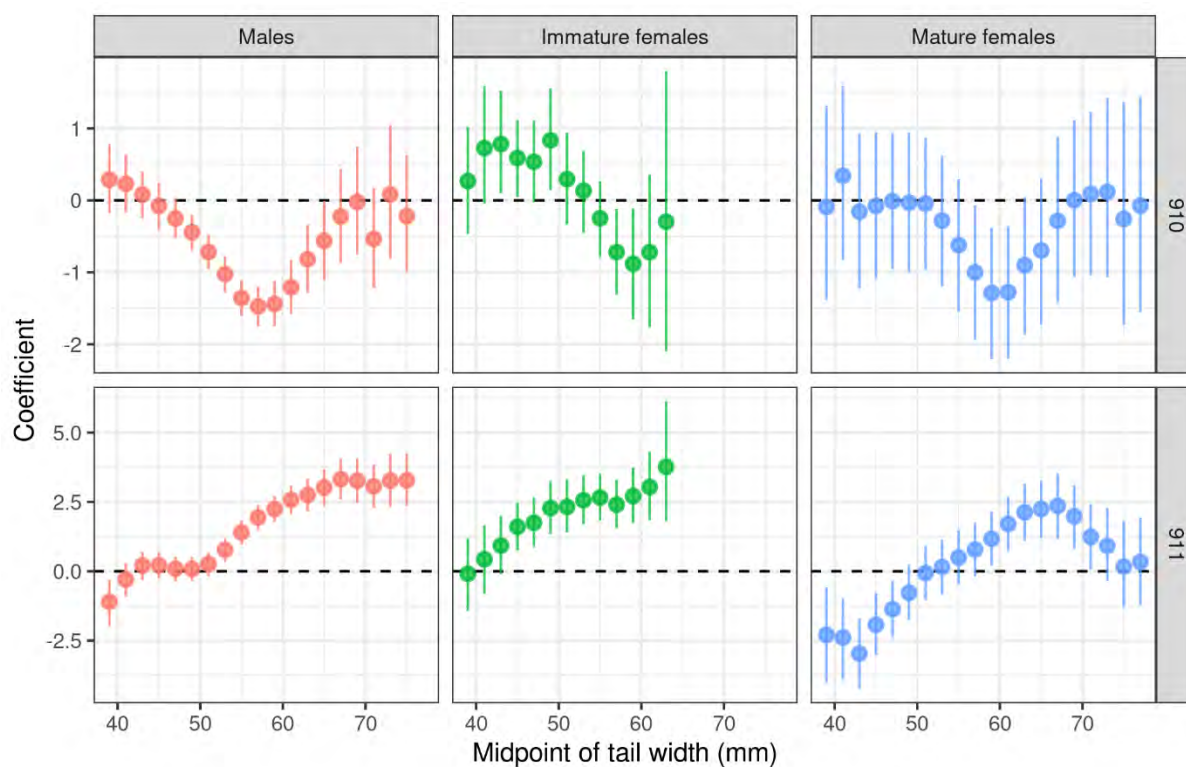


Figure 13: CRA 3: the population level effect of statistical area by TW bin for each sex category, showing the mean and 95% credible intervals. Statistical area 909 is not shown as it is the reference level.

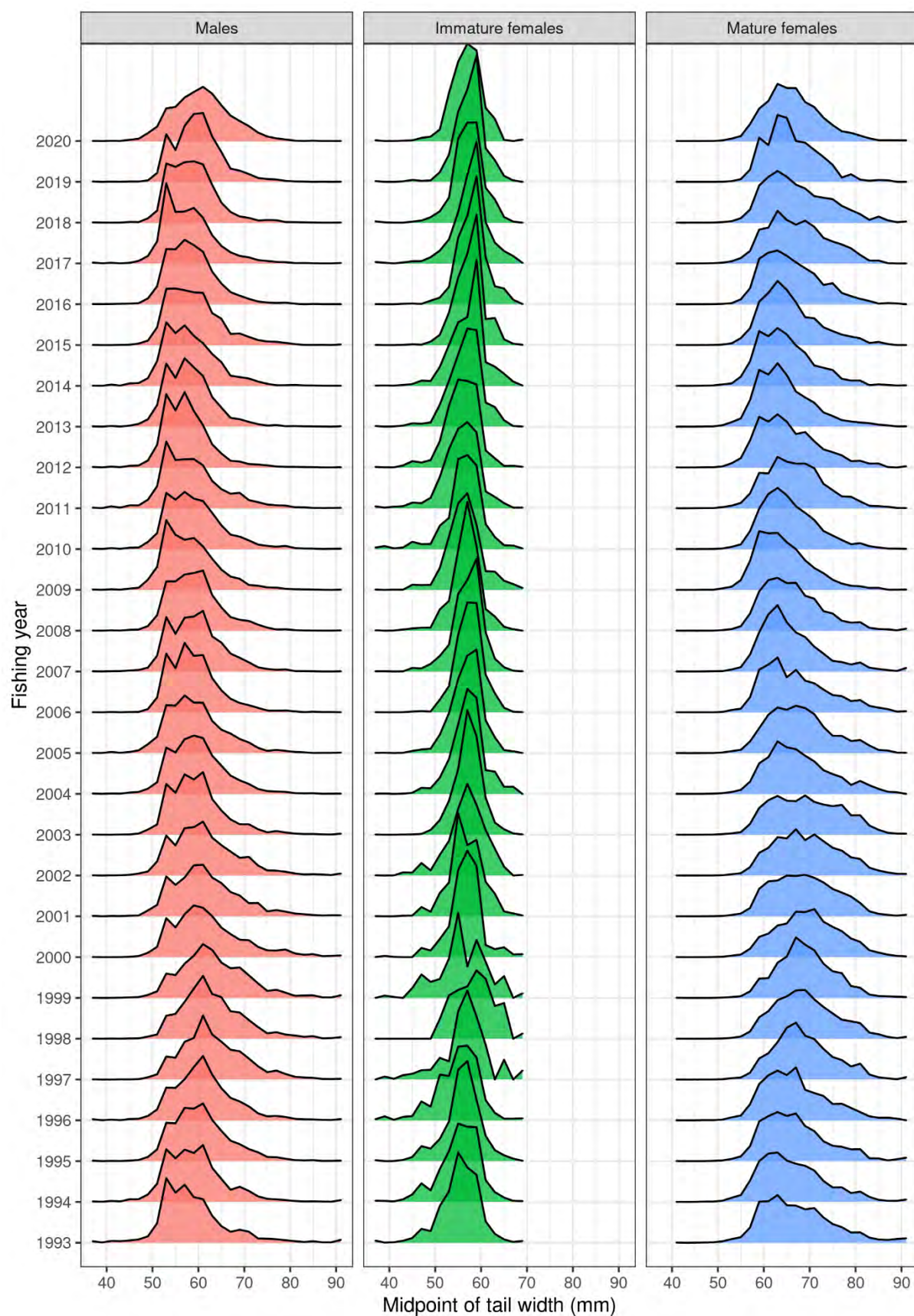


Figure 14: CRA 2: the conditional effect of fishing year by TW bin for each sex category.

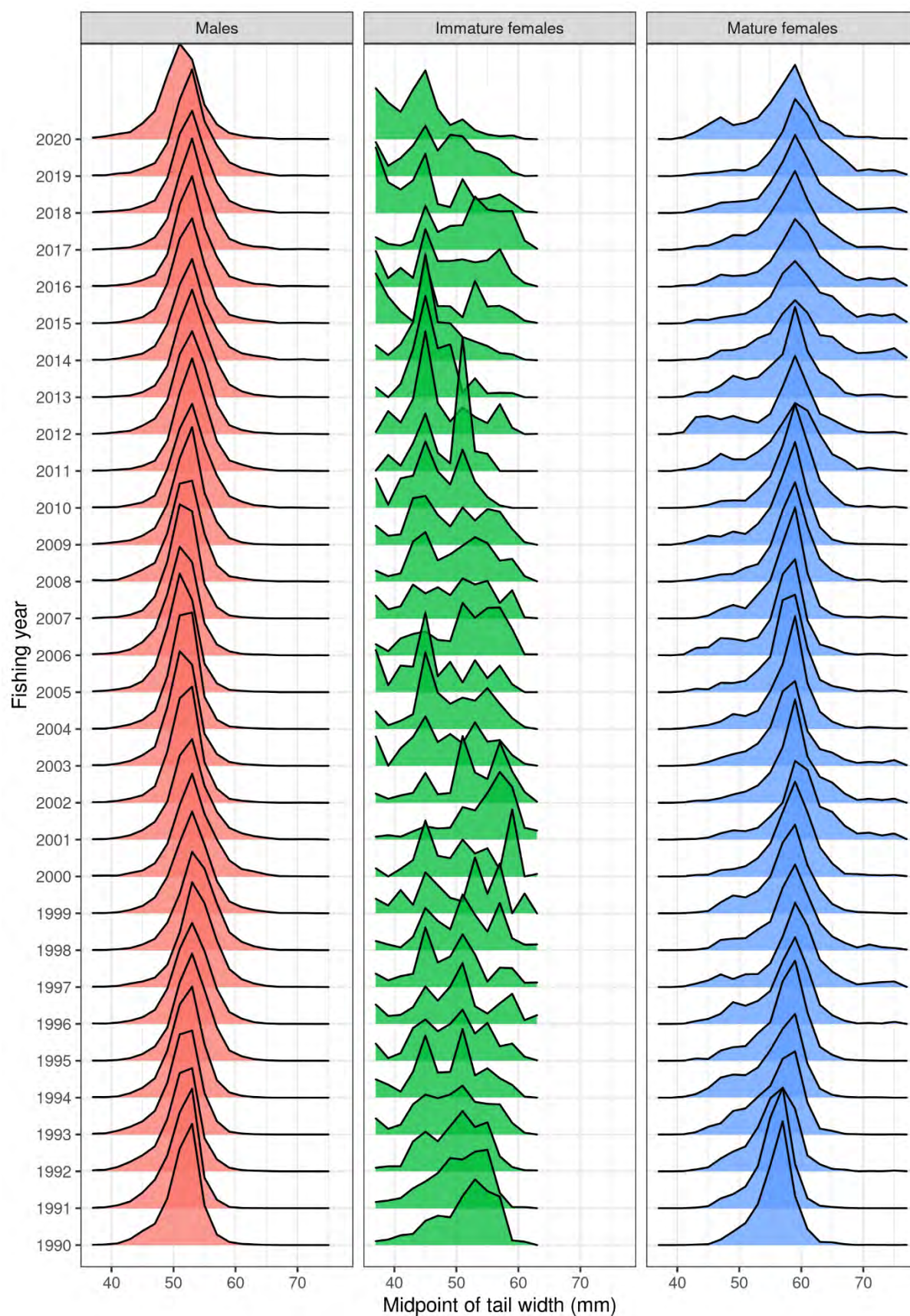


Figure 15: CRA 3: the conditional effect of fishing year by TW bin for each sex category.

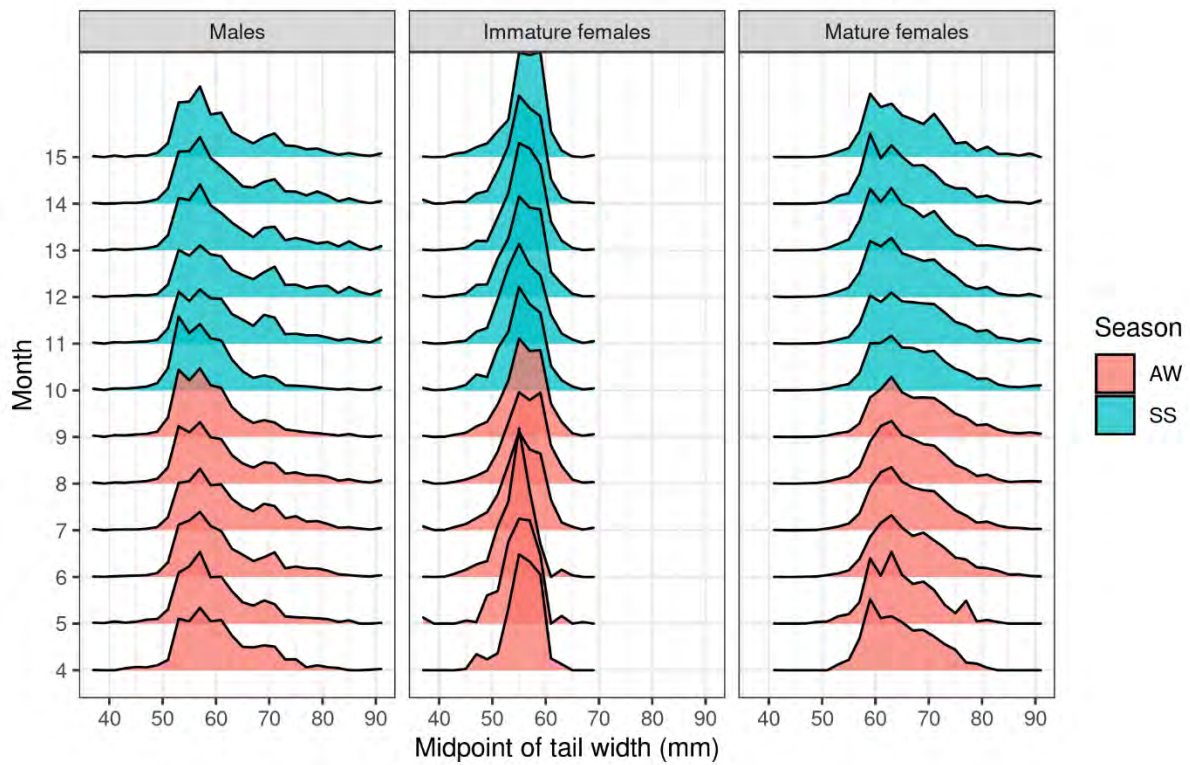


Figure 16: CRA 2: the conditional effect of fishing month (4 = April, 15 = March) by TW bin for each sex category.

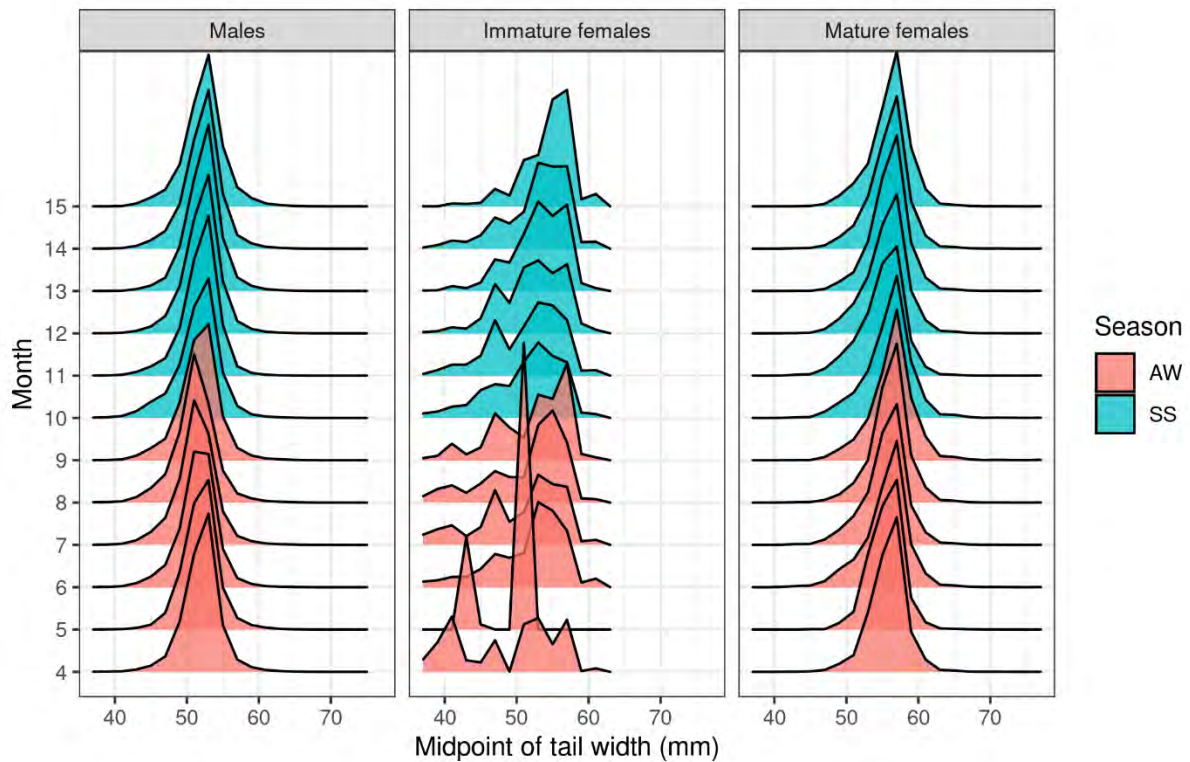


Figure 17: CRA 3: the conditional effect of fishing month (4 = April, 15 = March) by TW bin for each sex category.

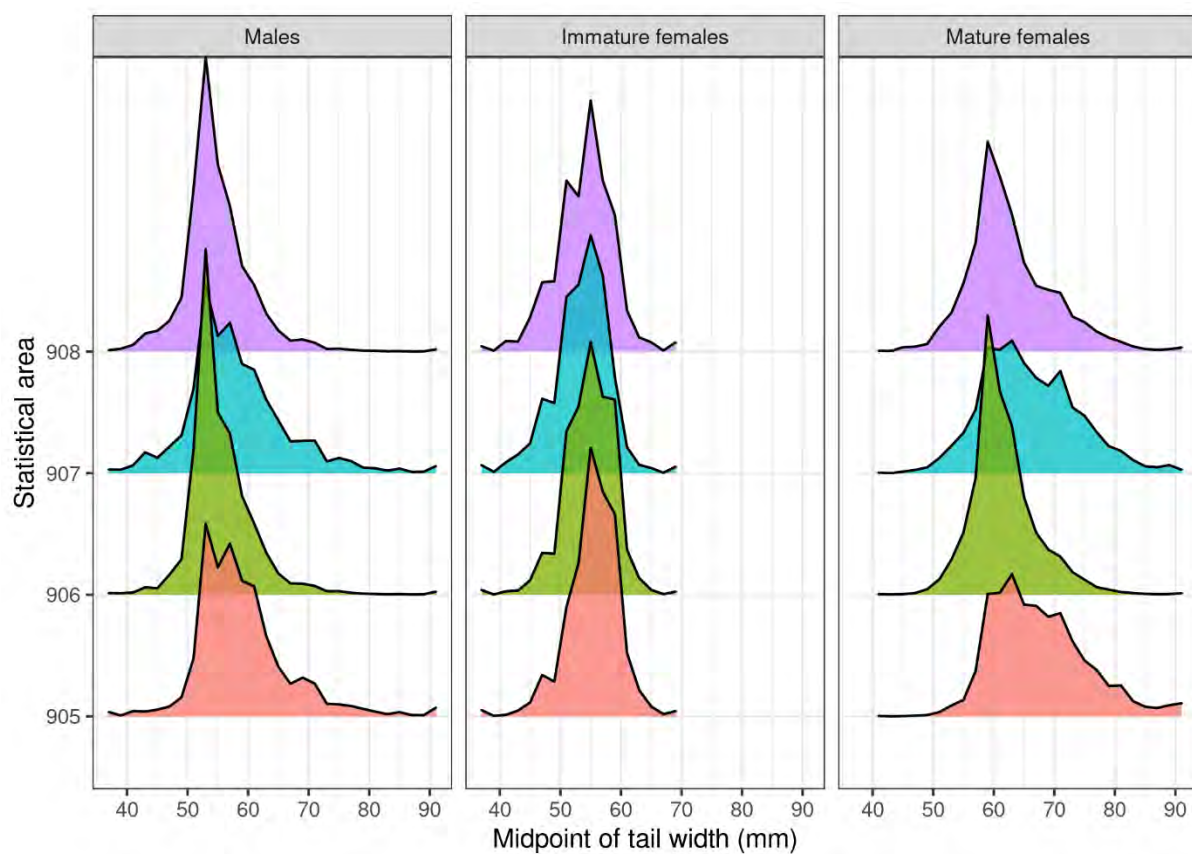


Figure 18: CRA 2: the conditional effect of statistical area by TW bin for each sex category.

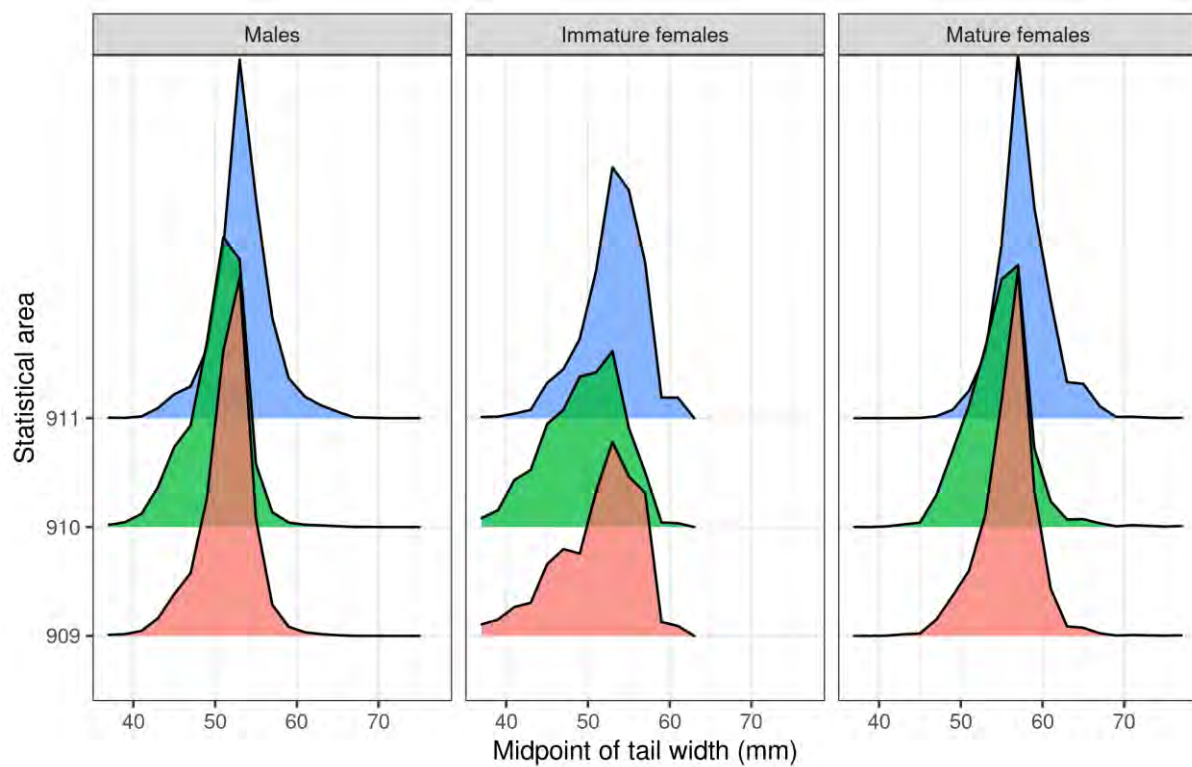


Figure 19: CRA 3: the conditional effect of statistical area by TW bin for each sex category.

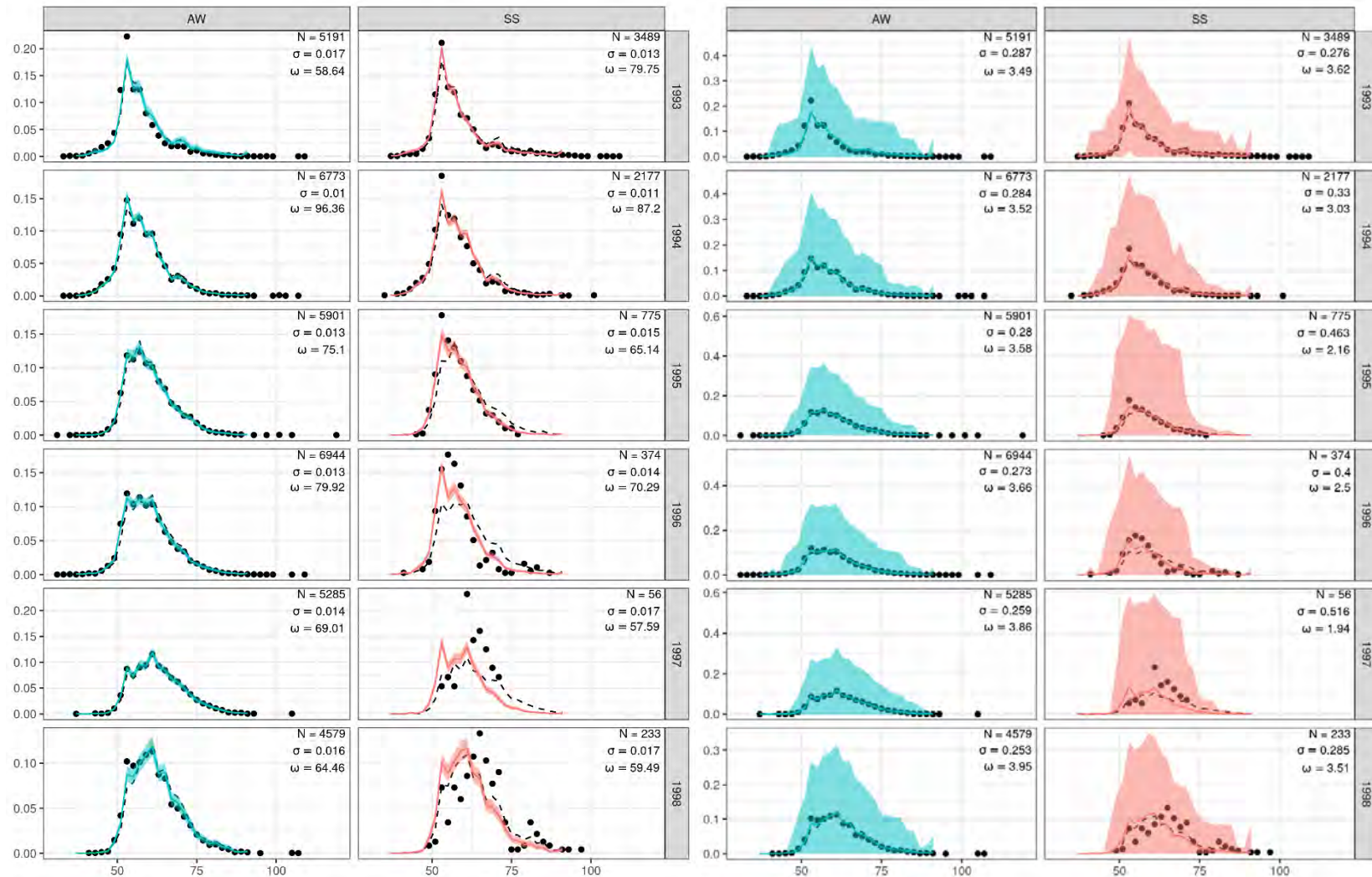


Figure 20: CRA 2: scaled LF distributions by fishing year, season, and TW bin for males (Equation 13) where the solid line is the mean and the shaded region is the 95% credible interval of the posterior distribution of the mean [left] or the posterior predictive distribution [right]. The aggregated data (points, Equation 10) and unscaled predictions of the LF (dashed line) for the same stratum are also shown. Each panel also includes text which provides: the number of lobsters measured; the SD of the predicted LF for each stratum (Equation 13); and the data set weight for each stratum (Equation 14).

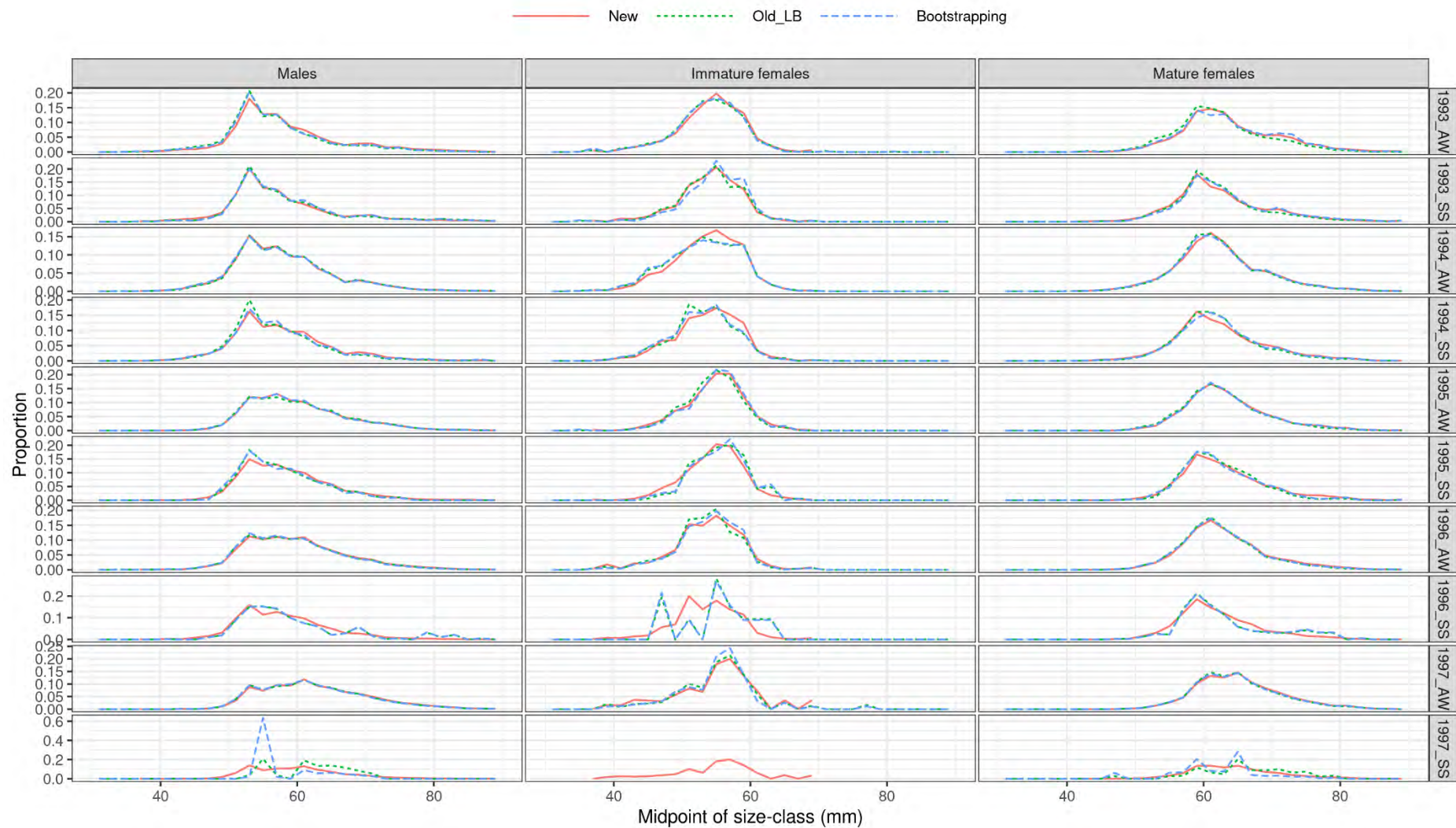


Figure 21: CRA 2: comparison between the old and new scaled LF distributions, and scaled LF distributions derived by bootstrapping, by fishing year, season (AW = autumn/winter, SS = spring/summer), sex category, data source (CS = catch sampling, LB = logbook), and TW bin (Equation 12) for 1993 to 1997.

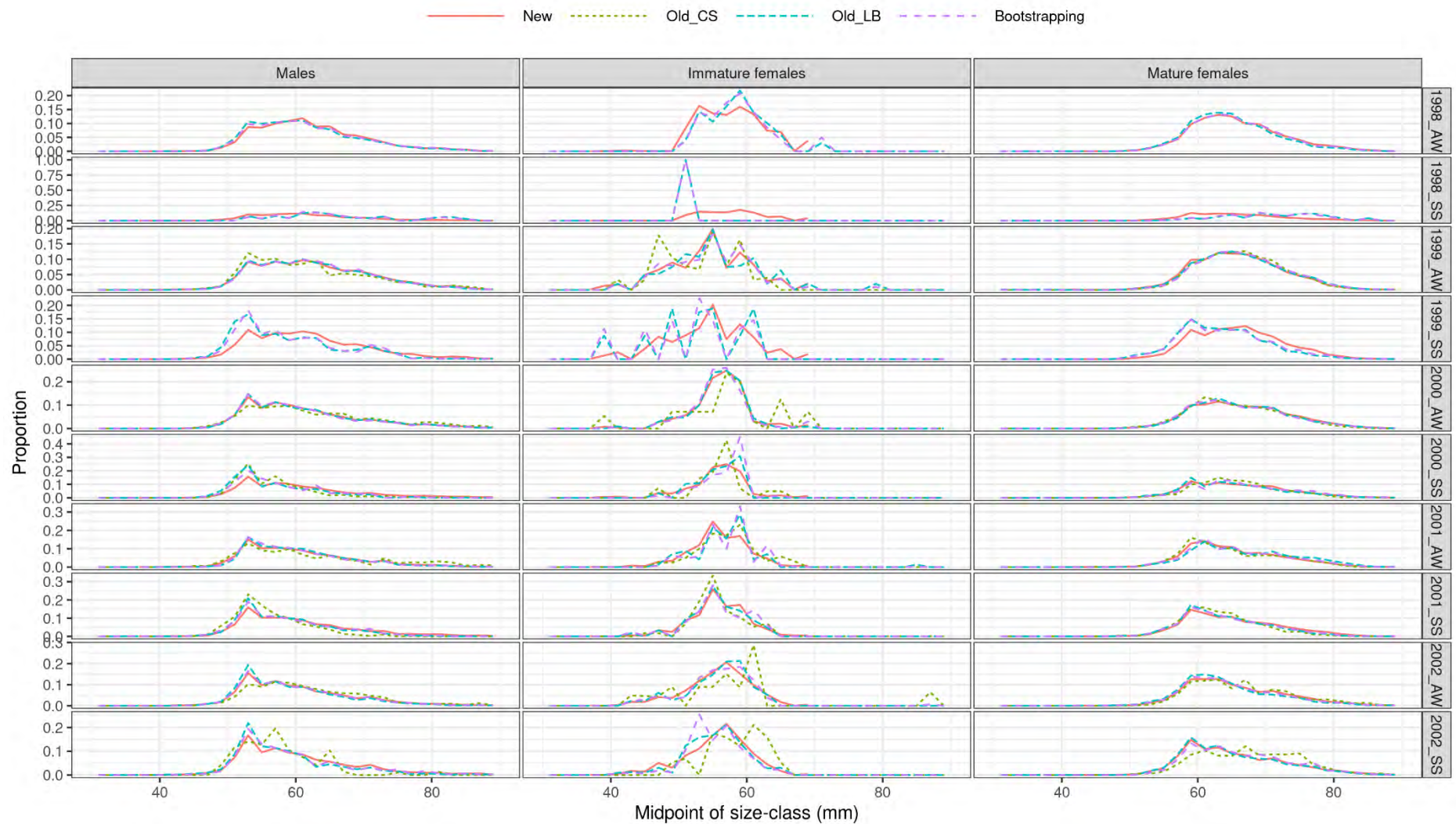


Figure 22: CRA 2: comparison between the old and new scaled LF distributions, and scaled LF distributions derived by bootstrapping, by fishing year, season (AW = autumn/winter, SS = spring/summer), sex category, data source (CS = catch sampling, LB = logbook), and TW bin (Equation 12) for 1998 to 2002.

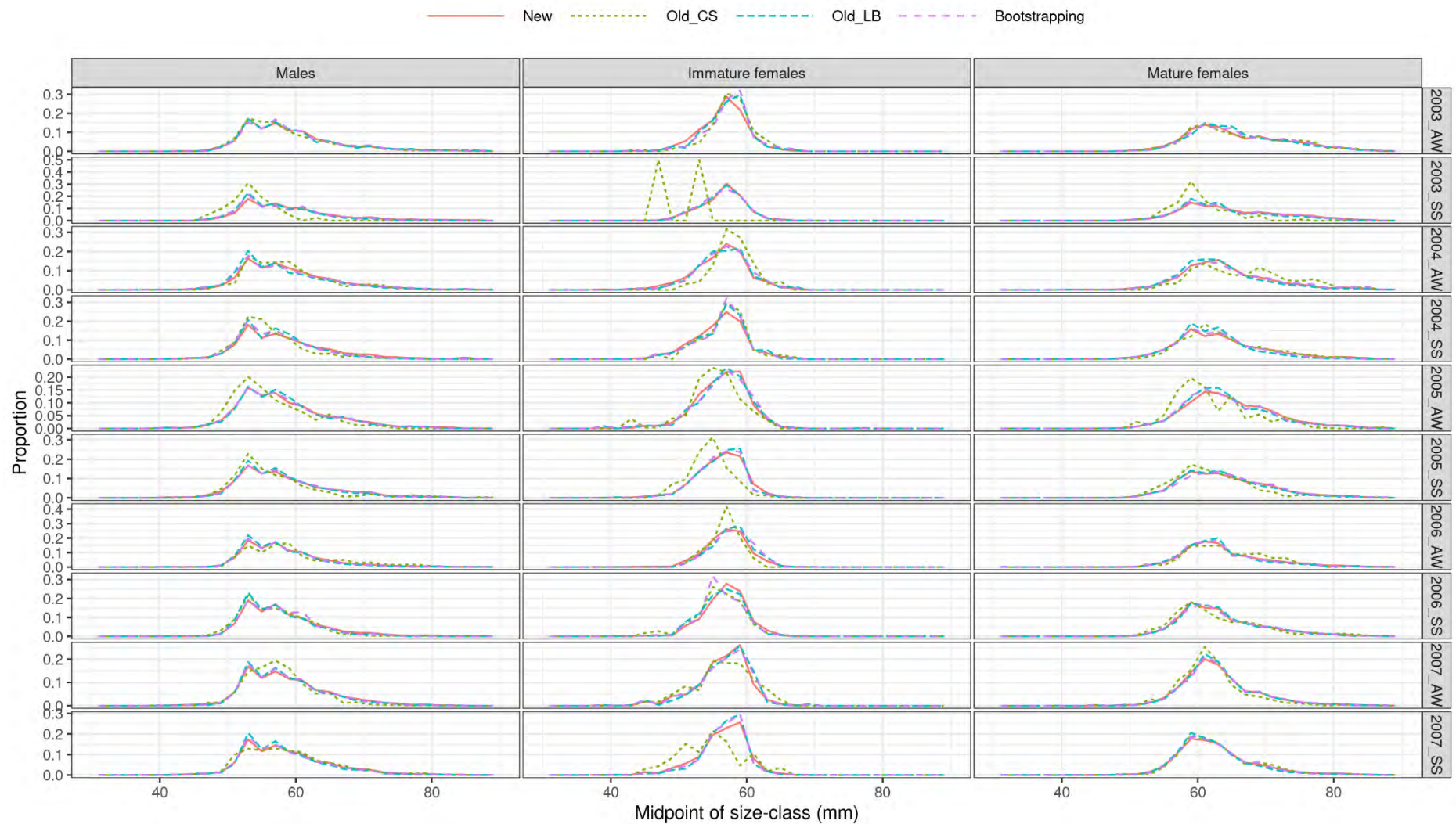


Figure 23: CRA 2: comparison between the old and new scaled LF distributions, and scaled LF distributions derived by bootstrapping, by fishing year, season (AW = autumn/winter, SS = spring/summer), sex category, data source (CS = catch sampling, LB = logbook), and TW bin (Equation 12) for 2003 to 2007.

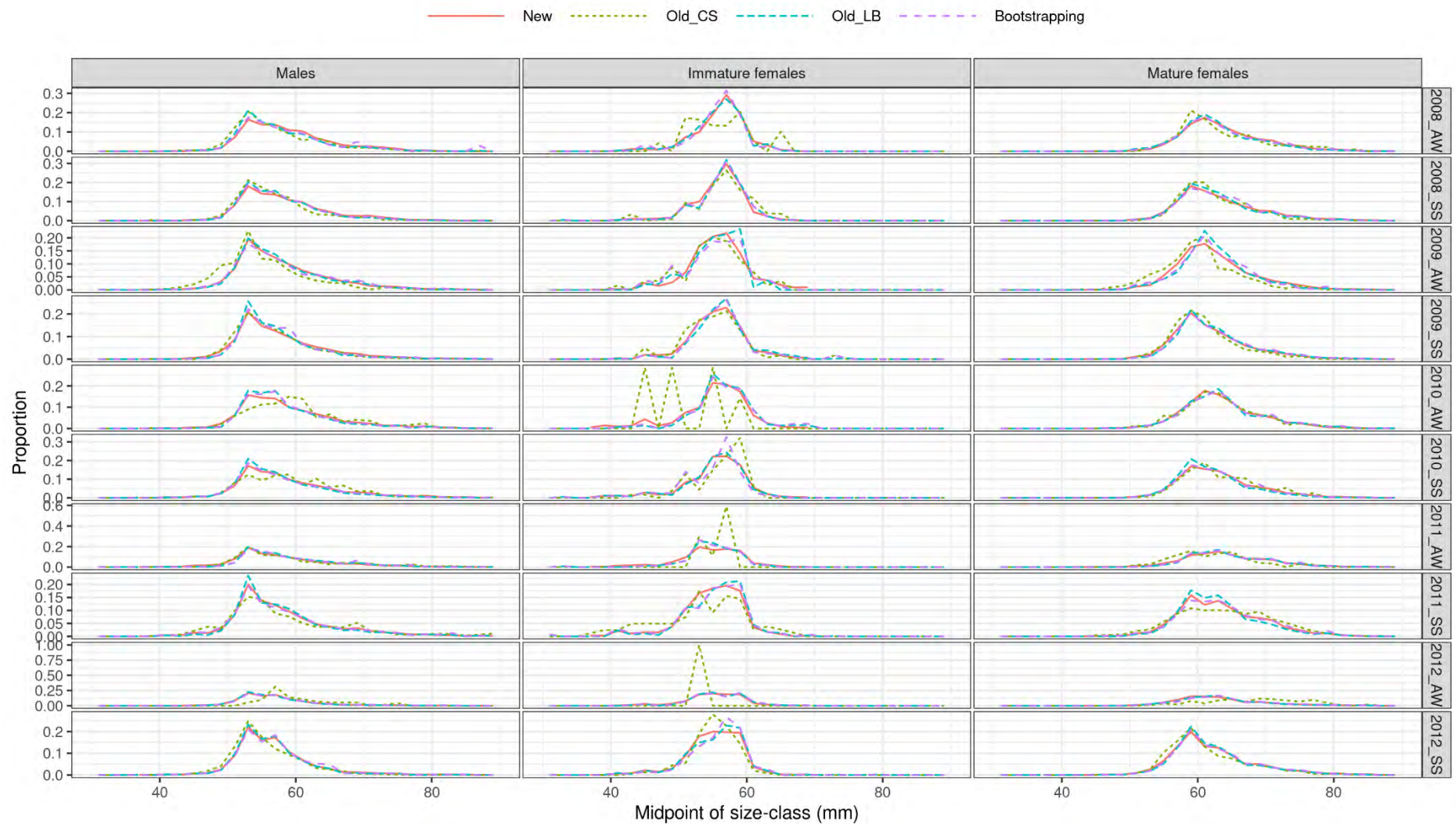


Figure 24: CRA 2: comparison between the old and new scaled LF distributions, and scaled LF distributions derived by bootstrapping, by fishing year, season (AW = autumn/winter, SS = spring/summer), sex category, data source (CS = catch sampling, LB = logbook), and TW bin (Equation 12) for 2008 to 2012.

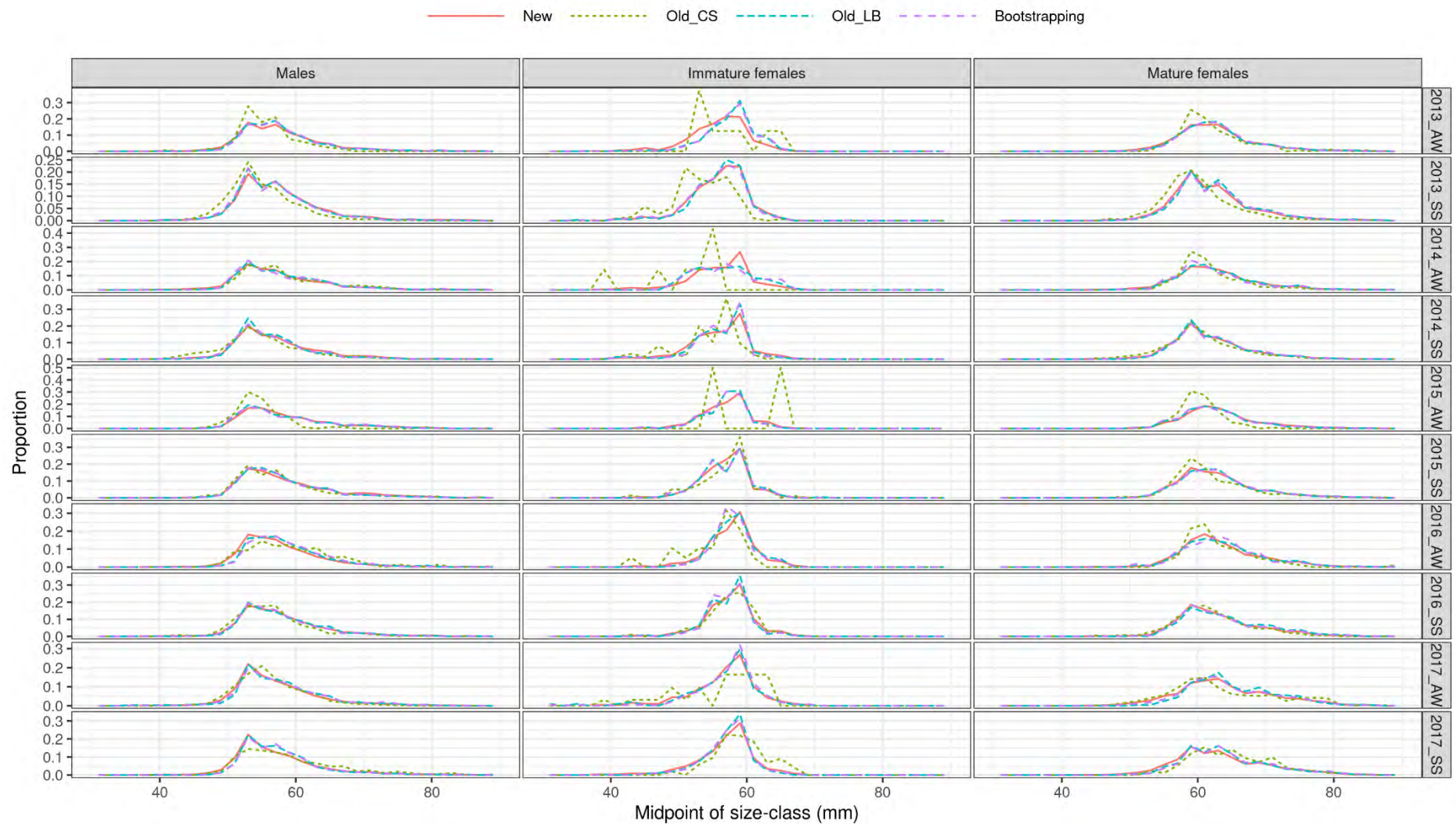


Figure 25: CRA 2: comparison between the old and new scaled LF distributions, and scaled LF distributions derived by bootstrapping, by fishing year, season (AW = autumn/winter, SS = spring/summer), sex category, data source (CS = catch sampling, LB = logbook), and TW bin (Equation 12) for 2013 to 2017.

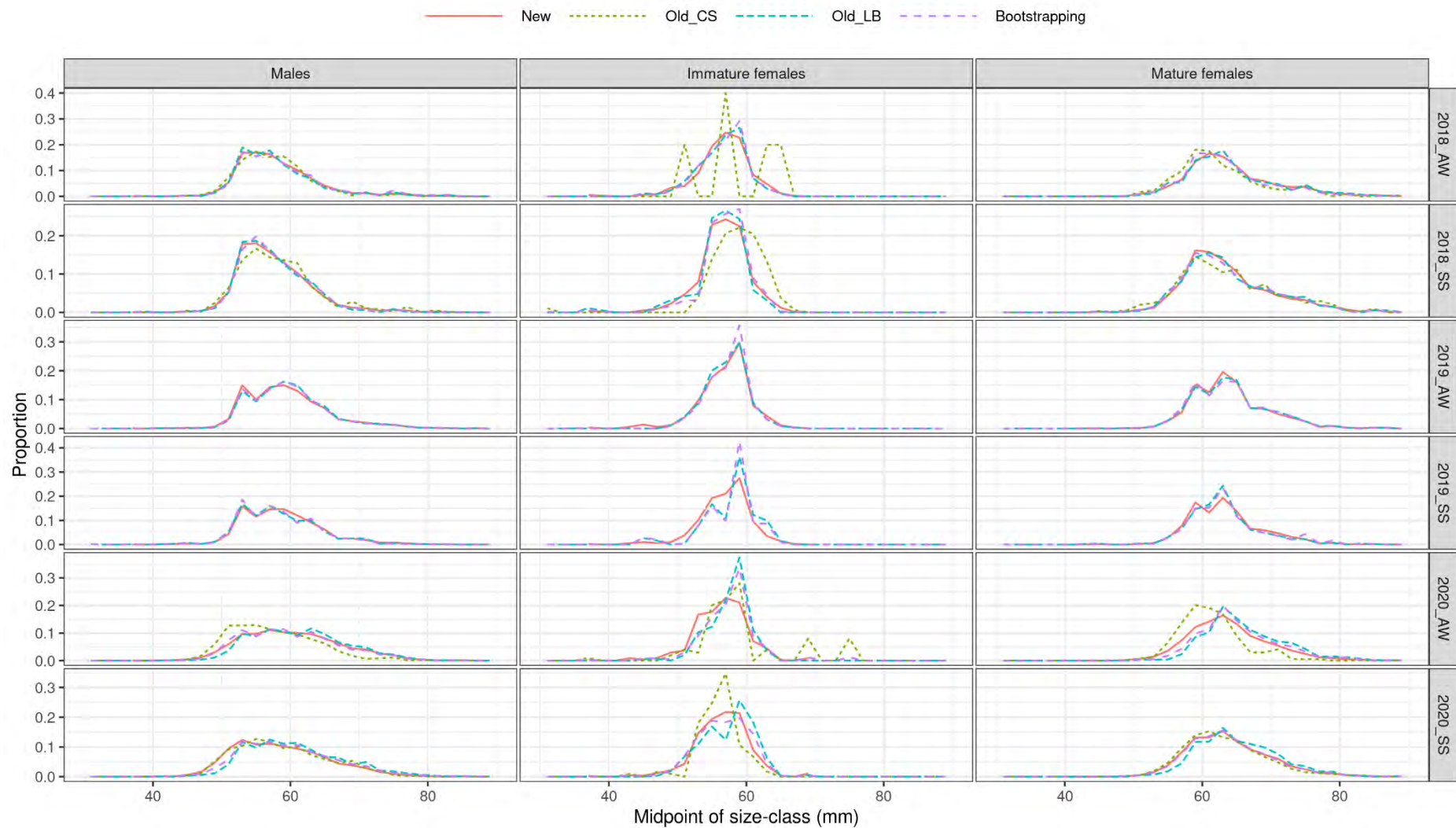


Figure 26: CRA 2: comparison between the old and new scaled LF distributions, and scaled LF distributions derived by bootstrapping, by fishing year, season (AW = autumn/winter, SS = spring/summer), sex category, data source (CS = catch sampling, LB = logbook), and TW bin (Equation 12) for 2018 to 2020.

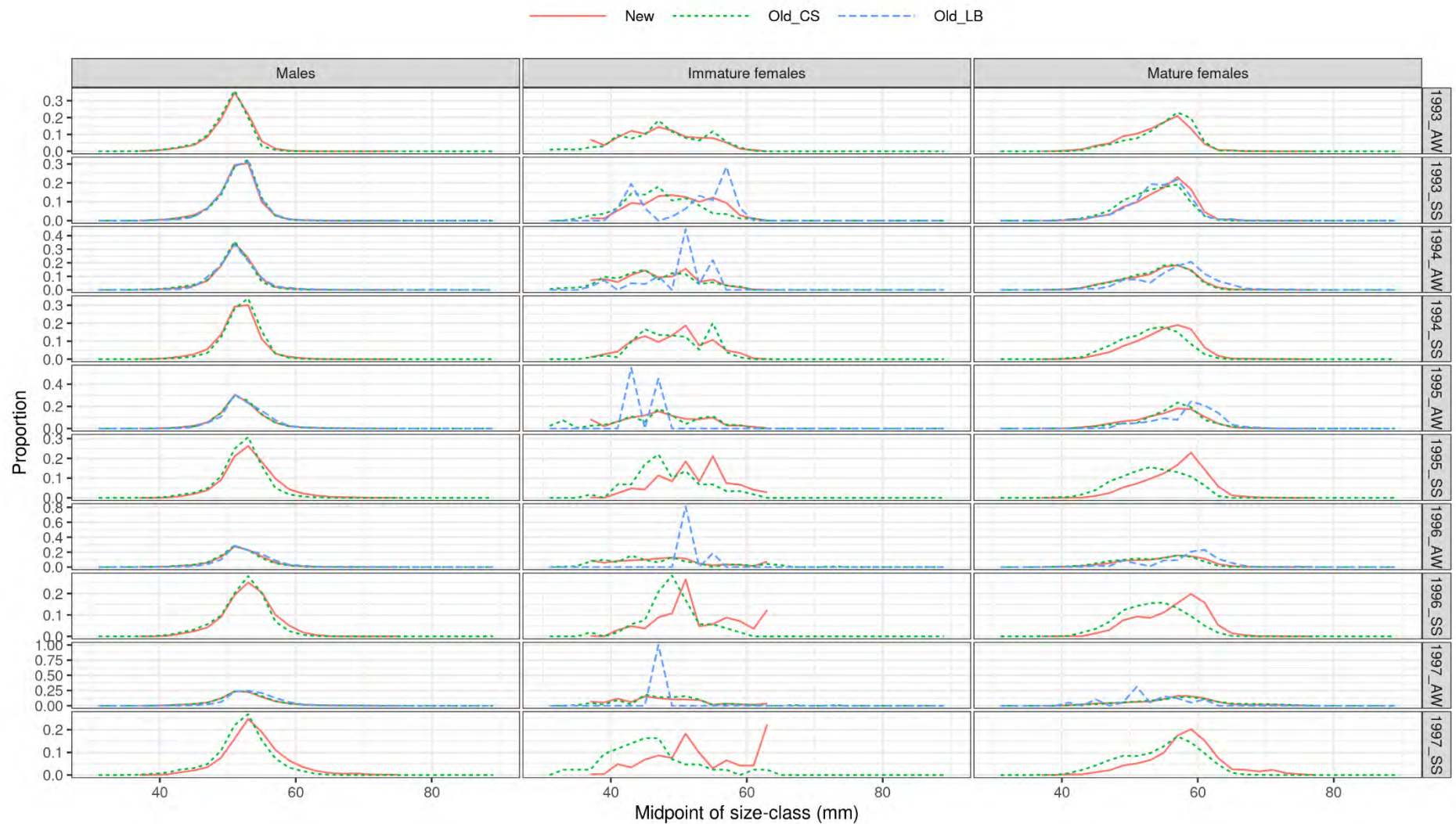


Figure 27: CRA 3 region 1: comparison between the old (dashed lines) and new (solid line) scaled LF distributions by fishing year, season (AW = autumn/winter, SS = spring/summer), sex category, data source (CS = catch sampling, LB = logbook), and TW bin (Equation 12) for 1993 to 1997.

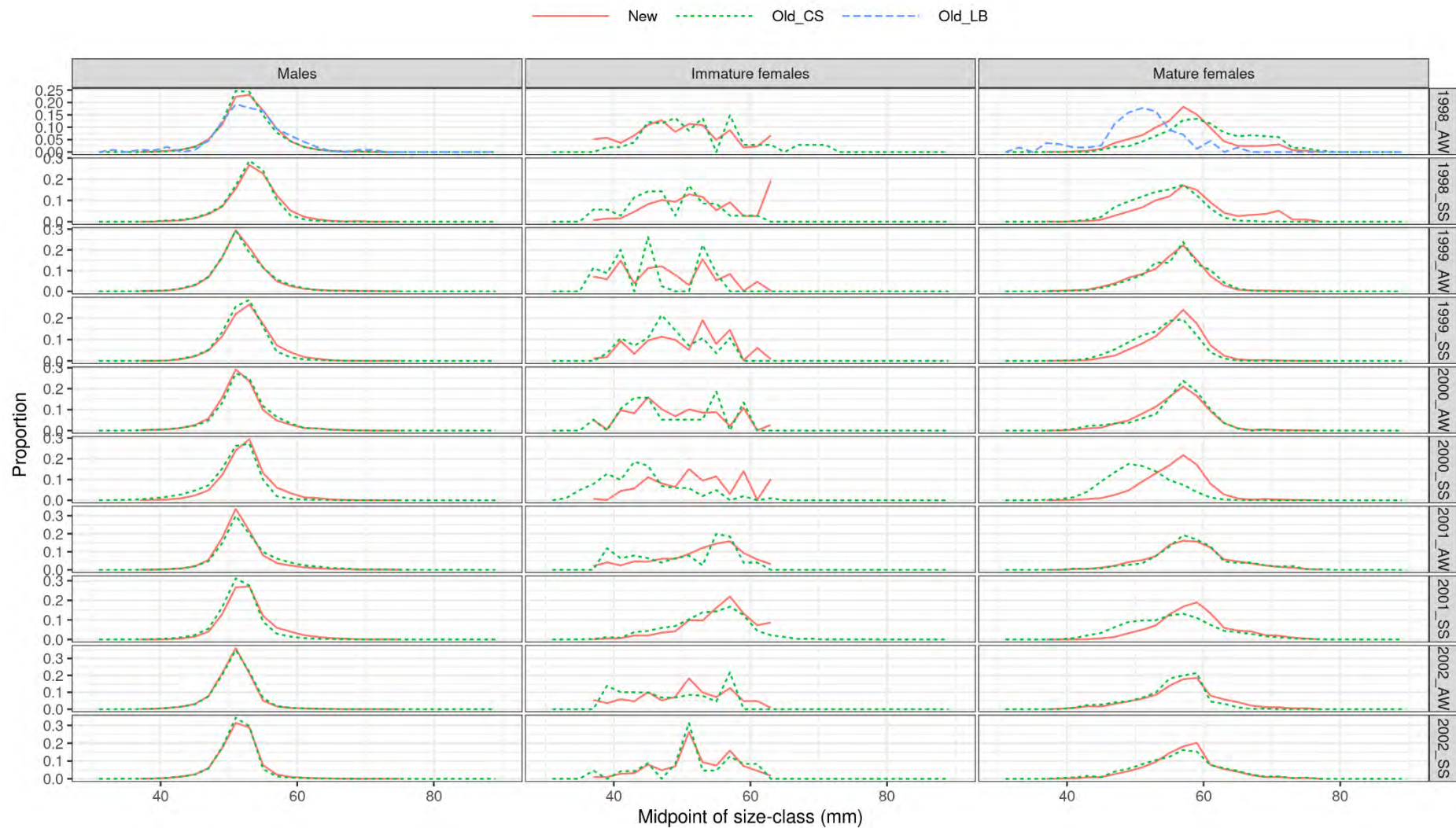


Figure 28: CRA 3 region 1: comparison between the old (dashed lines) and new (solid line) scaled LF distributions by fishing year, season (AW = autumn/winter, SS = spring/summer), sex category, data source (CS = catch sampling, LB = logbook), and TW bin (Equation 12) for 1998 to 2002.

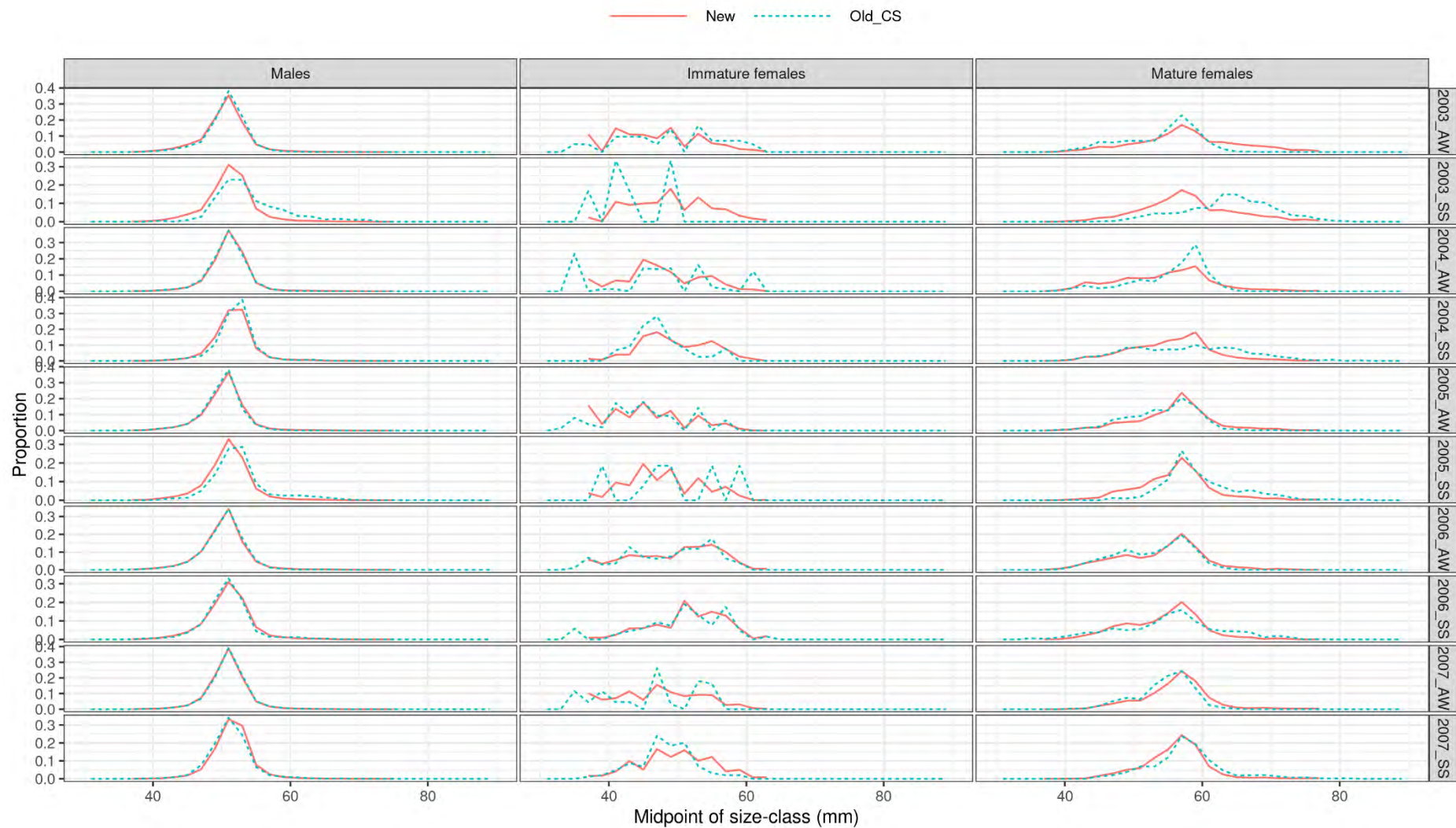


Figure 29: CRA 3 region 1: comparison between the old (dashed lines) and new (solid line) scaled LF distributions by fishing year, season (AW = autumn/winter, SS = spring/summer), sex category, data source (CS = catch sampling, LB = logbook), and TW bin (Equation 12) for 2003 to 2007.

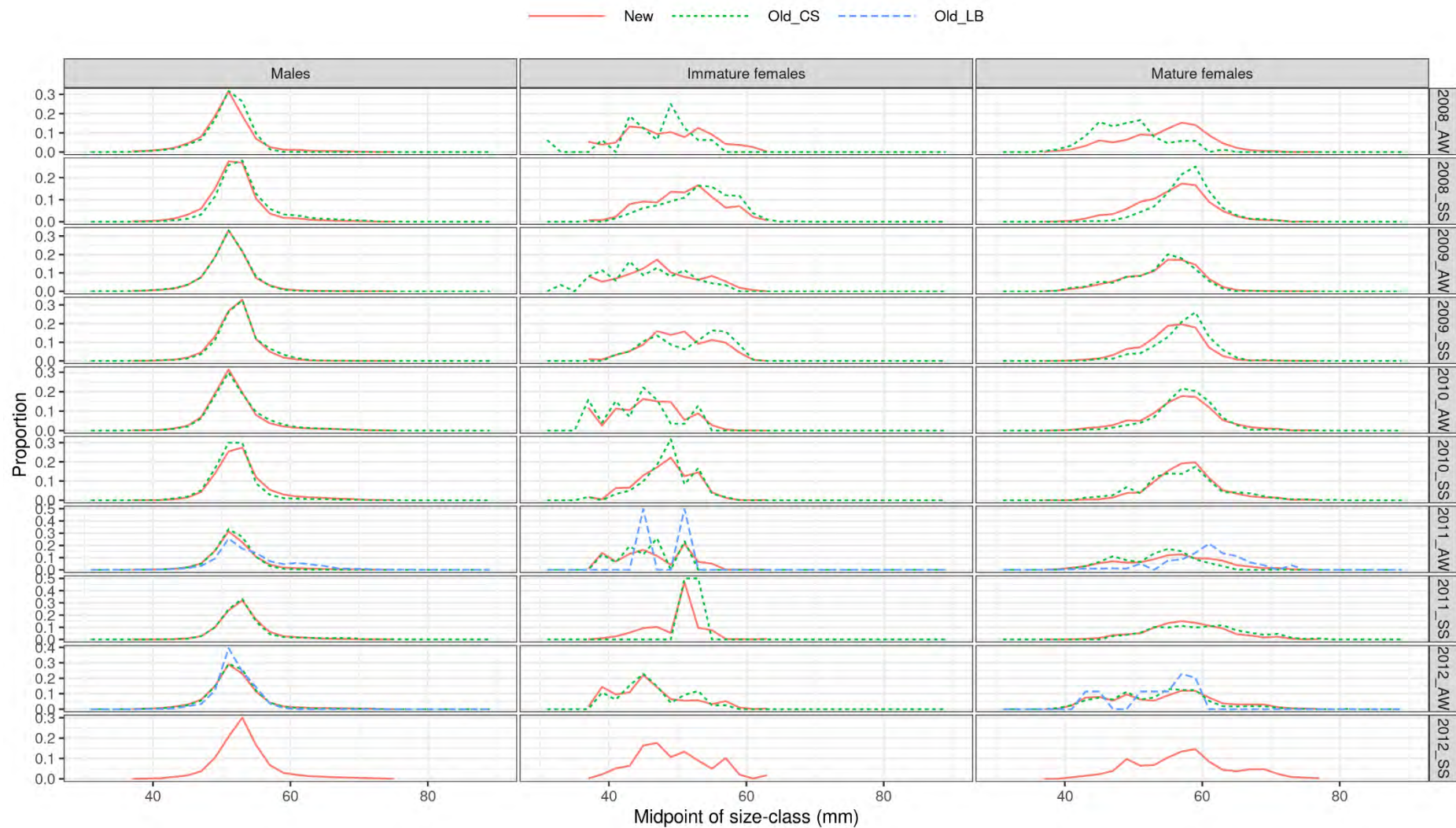


Figure 30: CRA 3 region 1: comparison between the old (dashed lines) and new (solid line) scaled LF distributions by fishing year, season (AW = autumn/winter, SS = spring/summer), sex category, data source (CS = catch sampling, LB = logbook), and TW bin (Equation 12) for 2008 to 2012.

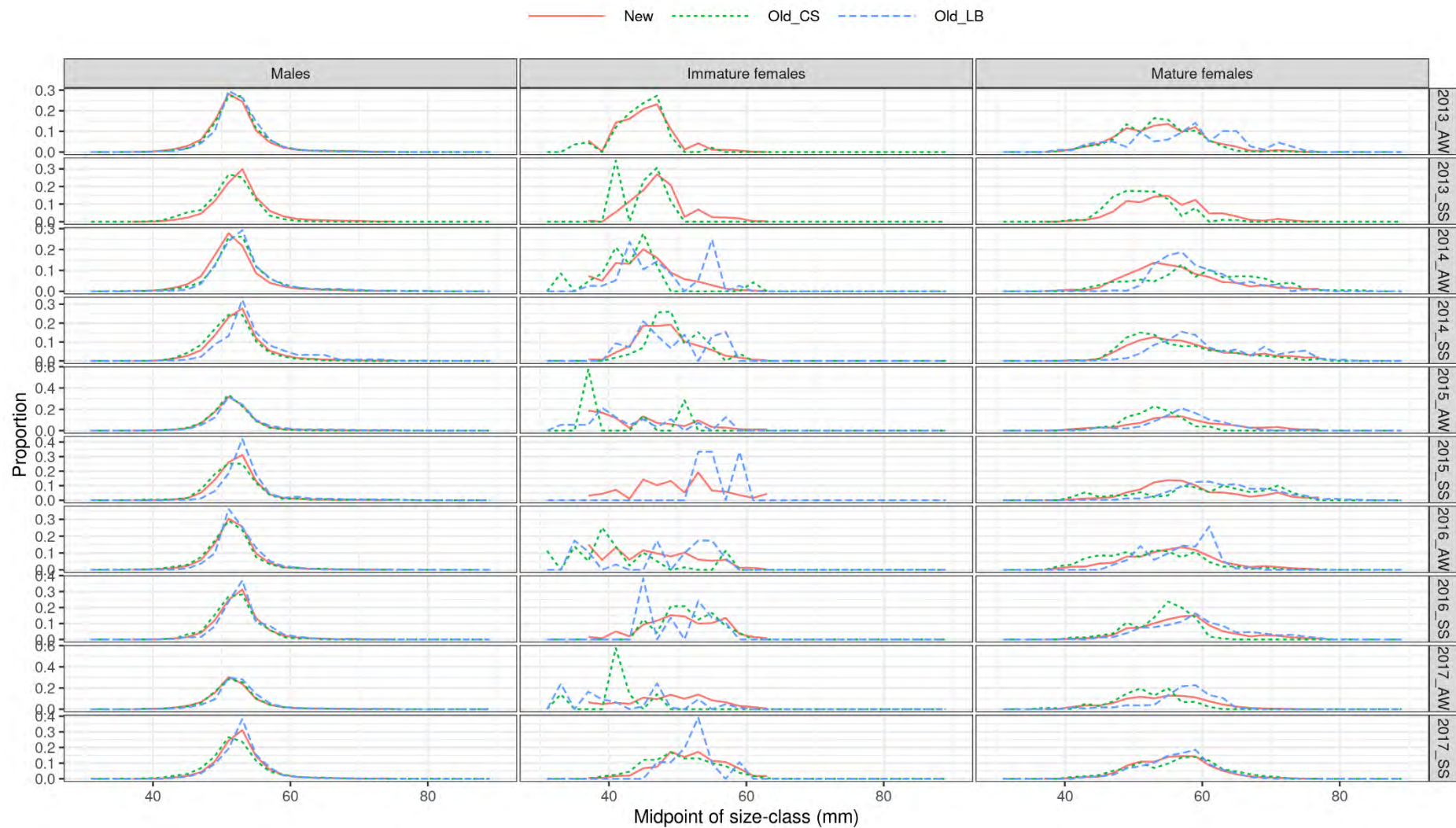


Figure 31: CRA 3 region 1: comparison between the old (dashed lines) and new (solid line) scaled LF distributions by fishing year, season (AW = autumn/winter, SS = spring/summer), sex category, data source (CS = catch sampling, LB = logbook), and TW bin (Equation 12) for 2013 to 2017.

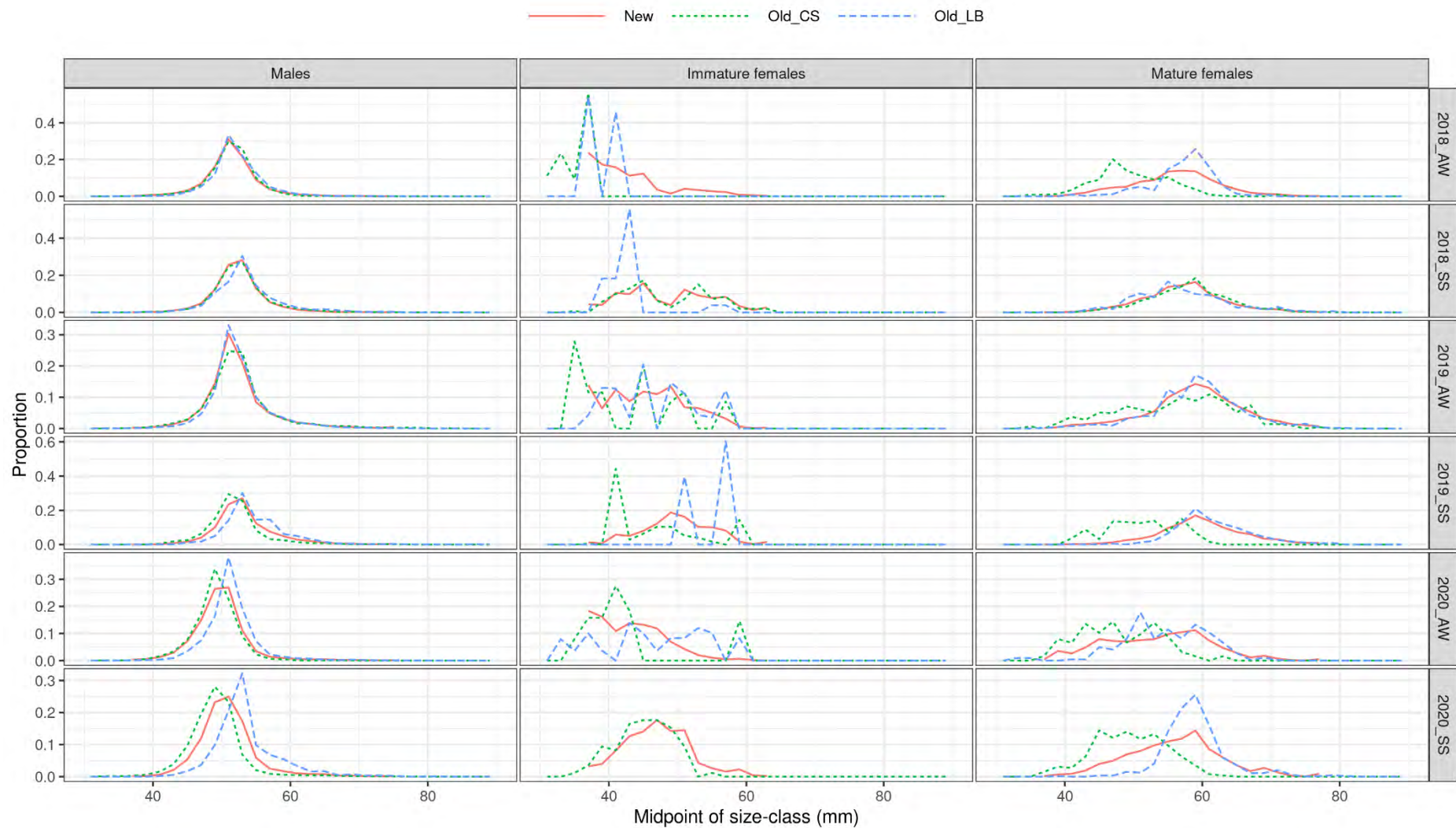


Figure 32: CRA 3 region 1: comparison between the old (dashed lines) and new (solid line) scaled LF distributions by fishing year, season (AW = autumn/winter, SS = spring/summer), sex category, data source (CS = catch sampling, LB = logbook), and TW bin (Equation 12) for 2018 to 2020.

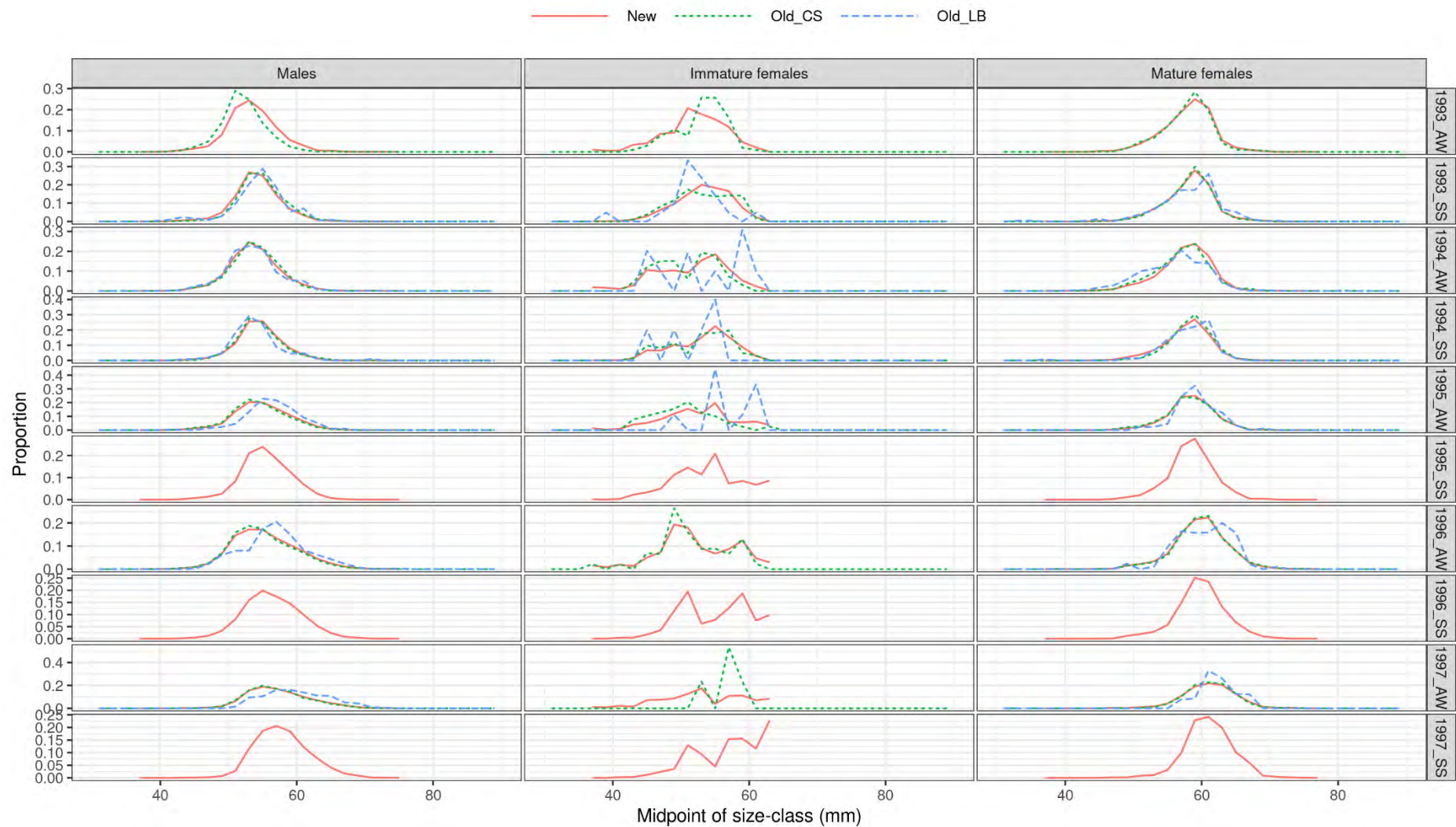


Figure 33: CRA 3 region 2: comparison between the old (dashed lines) and new (solid line) scaled LF distributions by fishing year, season (AW = autumn/winter, SS = spring/summer), sex category, data source (CS = catch sampling, LB = logbook), and TW bin (Equation 12) for 1993 to 1997.

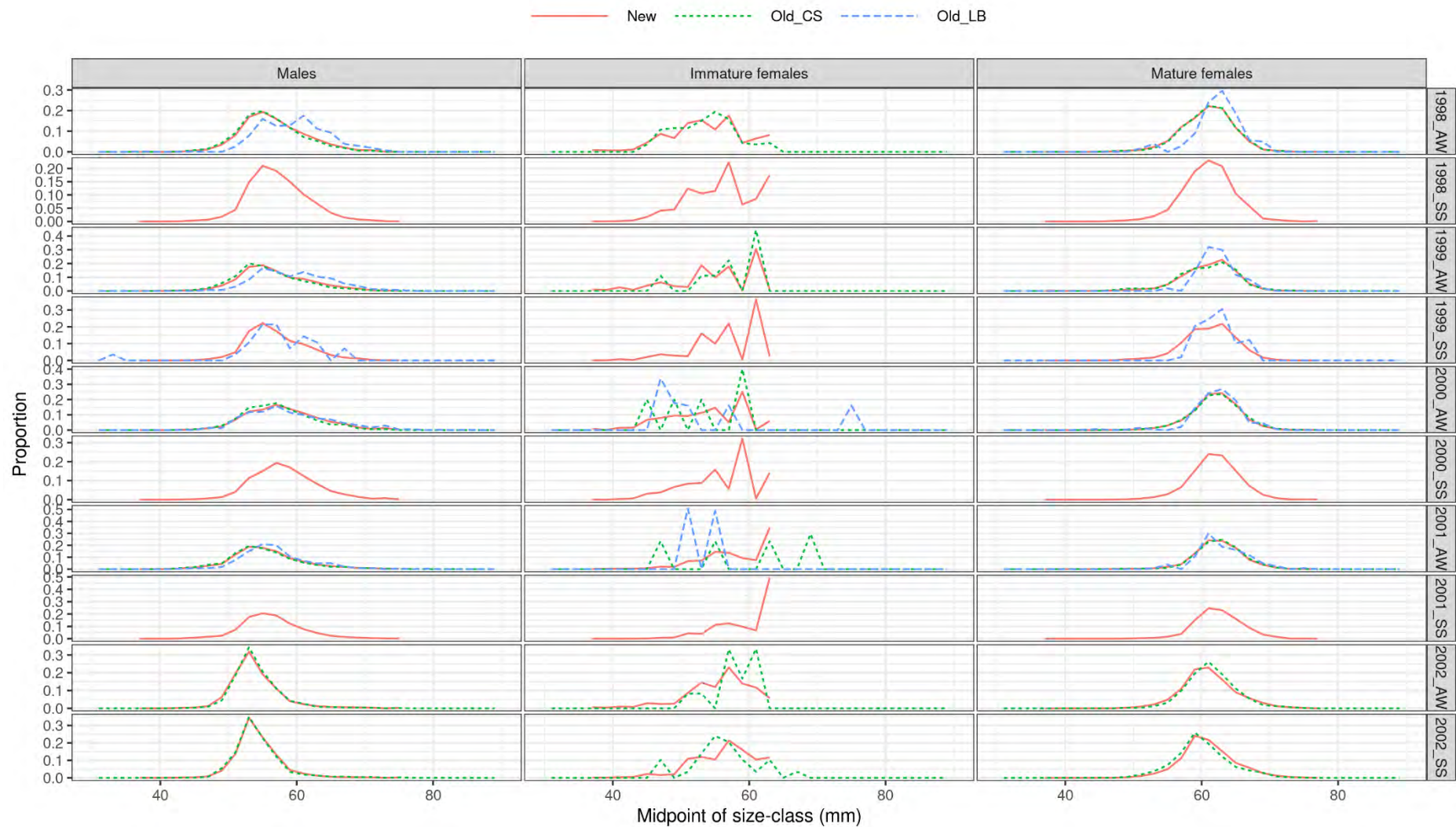


Figure 34: CRA 3 region 2: comparison between the old (dashed lines) and new (solid line) scaled LF distributions by fishing year, season (AW = autumn/winter, SS = spring/summer), sex category, data source (CS = catch sampling, LB = logbook), and TW bin (Equation 12) for 1998 to 2002.

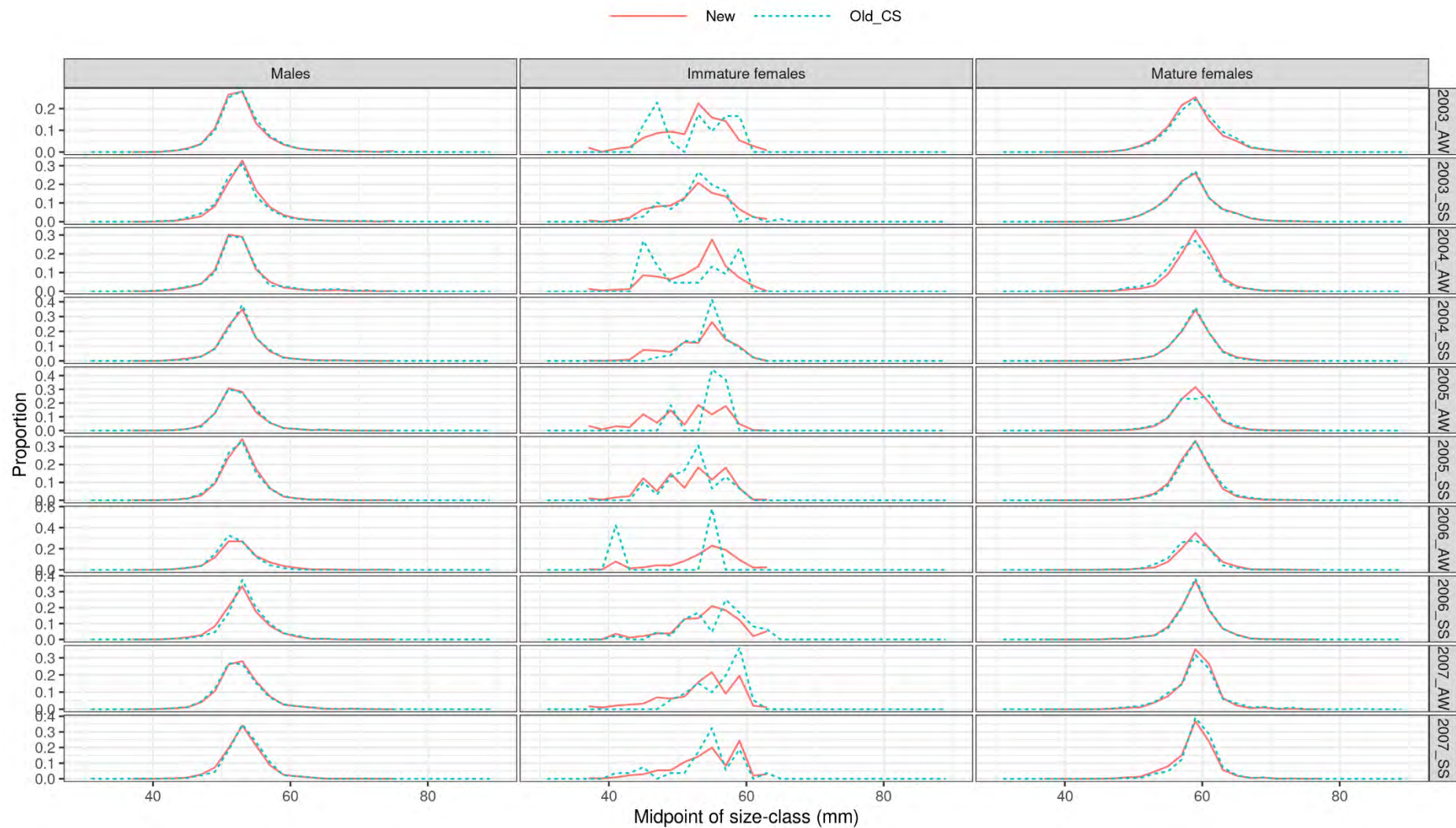


Figure 35: CRA 3 region 2: comparison between the old (dashed lines) and new (solid line) scaled LF distributions by fishing year, season (AW = autumn/winter, SS = spring/summer), sex category, data source (CS = catch sampling, LB = logbook), and TW bin (Equation 12) for 2003 to 2007.

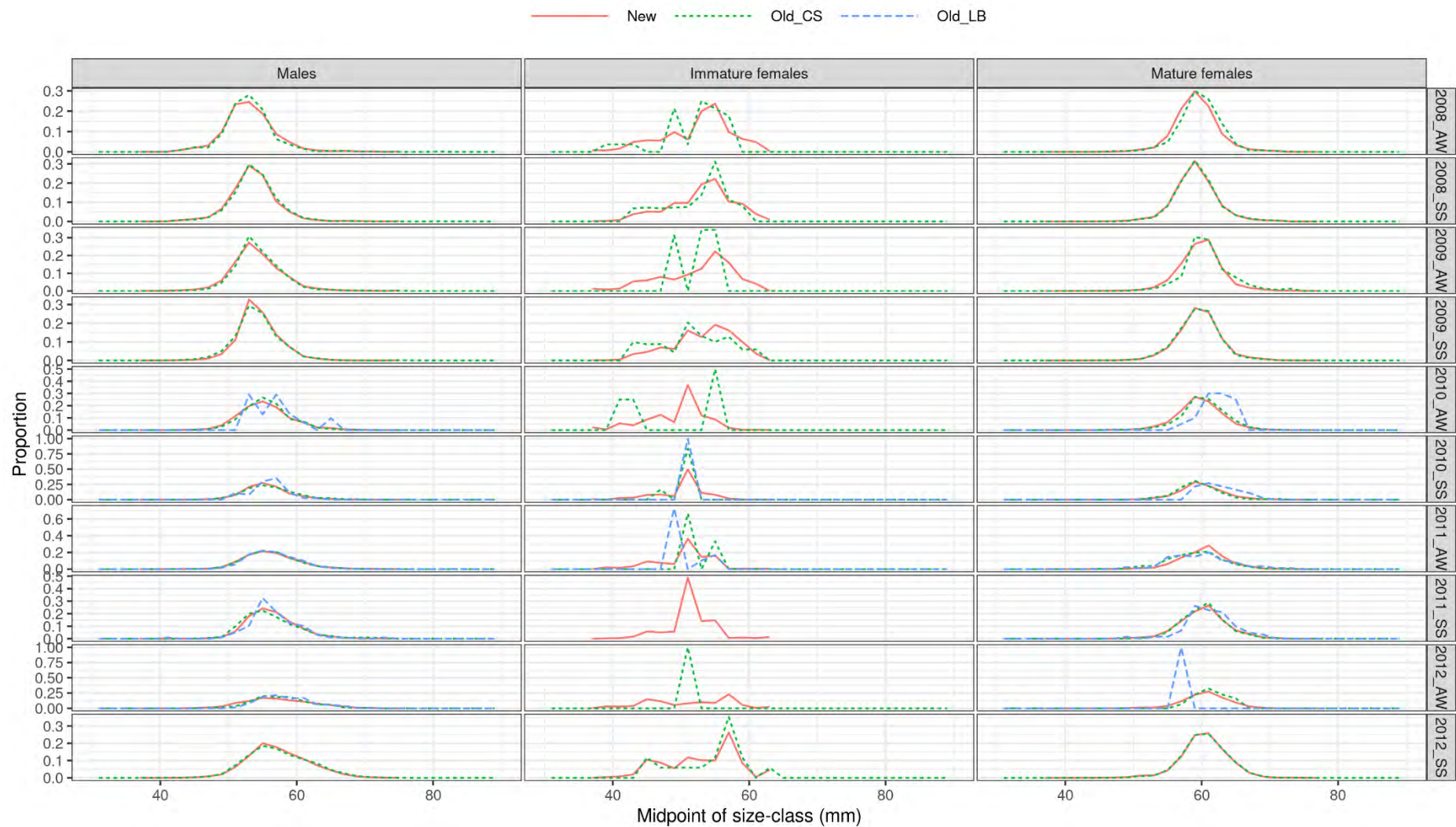


Figure 36: CRA 3 region 2: comparison between the old (dashed lines) and new (solid line) scaled LF distributions by fishing year, season (AW = autumn/winter, SS = spring/summer), sex category, data source (CS = catch sampling, LB = logbook), and TW bin (Equation 12) for 2008 to 2012.

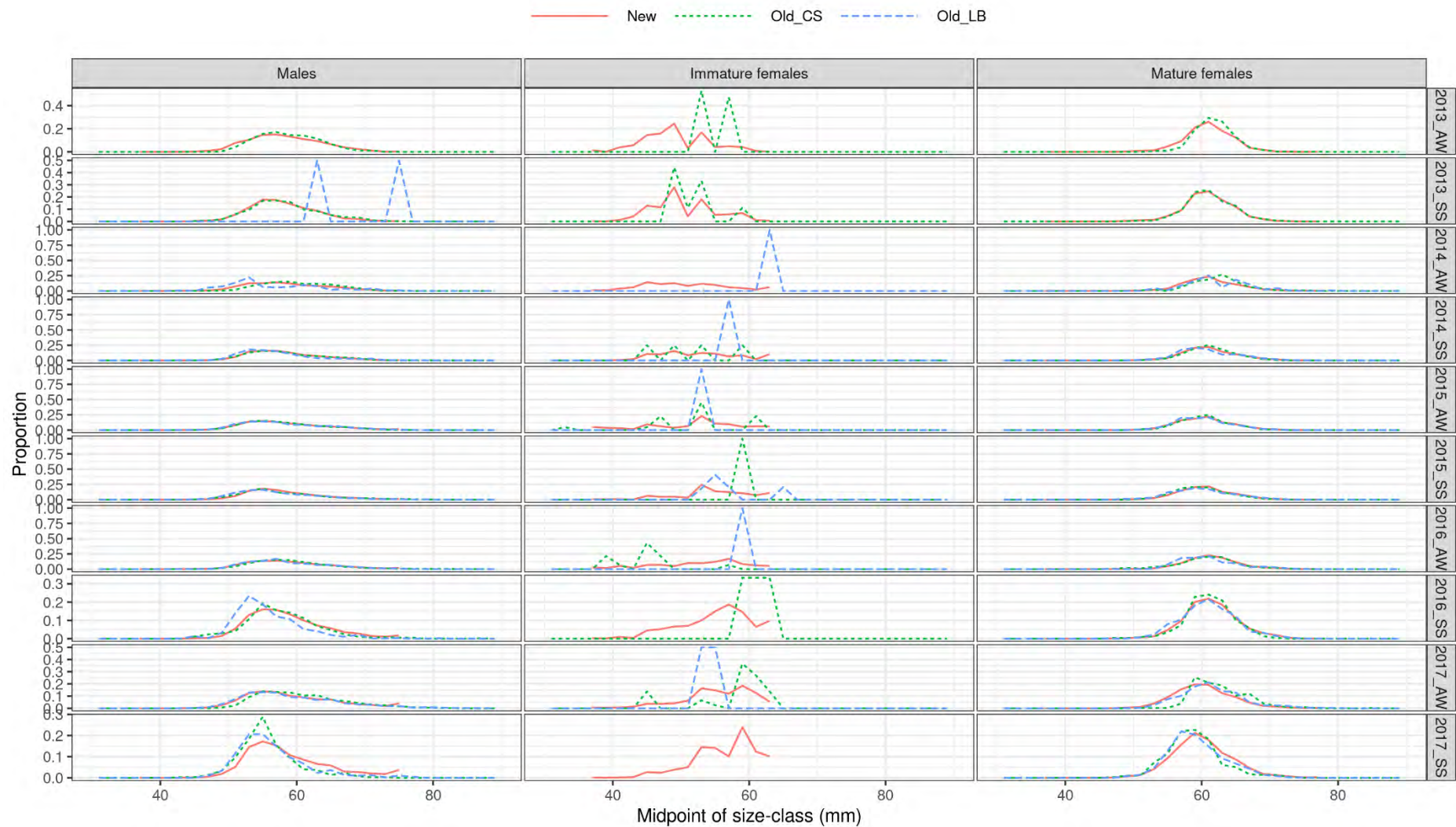


Figure 37: CRA 3 region 2: comparison between the old (dashed lines) and new (solid line) scaled LF distributions by fishing year, season (AW = autumn/winter, SS = spring/summer), sex category, data source (CS = catch sampling, LB = logbook), and TW bin (Equation 12) for 2013 to 2017.

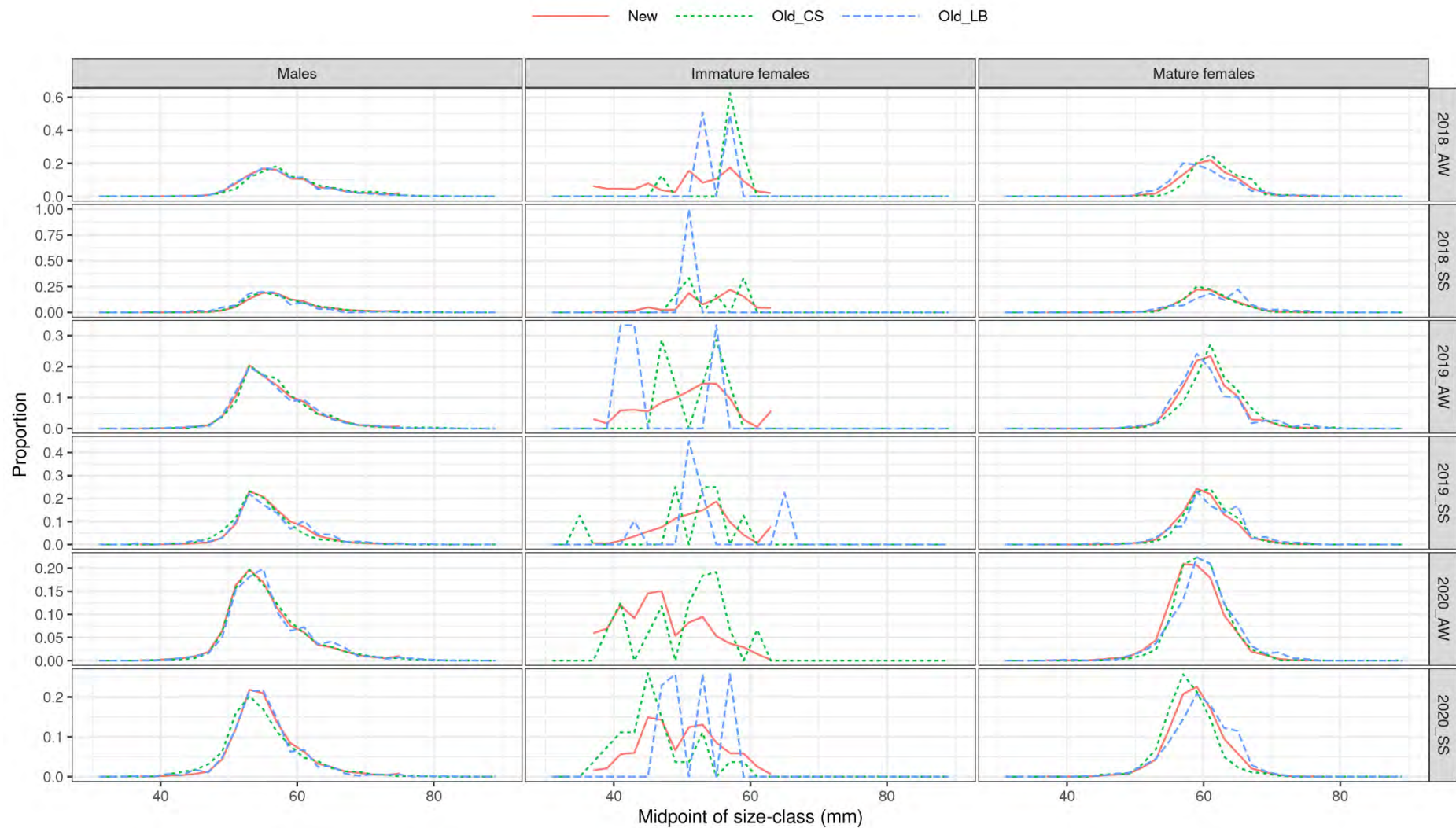


Figure 38: CRA 3 region 2: comparison between the old (dashed lines) and new (solid line) scaled LF distributions by fishing year, season (AW = autumn/winter, SS = spring/summer), sex category, data source (CS = catch sampling, LB = logbook), and TW bin (Equation 12) for 2018 to 2020.

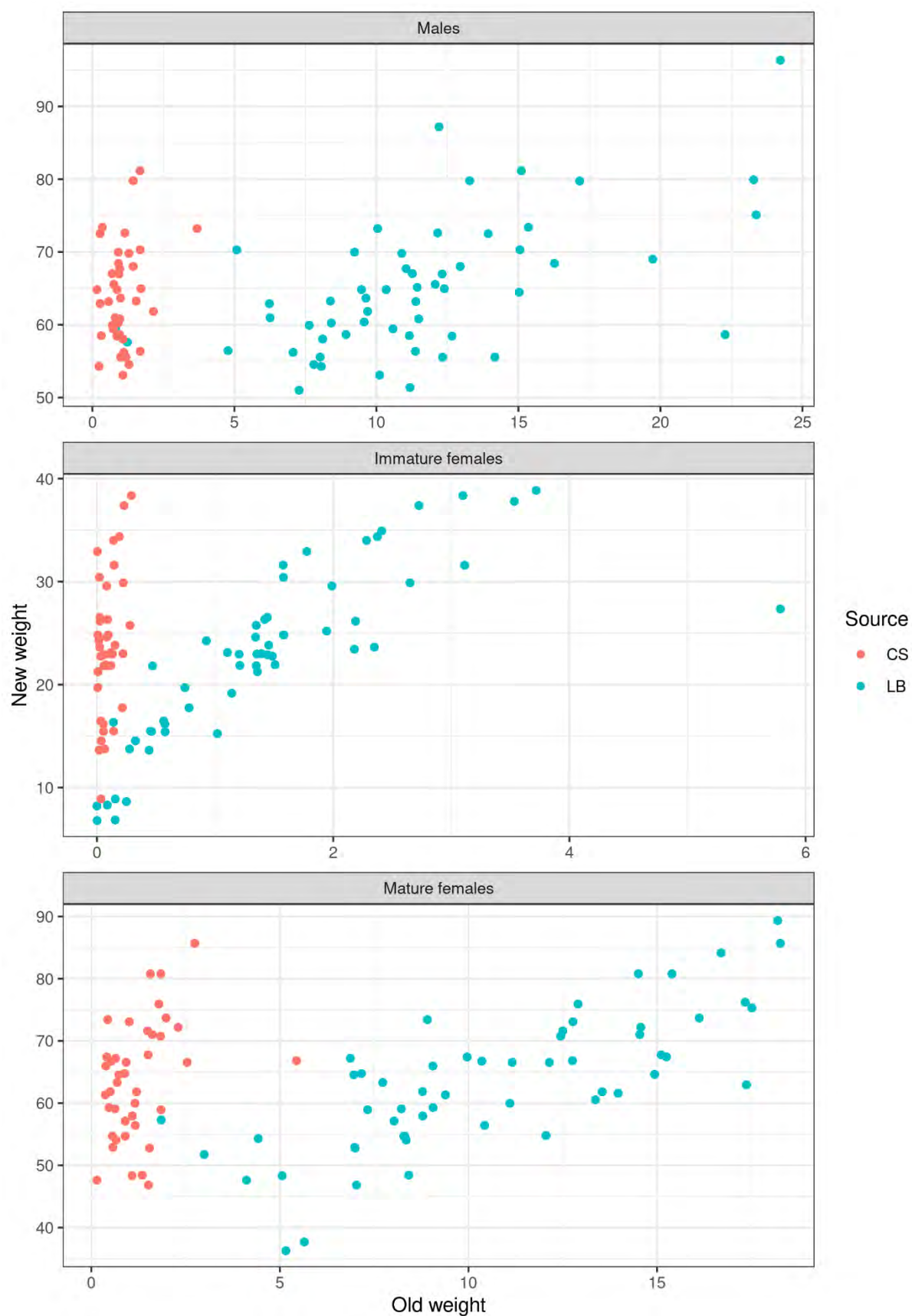


Figure 39: CRA 2: LF dataset weights by sex category and data source (CS = catch sampling, LB=logbook) generated using the previous methodology (Equation 4) and the new methodology (Equation 14).

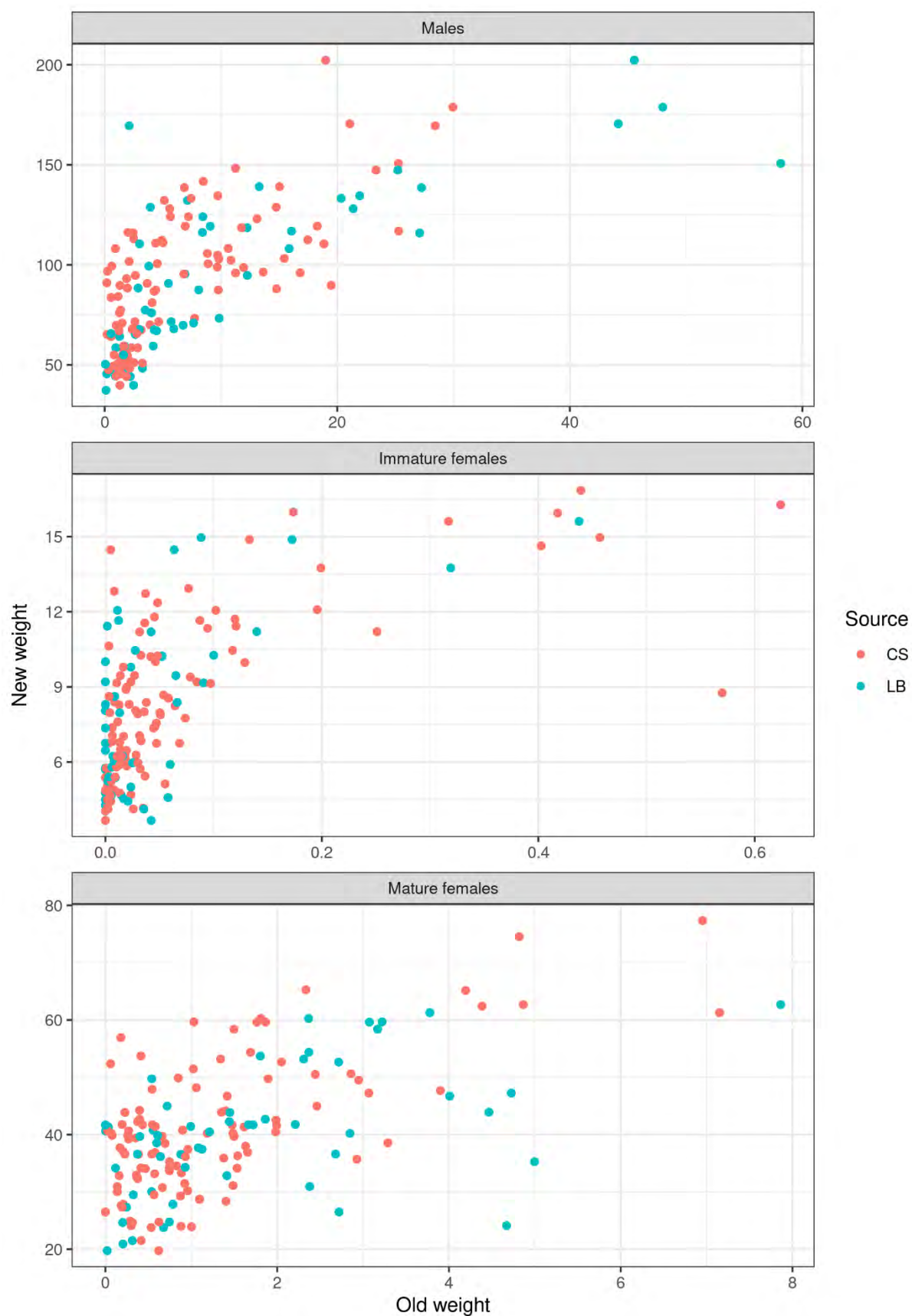


Figure 40: CRA 3: LF dataset weights by sex category and data source (CS = catch sampling, LB = logbook) generated using the old methodology (Equation 4) and the new methodology (Equation 14).

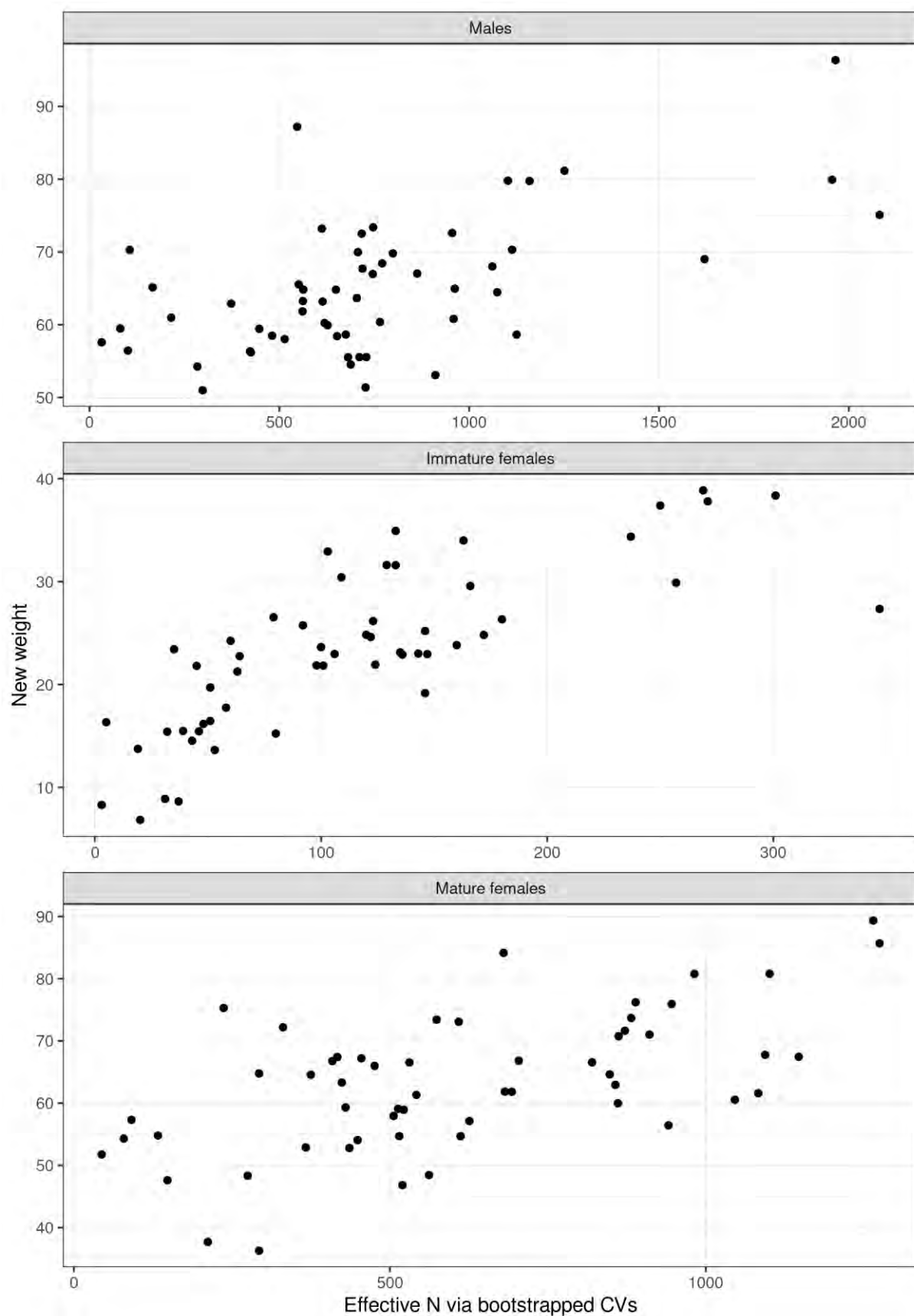


Figure 41: CRA 2: effective sample size by sex category generated using bootstrapping (Equation 15) and LF dataset weights generated using the new methodology (Equation 14).

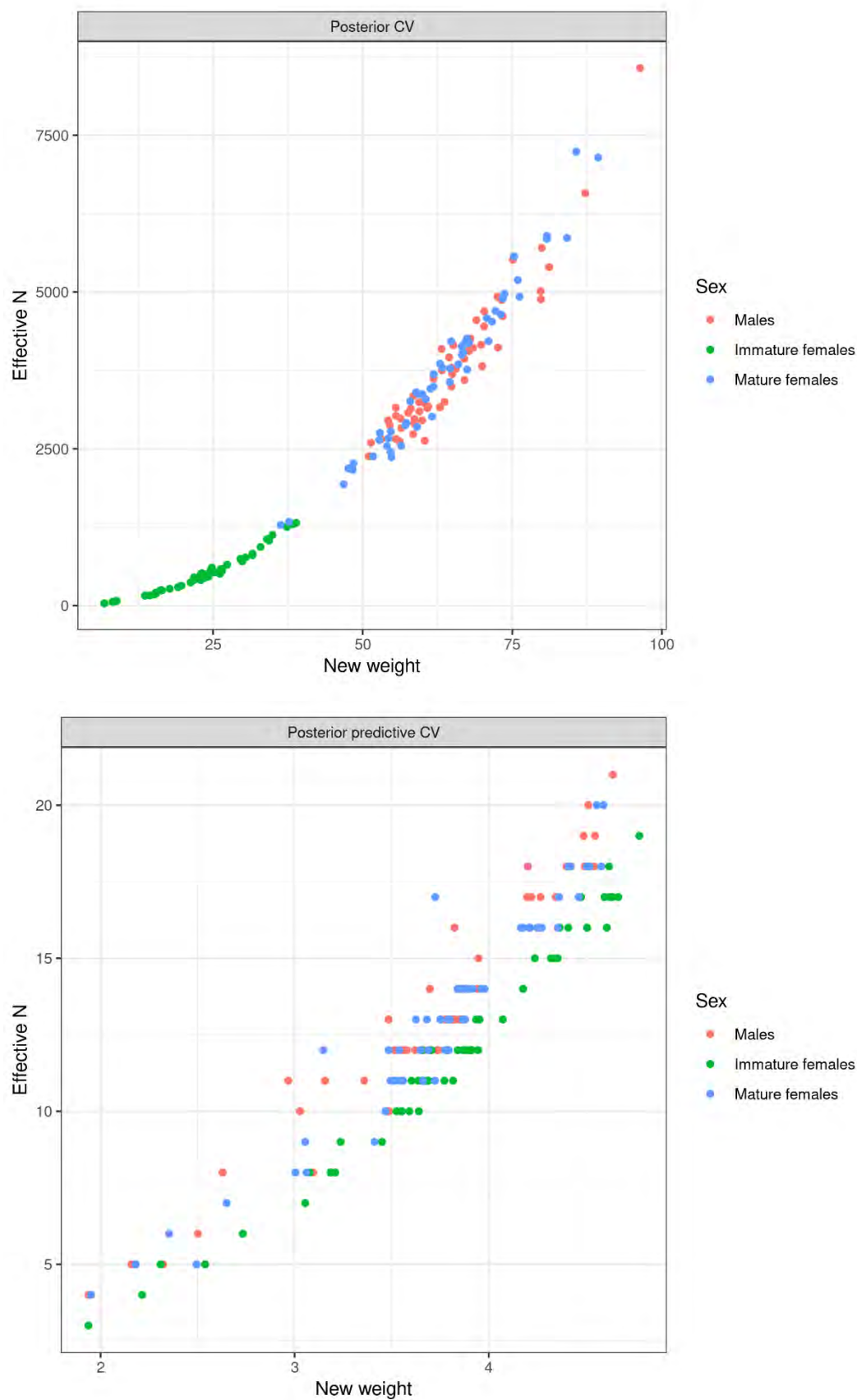


Figure 42: CRA 2: effective sample size by sex category generated using Equation 16 and LF dataset weights generated using the new methodology (Equation 14) derived from the posterior distribution [top] or the posterior predictive distribution [bottom].

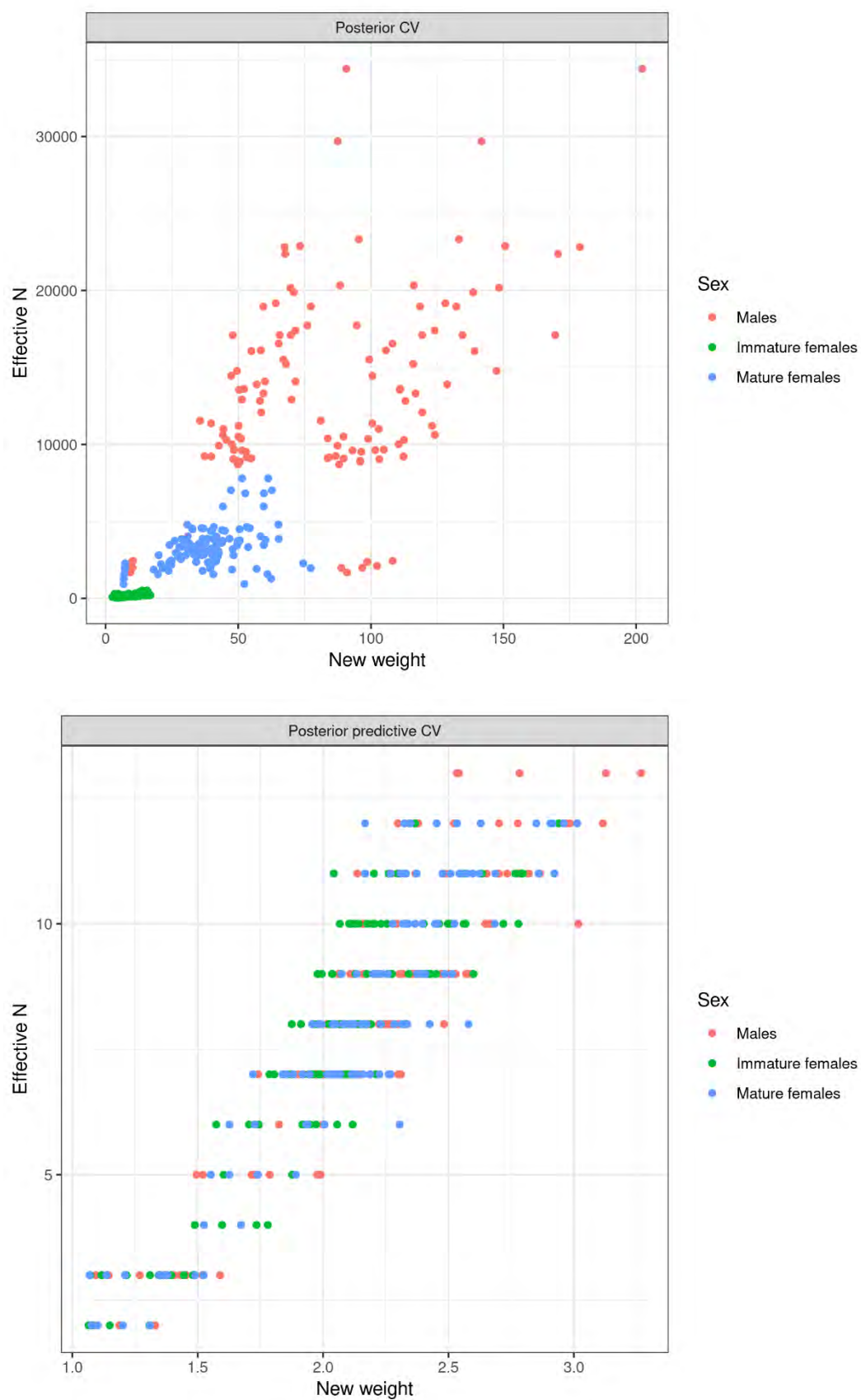


Figure 43: CRA 3: effective sample size by sex category generated using Equation 16 and LF dataset weights generated using the new methodology (Equation 14) derived from the posterior distribution [top] or the posterior predictive distribution [bottom].

9. APPENDIX I: DISTRIBUTIONS FOR LF DATA

Francis (2014) lists four desirable properties of a distribution used for composition data (i.e., length frequency or age frequency data):

1. it must allow substantial correlations;
2. it should be self-weighting;
3. it should be proper; and
4. it should be parsimonious.

He then identified several distributions that had some of these properties (see table 2 of Francis 2014). This table is repeated here for some of those distributions but an additional column has been added that provides the support for each distribution presented (Table 7). Each of the distributions in Table 7 are explored further below.

Table 7: A selection of likelihoods that have been used for fitting to length frequency or age frequency data, and their performance with respect to four desirable properties. The entries ‘Possibly’ and ‘Usually’ mean that the performance depends on how the likelihoods are implemented: ‘Possibly’ indicates that substantial correlations are possible, but not in current implementations; ‘Usually’ means that the likelihood is parsimonious in current implementations, but may not always be. ‘Partially’ indicates that the likelihood is not proper or self-weighting relative to all weighting parameters. This table was largely copied from Francis (2014).

Distribution	Support	Substantial correlations	Self-weighting	Proper	Parsimonious
Multinomial	$[0, \infty]$	No	No	No	Yes
Dirichlet	$(0,1)$	No	Yes	Yes	Yes
Dirichlet-multinomial	$[0, \infty]$	No	Partially	Partially	Yes
Logistic-normal	$(0,1)$	Possibly	Yes	Yes	Usually

9.1 Multinomial

The multinomial distribution is a generalisation of the **binomial** distribution. It describes the probability of obtaining a specific number of counts for each of k categories for n independent trials. When $k = 2$ and $n = 1$, the multinomial distribution is the **Bernoulli** distribution. When $k = 2$ and $n > 1$, it is the binomial distribution. When $k > 2$ and $n = 1$, it is the **categorical** distribution. The multinomial distribution has parameters

$$p_1, \dots, p_k \text{ where } \sum_{i=1}^k p_i = 1 \text{ and } p_i \geq 0$$

The probability mass function is

$$f(x_1, \dots, x_k; p_1, \dots, p_k) = \frac{n!}{x_1! \dots x_k!} p_1^{x_1} \dots p_k^{x_k} = \frac{\Gamma(\sum_{i=1}^k x_i + 1)}{\prod_{i=1}^k \Gamma(x_i + 1)} \prod_{i=1}^k p_i^{x_i}$$

The second form shows its resemblance to the **Dirichlet** distribution (Section 9.2), which is its conjugate prior. From this it is clear that the multinomial distribution is for categorical outcomes (e.g., counts for each side of a dice roll) and not for continuous data (e.g., counts/proportions of ages of fish or counts/proportions within length classes of shellfish). Despite this, the multinomial is a popular choice in fisheries research and may still be a reasonable choice when fitting to count data.

A data set of 1000 draws from a multinomial distribution using a fixed proportion $\mathbf{p}$ and n of 10, 100, and 1000 was generated using

$$\mathbf{x} \sim \frac{1}{n} \text{multinomial}(n, \mathbf{p})$$

and these draws are presented in Figure 44.

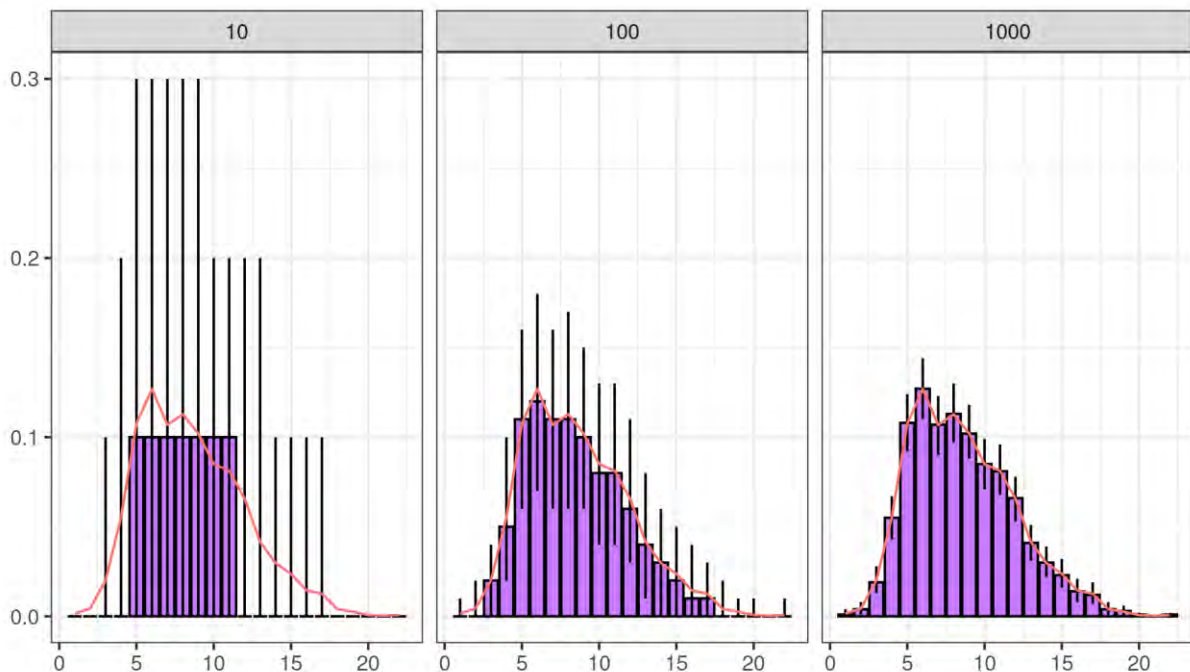


Figure 44: Median and 90% credible intervals of 1000 draws from a multinomial distribution with $n = 10$, $n = 100$, or $n = 1000$ and proportion in red.

9.2 Dirichlet

The Dirichlet distribution is a multivariate generalisation of the **beta** distribution. It is a family of continuous multivariate probability distributions parameterised by a vector α of positive reals. Dirichlet distributions are commonly used as prior distributions in Bayesian statistics, and in fact the Dirichlet distribution is the conjugate prior of the **categorical** distribution and **multinomial** distribution. The Dirichlet distribution of order $k \geq 2$ with parameters $\alpha_1, \dots, \alpha_k > 0$ has a probability density function

$$f(x_1, \dots, x_k; \alpha_1, \dots, \alpha_k) = \frac{\Gamma(\sum_{i=1}^k \alpha_i)}{\prod_{i=1}^k \Gamma(\alpha_i)} \prod_{i=1}^k x_i^{\alpha_i-1}$$

where $\{x_i\}_{i=1}^{i=k}$ belong to the standard $k - 1$ simplex, or

$$\sum_{i=1}^k x_i = 1 \text{ and } x_i \geq 0 \text{ for all } i \in \{1, \dots, k\}$$

The support of the Dirichlet distribution is the set of k -dimensional vectors $\mathbf{x}$ whose entries are real numbers in the interval $(0,1)$ such that $\|\mathbf{x}\|_1 = 1$ (i.e., the sum of the vector is equal to 1).

A data set of 1000 draws from a Dirichlet distribution using a fixed proportion $\mathbf{p}$ and n of 10, 100, and 1000 was generated using

$$\mathbf{x} \sim \text{Dirichlet}(\alpha = n\mathbf{p})$$

and these draws are presented in Figure 45.

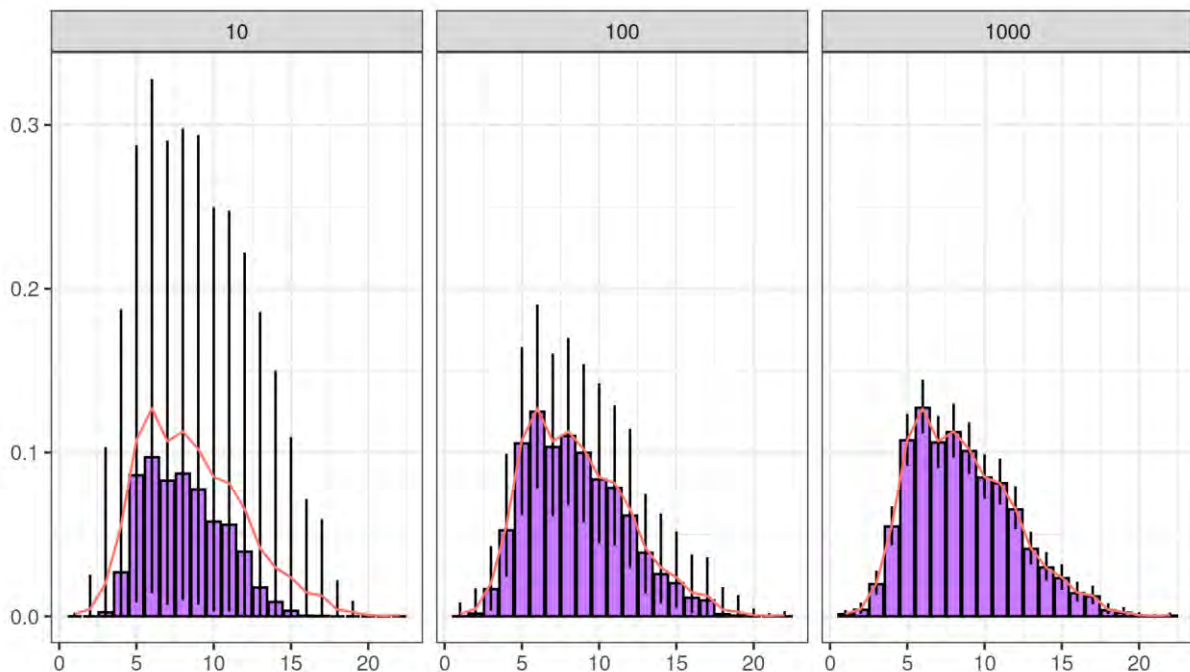


Figure 45: Median and 90% credible intervals of 1000 draws from a Dirichlet distribution with $n = 10$, $n = 100$, or $n = 1000$ and proportion in red.

9.3 Dirichlet-multinomial

In a model where a **Dirichlet** prior distribution is placed over a set of categorical-valued observations, the marginal joint distribution of the observations (i.e., the joint distribution of the observations, with the prior parameter marginalised out) is a Dirichlet-multinomial distribution. This distribution plays an important role in hierarchical Bayesian models, because when doing inference over such models using methods such as Gibbs sampling or variational Bayes, Dirichlet prior distributions are often marginalised out.

A data set of 1000 draws from a Dirichlet-multinomial distribution using a fixed proportion $\mathbf{p}$ and n of 10, 100, and 1000 was generated using

$$\mathbf{y} \sim \text{Dirichlet}(\boldsymbol{\alpha} = n\mathbf{p})$$

$$\mathbf{x} \sim \frac{1}{n} \text{multinomial}(n, \mathbf{y})$$

and these draws are presented in Figure 46.

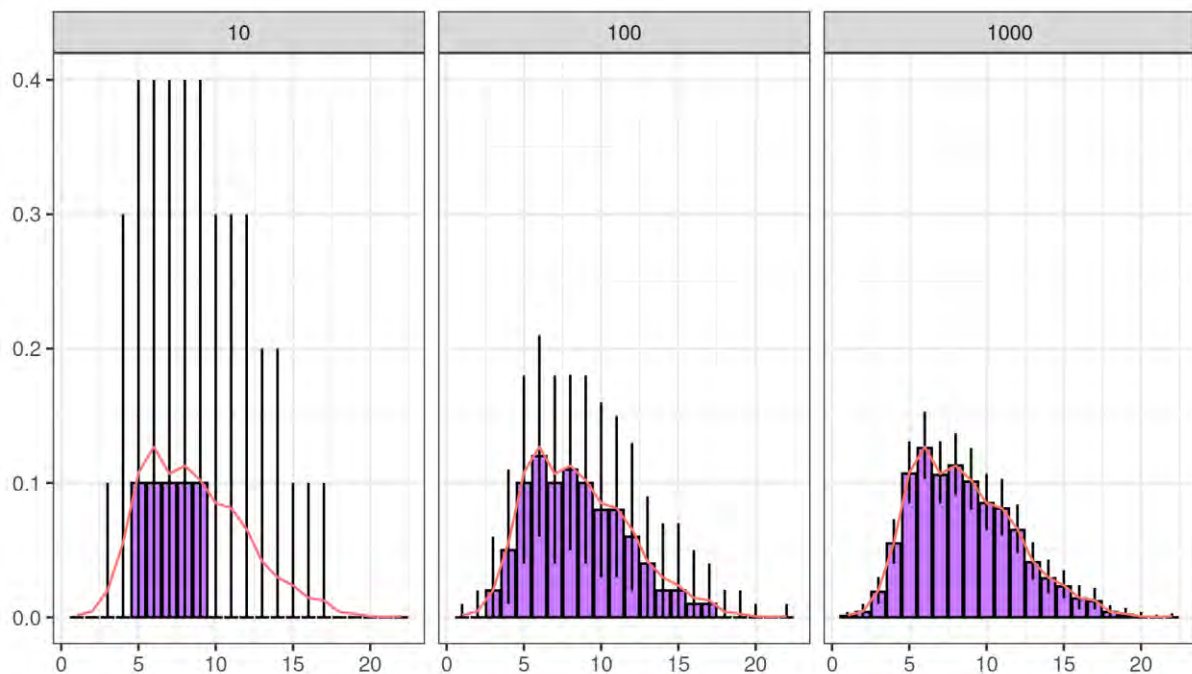


Figure 46: Median and 90% credible intervals of 1000 draws from a Dirichlet-multinomial distribution with $n = 10$, $n = 100$, or $n = 1000$ and proportion in red.

9.4 Logistic-normal

The logistic-normal distribution is a generalisation of the logit-normal distribution, being composed of a logistic transformation applied to a D -dimensional multivariate normal distribution (Aitchison & Shen 1980). The logistic-normal distribution is a more flexible alternative to the Dirichlet distribution in that it can capture correlations between components of probability vectors.

A data set of 1000 draws from a logistic-normal distribution using a fixed proportion $\mathbf{p}$ and σ of 1, 0.1, and 0.01 was generated using

$$\mathbf{y} \sim N_D(\log(\mathbf{p}), \Sigma) \text{ where } \Sigma = \sigma I_D$$

$$\mathbf{x} = \frac{e^{\mathbf{y}}}{\sum_{i=1}^D e^{y_i}}$$

and these draws are presented in (Figure 47).

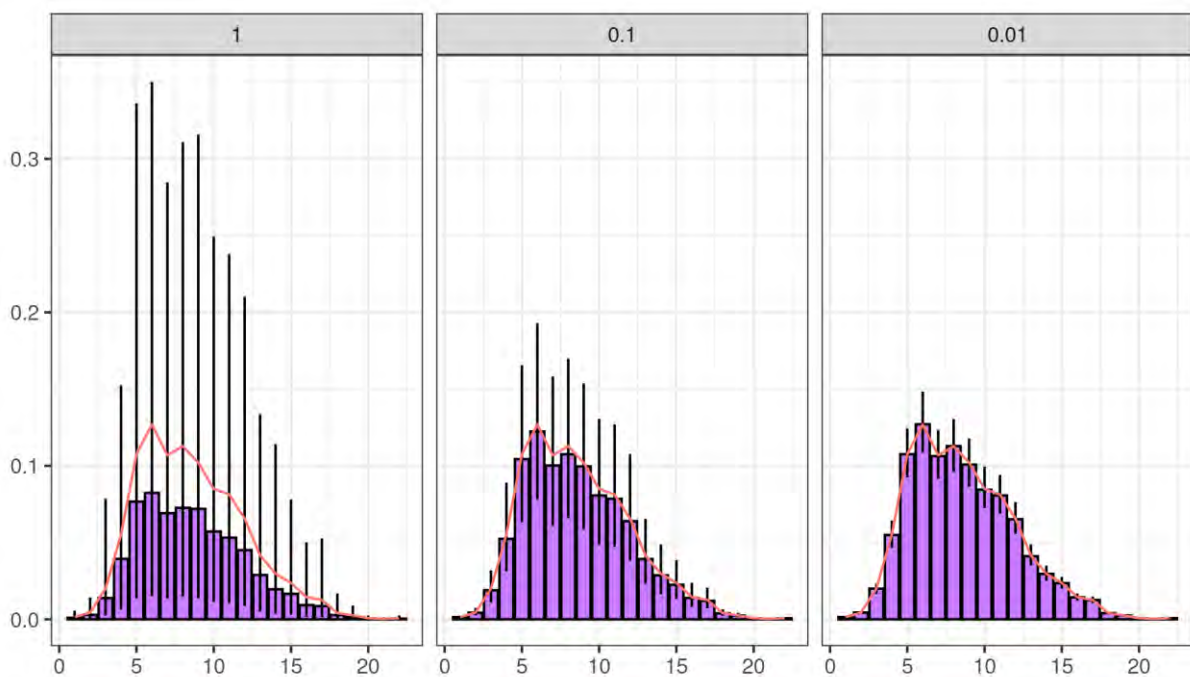


Figure 47: Median and 90% credible intervals of 1000 draws from a logistic-normal distribution with $\sigma = 10$, $\sigma = 100$, or $\sigma = 1000$ and proportion in red.

10. APPENDIX II: MCMC DIAGNOSTICS

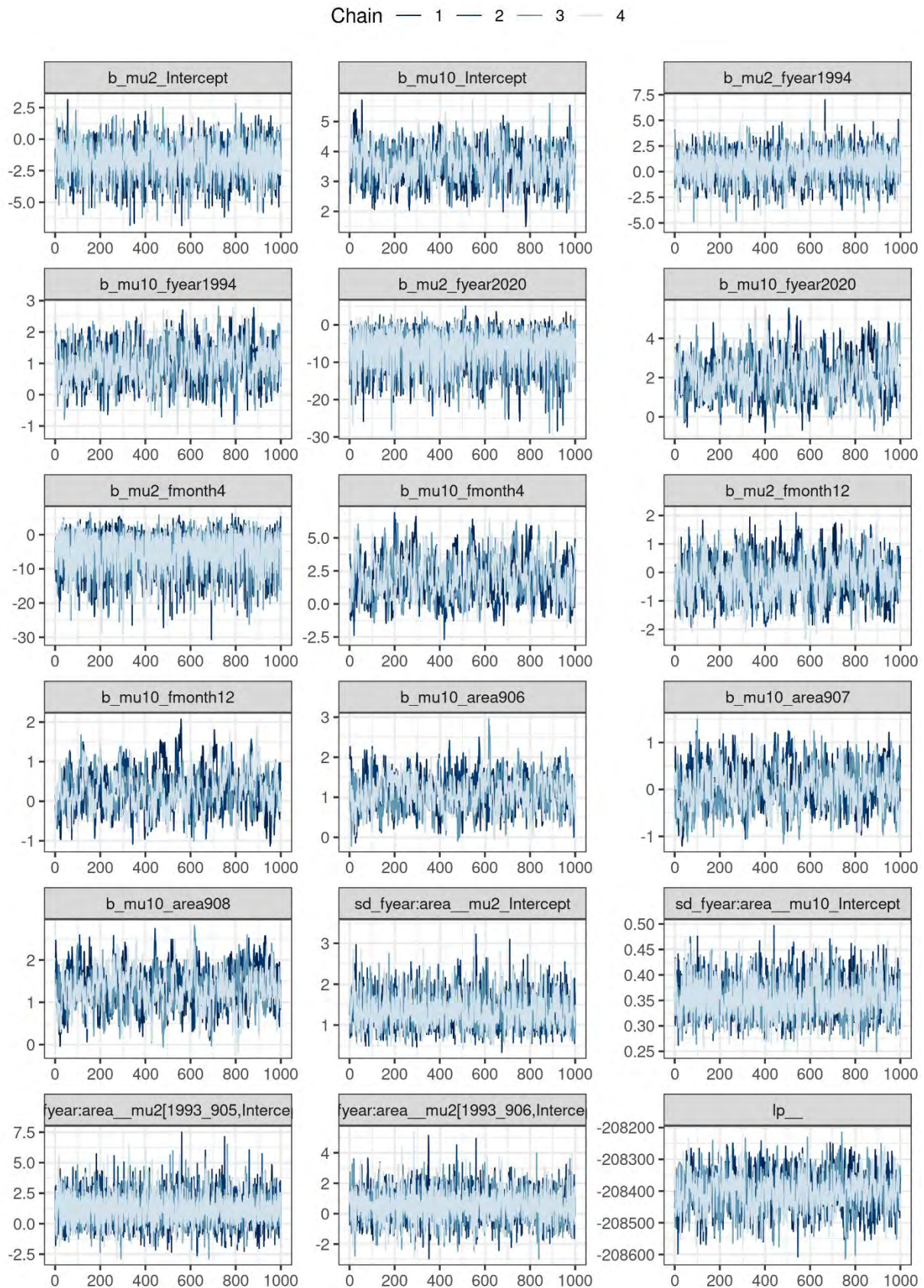


Figure 48: CRA 2 males: MCMC traces for a selection of parameters.

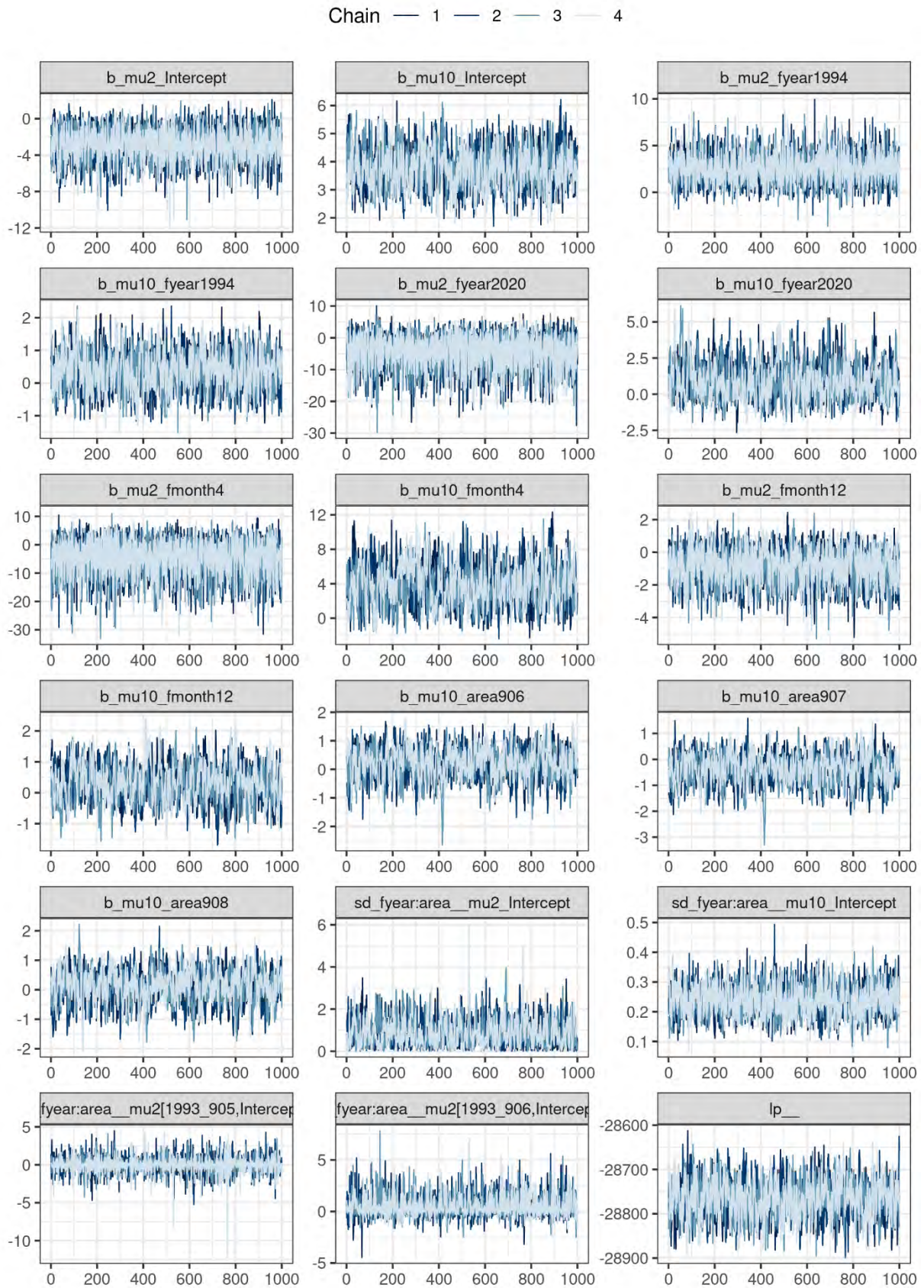


Figure 49: CRA 2 immature females: MCMC traces for a selection of parameters.

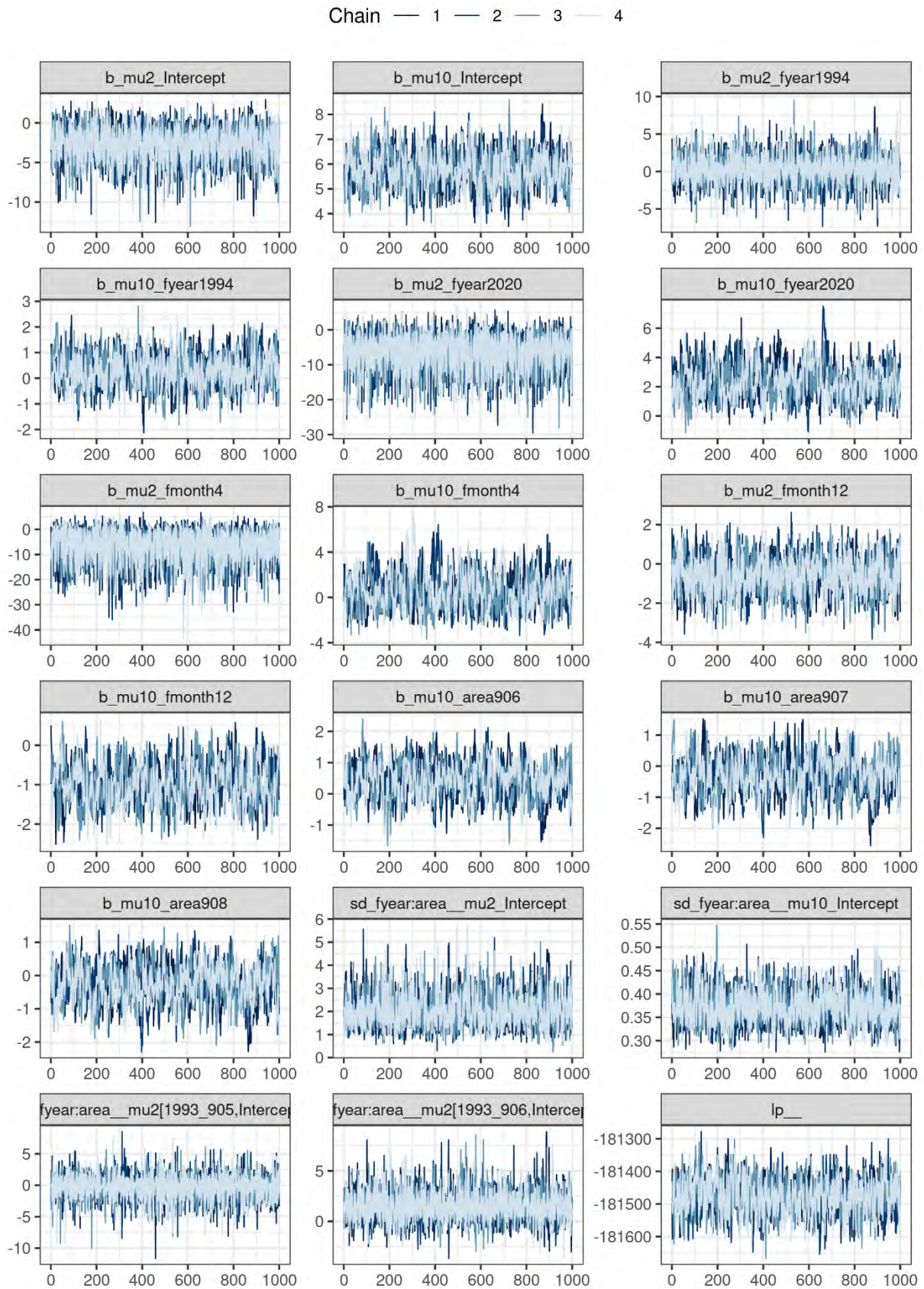


Figure 50: CRA 2 mature females: MCMC traces for a selection of parameters.

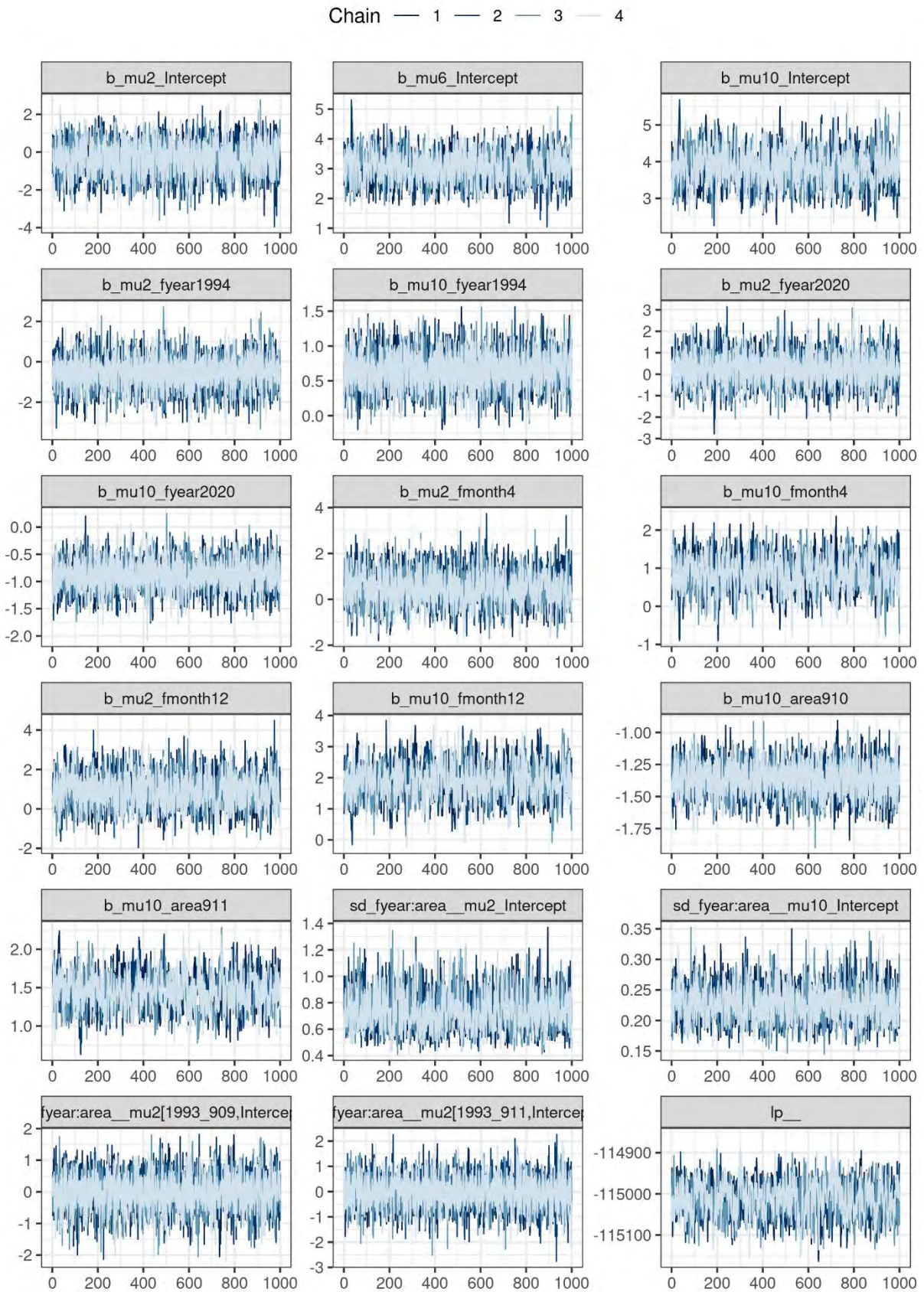


Figure 51: CRA 3 males: MCMC traces for a selection of parameters.

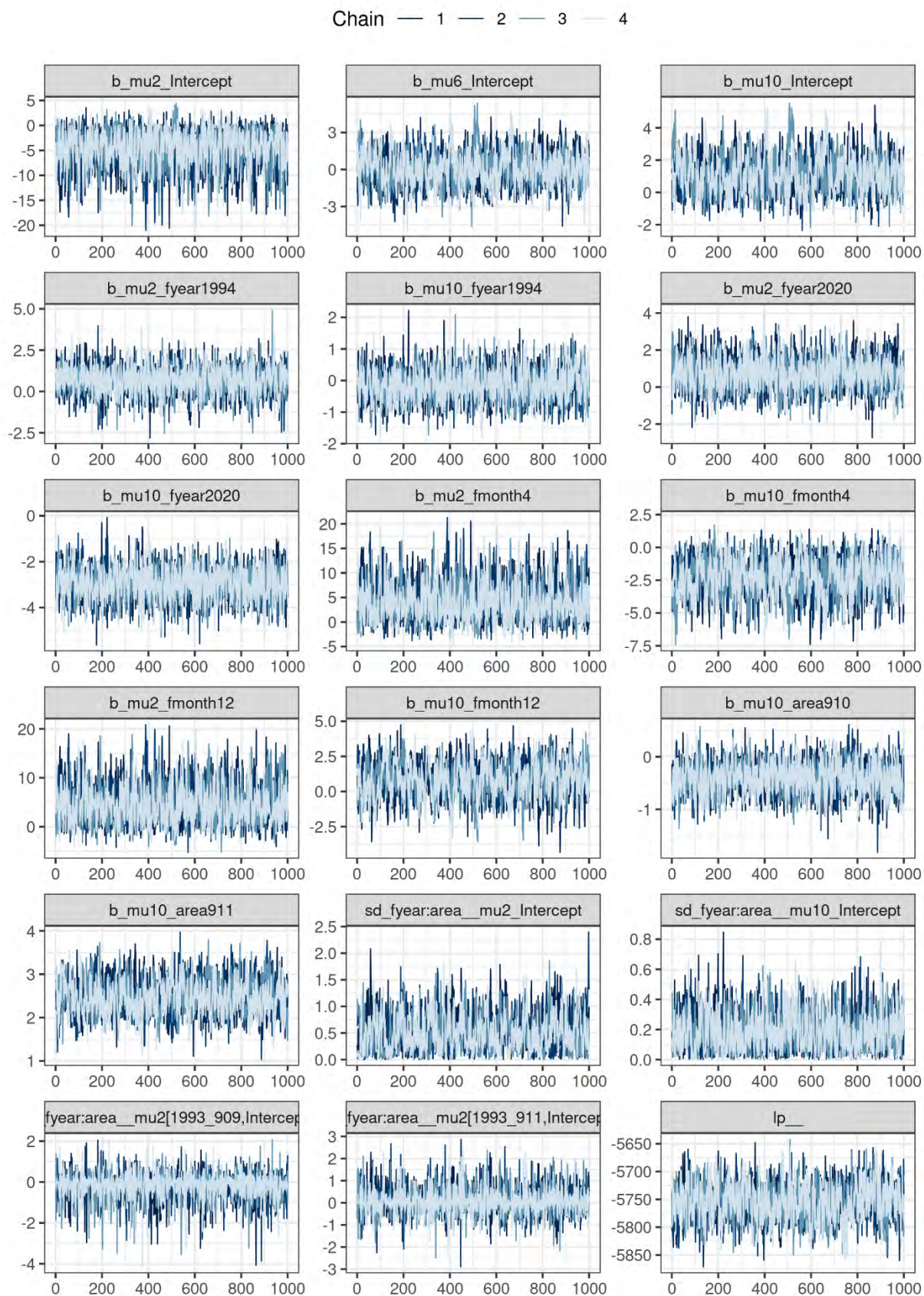


Figure 52: CRA 3 immature females: MCMC traces for a selection of parameters.

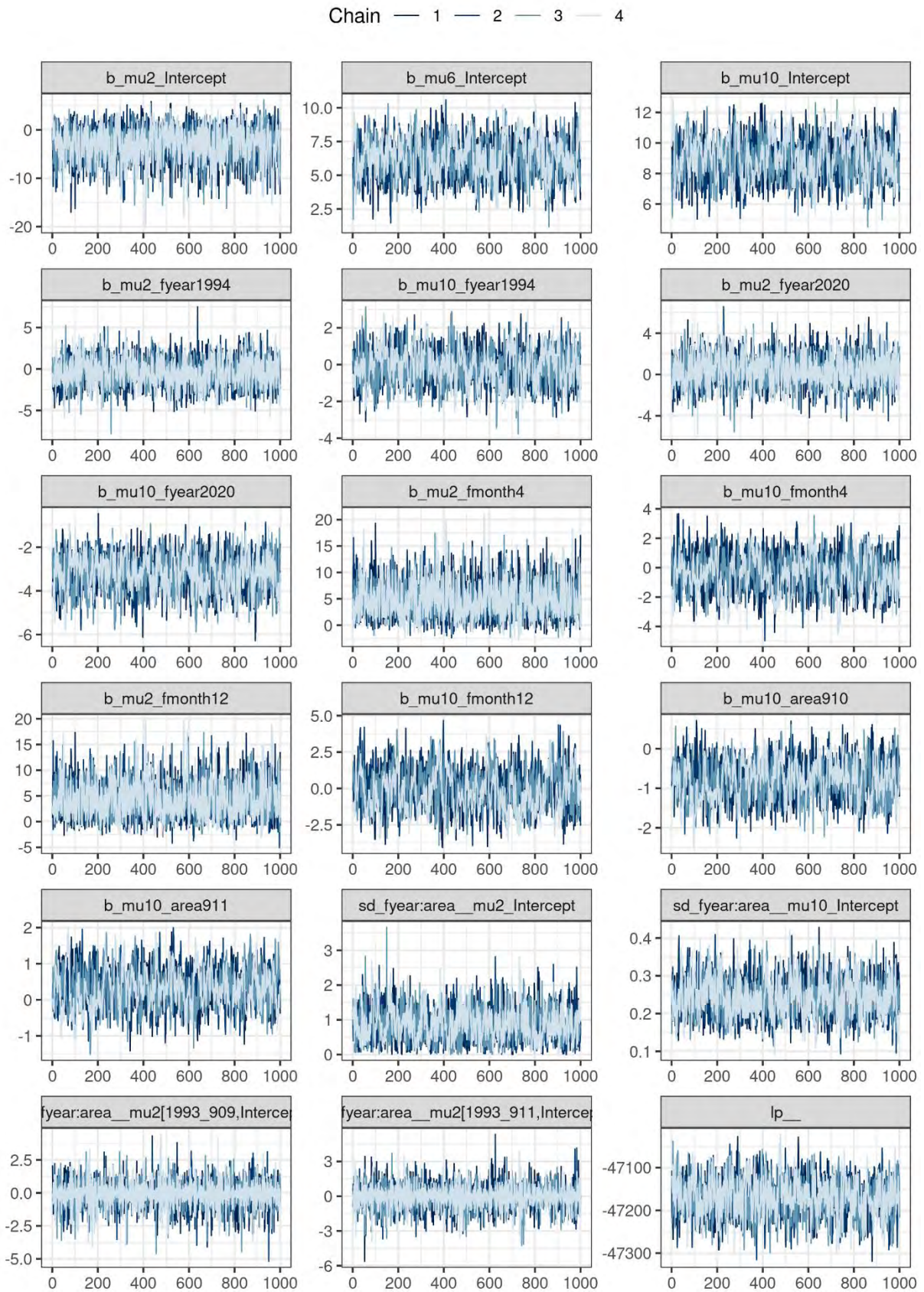


Figure 53: CRA 3 mature females: MCMC traces for a selection of parameters.

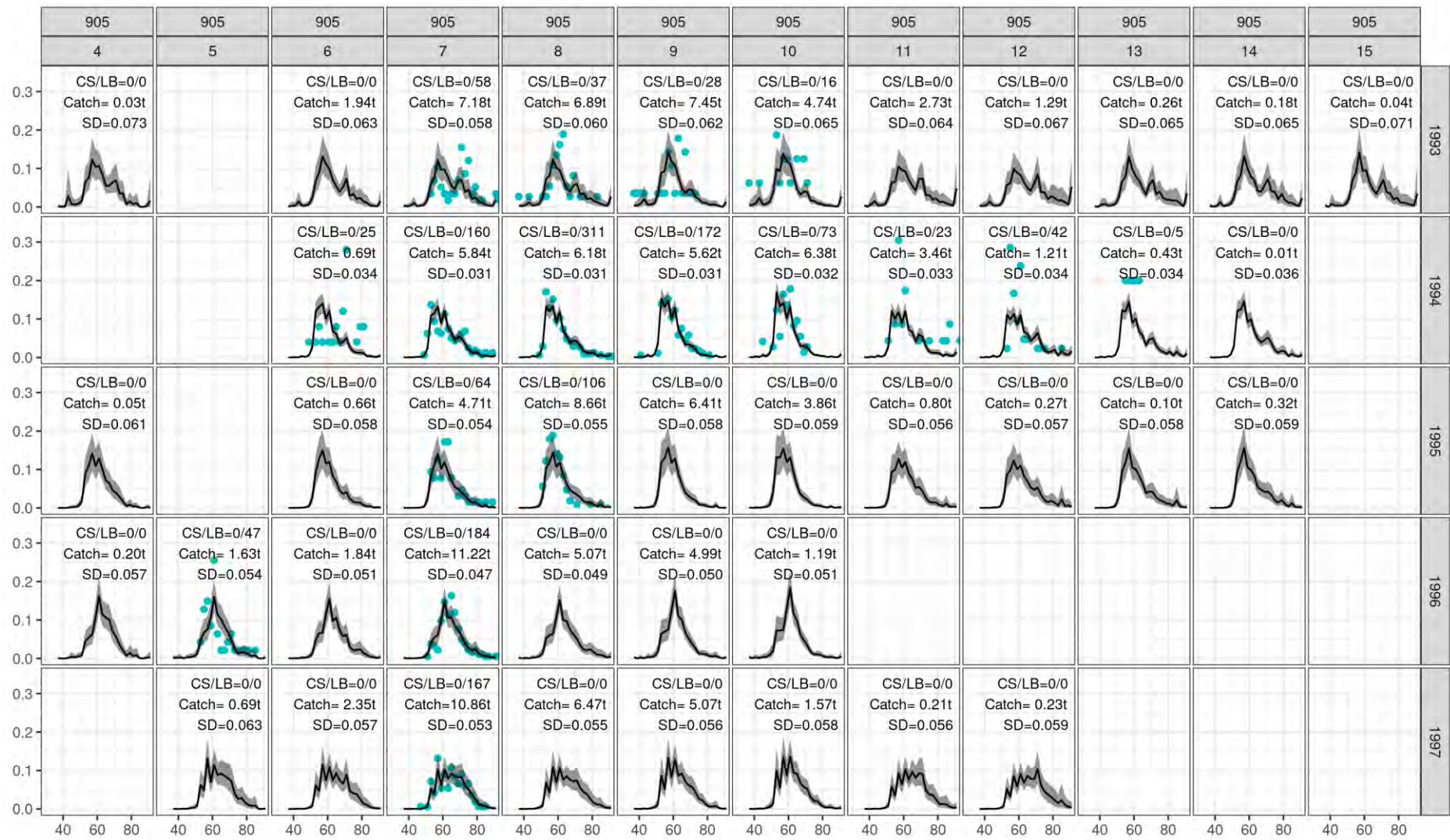
11. APPENDIX III: LF PREDICTIONS

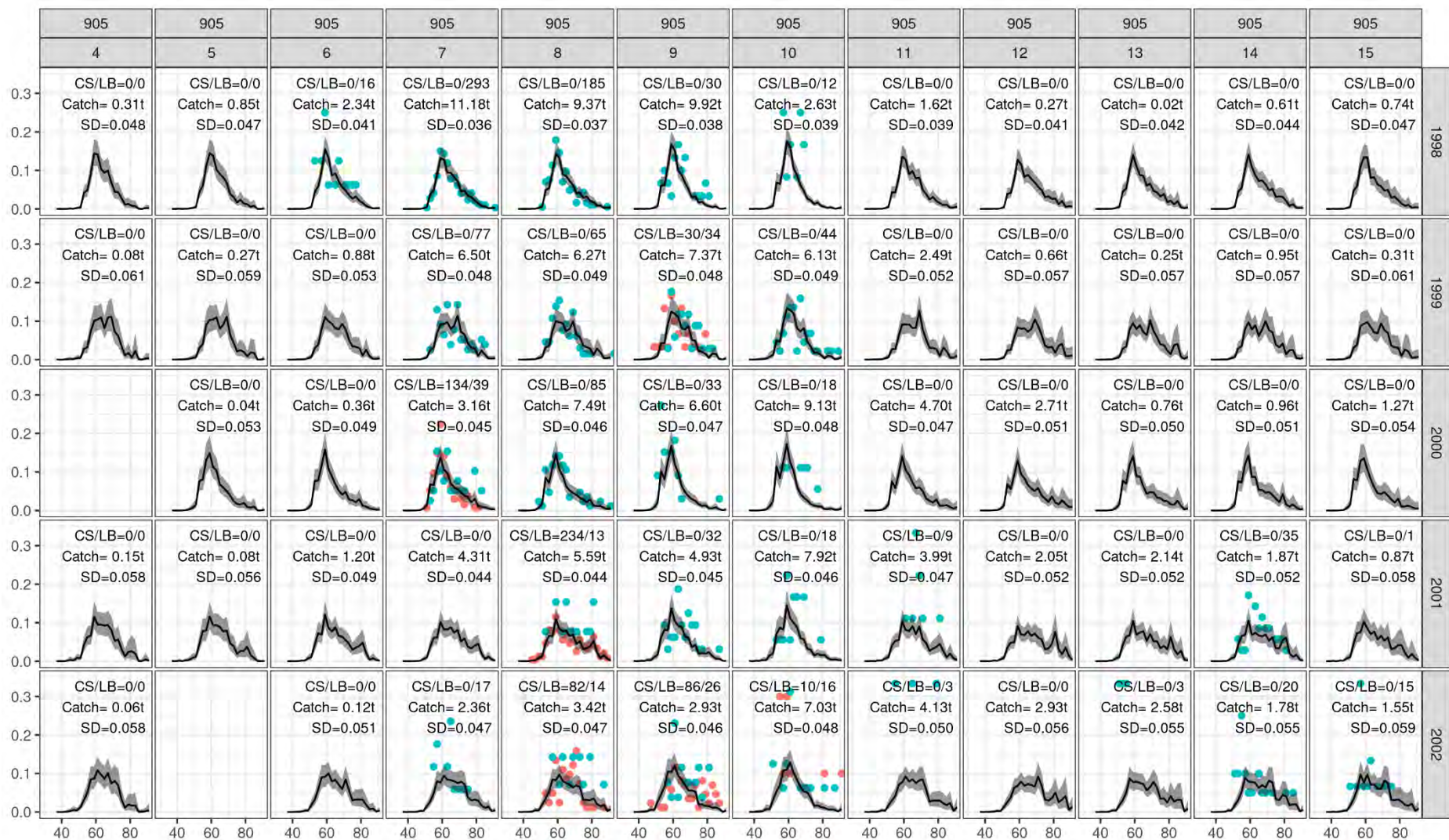
The figures in this appendix show the LF data aggregated by fishing year, month (4 = April, 5 = May, 6 = June, 7 = July, 8 = August, 9 = September, 10 = October, 11 = November, 12 = December, 13 = January, 14 = February, 15 = March), statistical area (CRA 2 includes 905, 906, 907, and 908; and CRA 3 includes 909, 910, and 911), data source (CS = catch sampling in red, LB = logbook in blue), and TW bin for each QMA and sex category (Equation 9). Predictions of the LF are also shown where the solid line is the mean and the shaded region is the 95% credible interval. Each panel also includes text which provides:

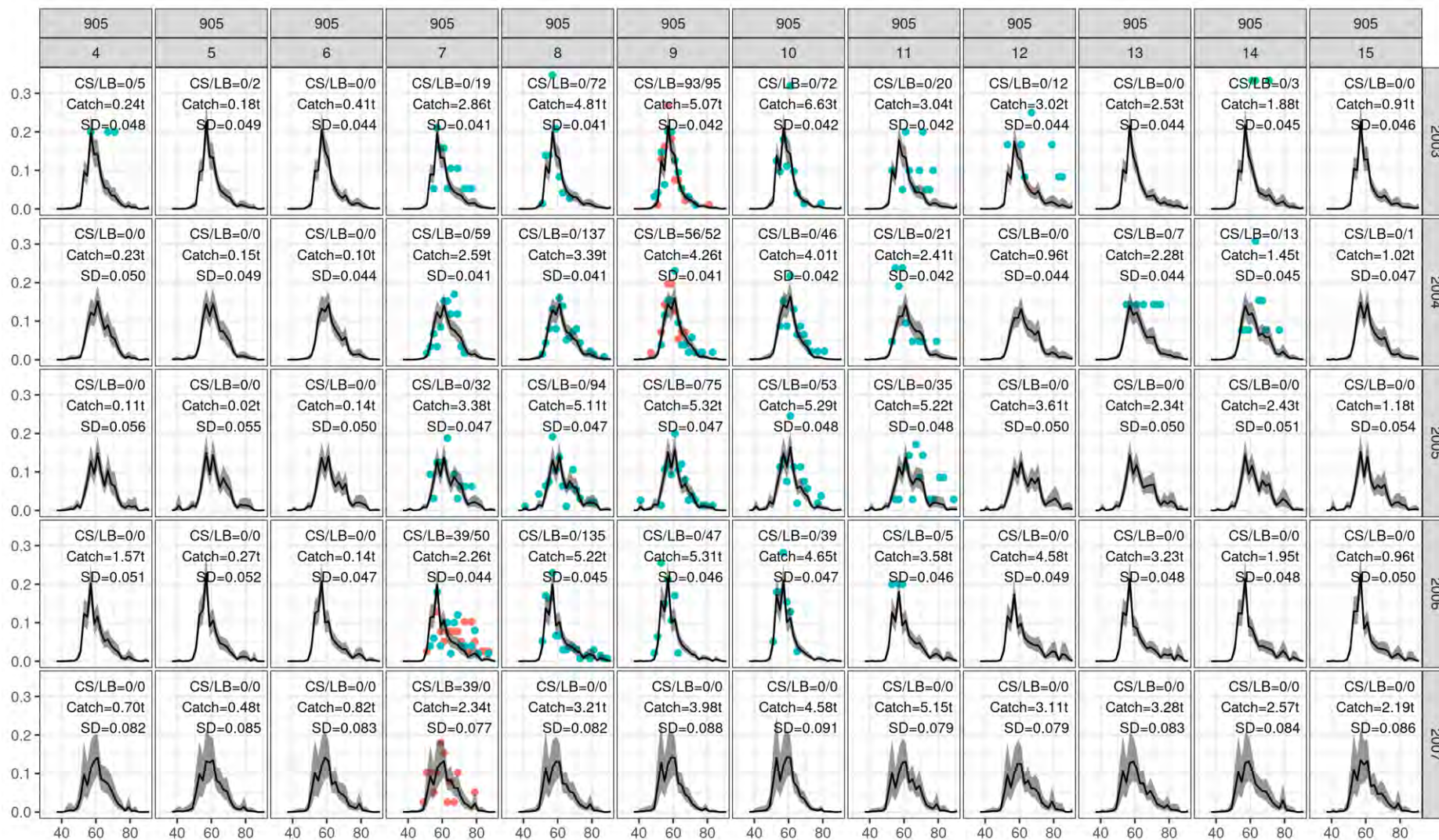
- the number of lobsters measured in the CS and LB programmes;
- the catch (tonnes) associated with each stratum; and
- the SD of the predicted LF for each stratum (Equation 11).

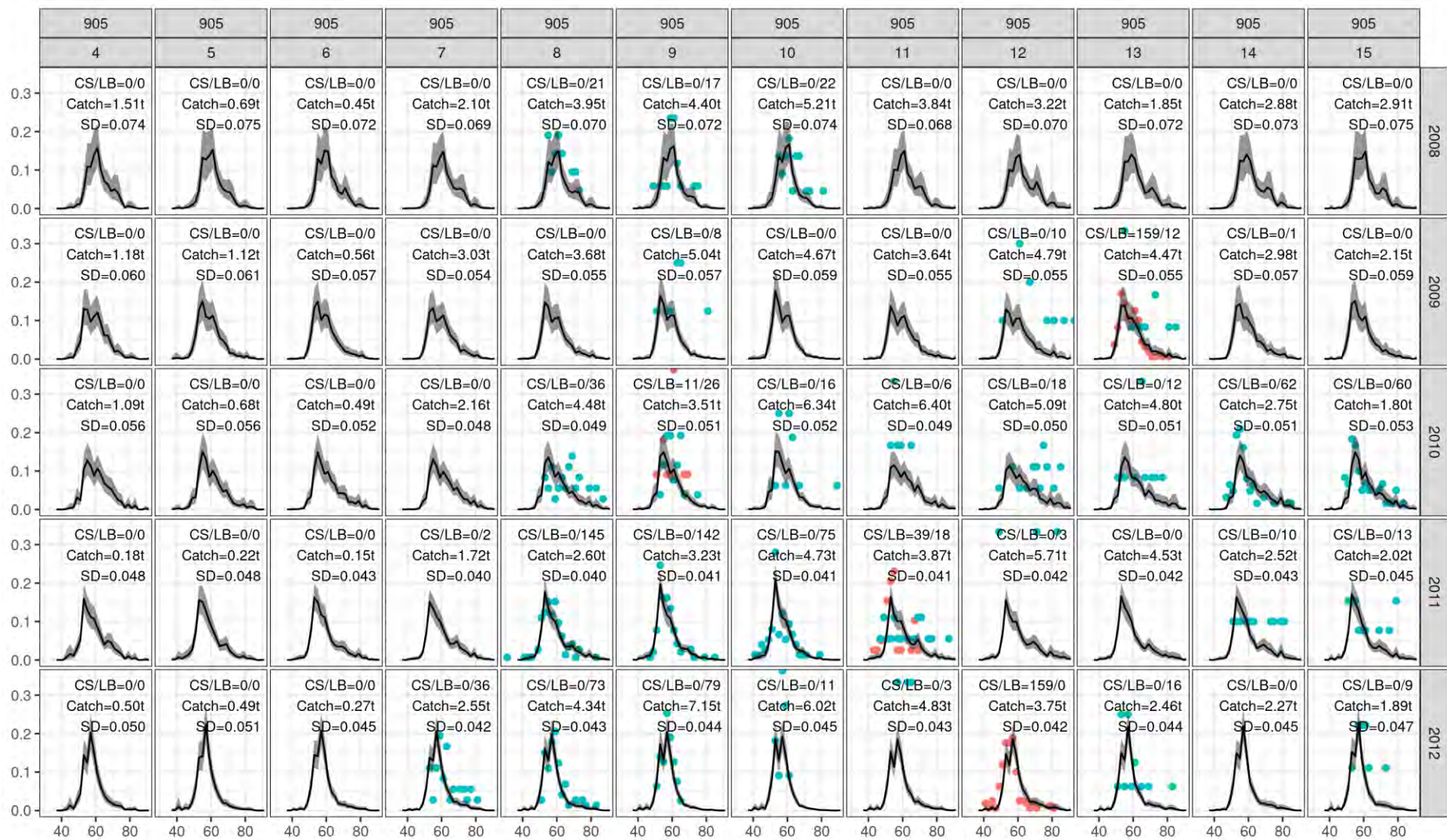
A prediction is not provided if the number of lobsters measured by the CS/LB and the catch was zero.

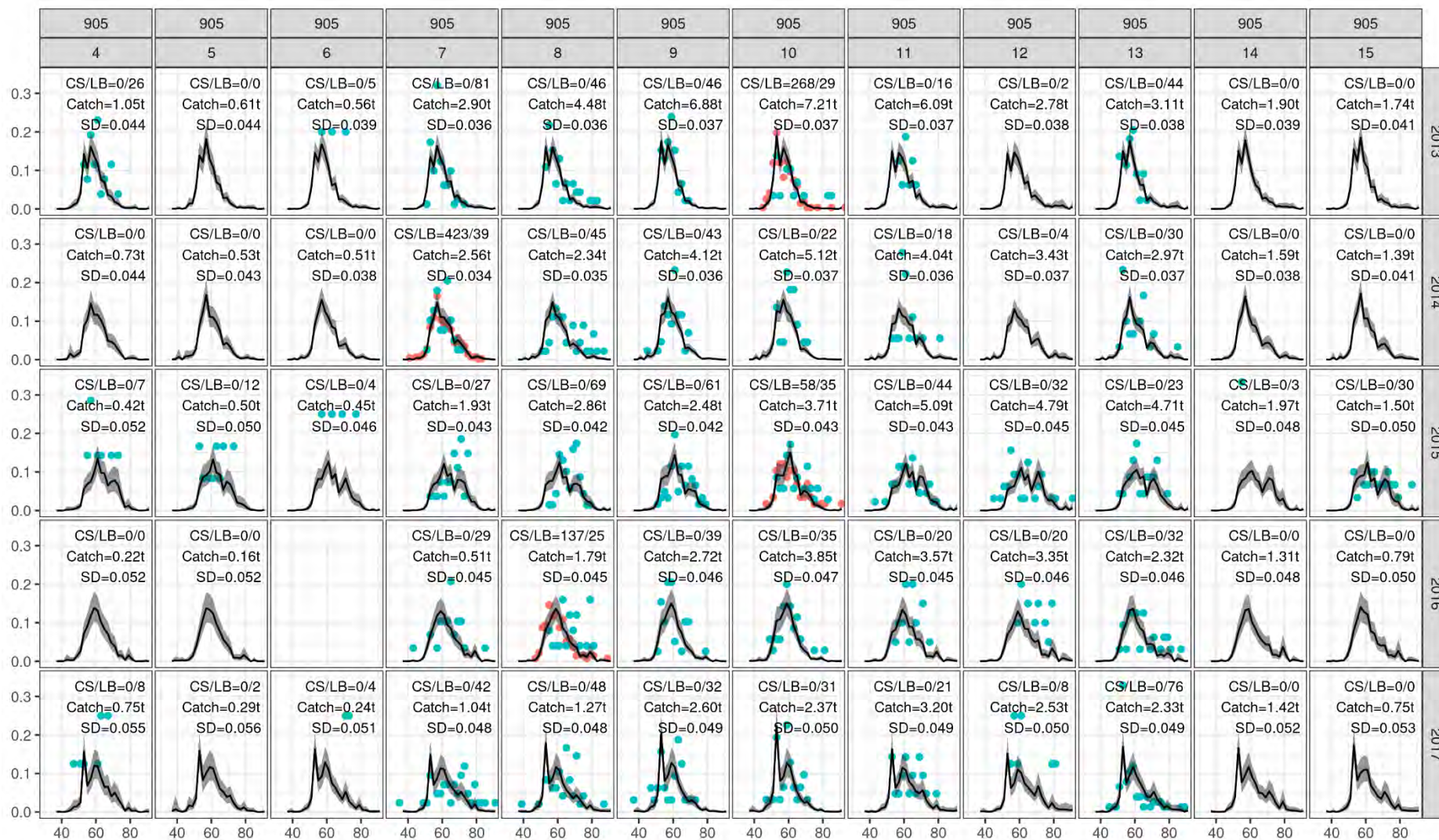
11.1 CRA 2 males

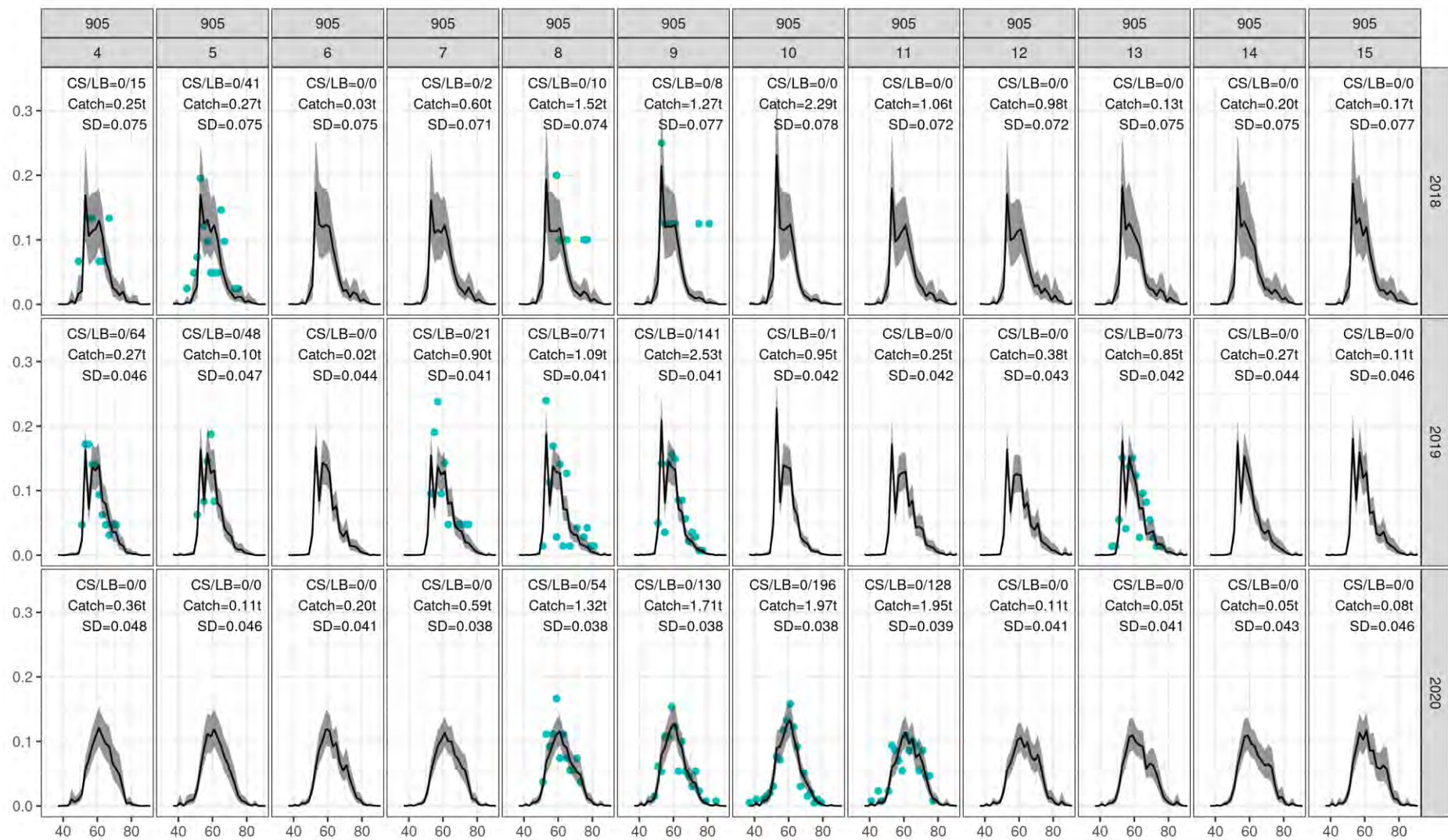


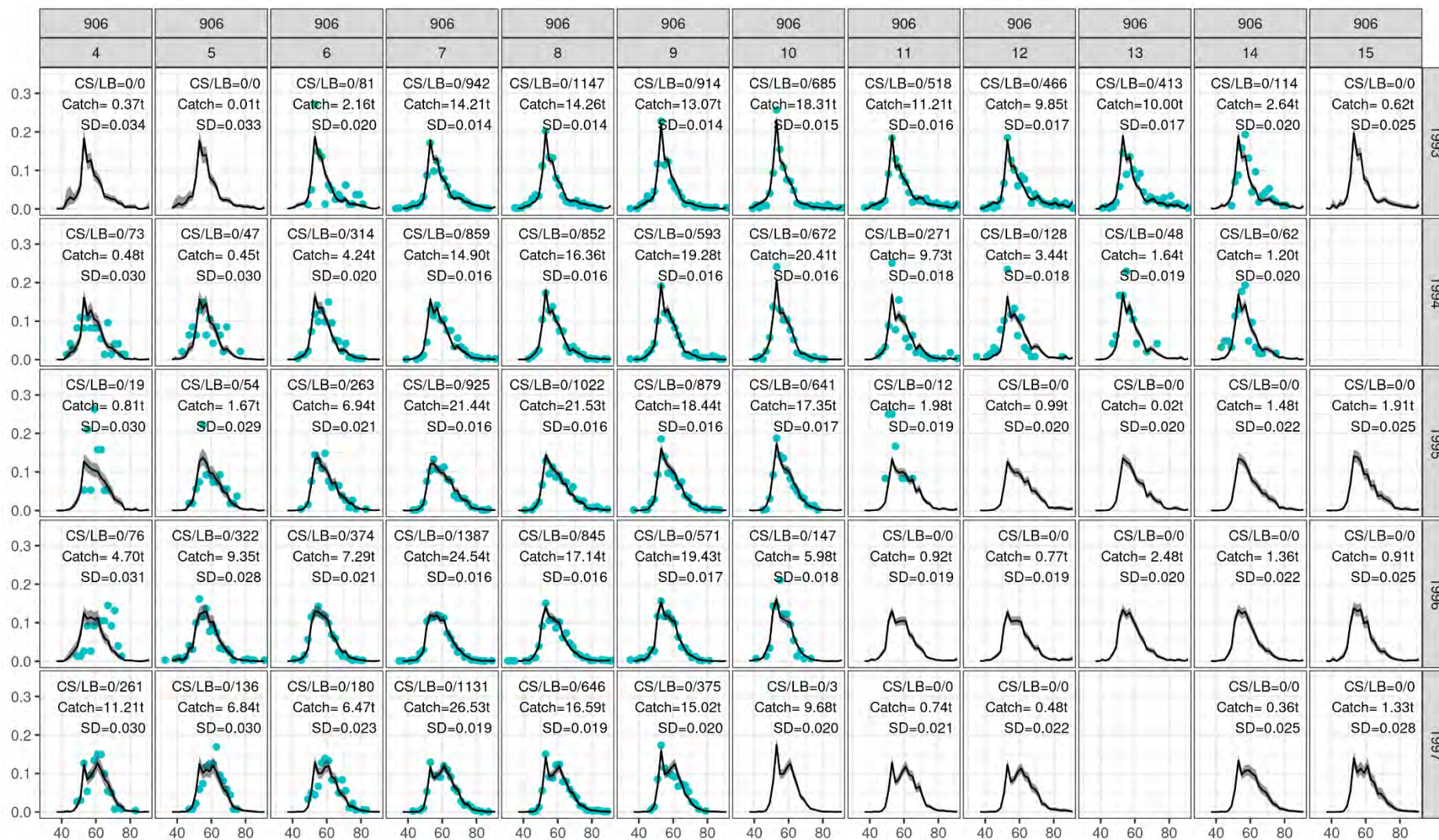


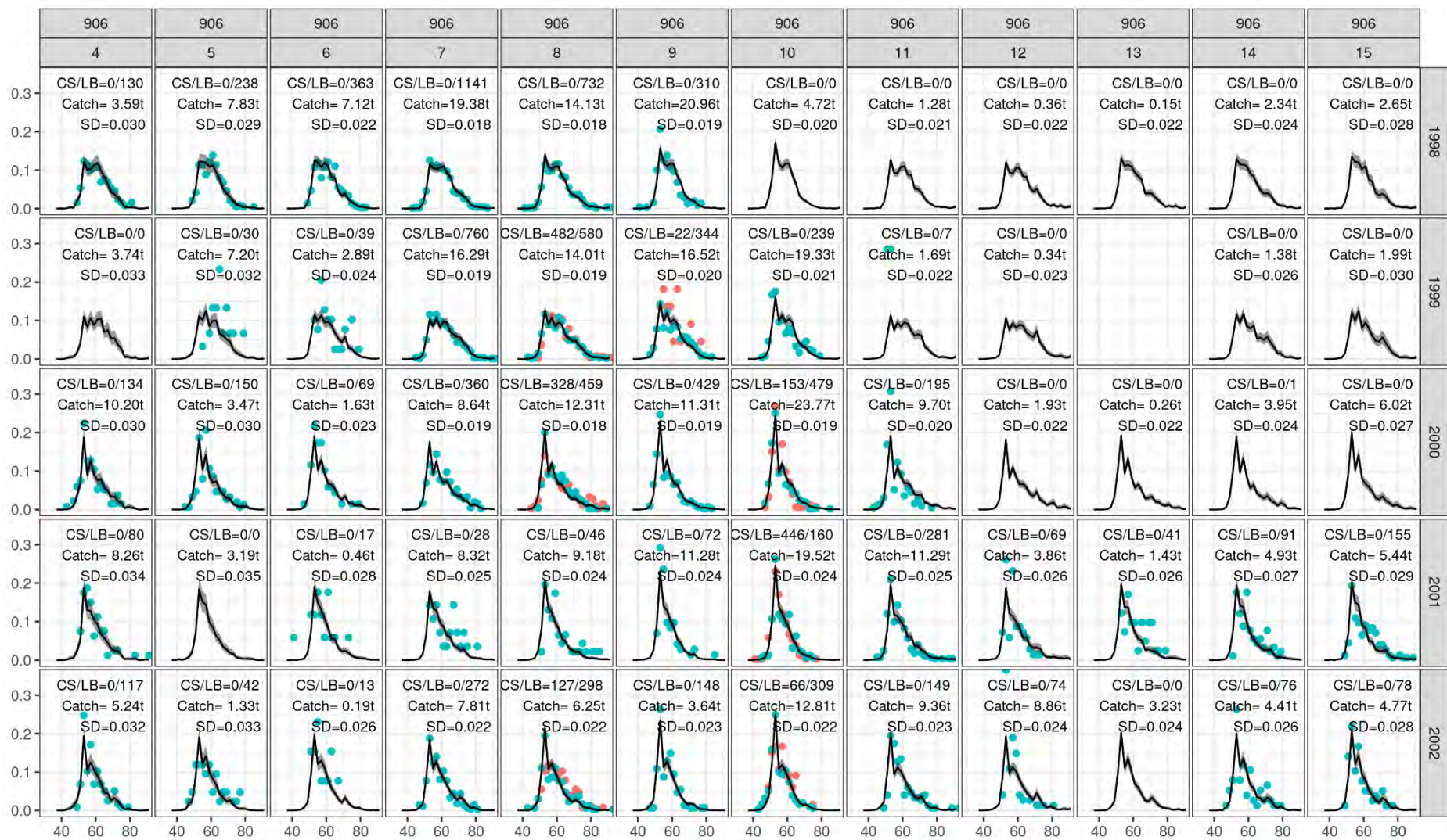


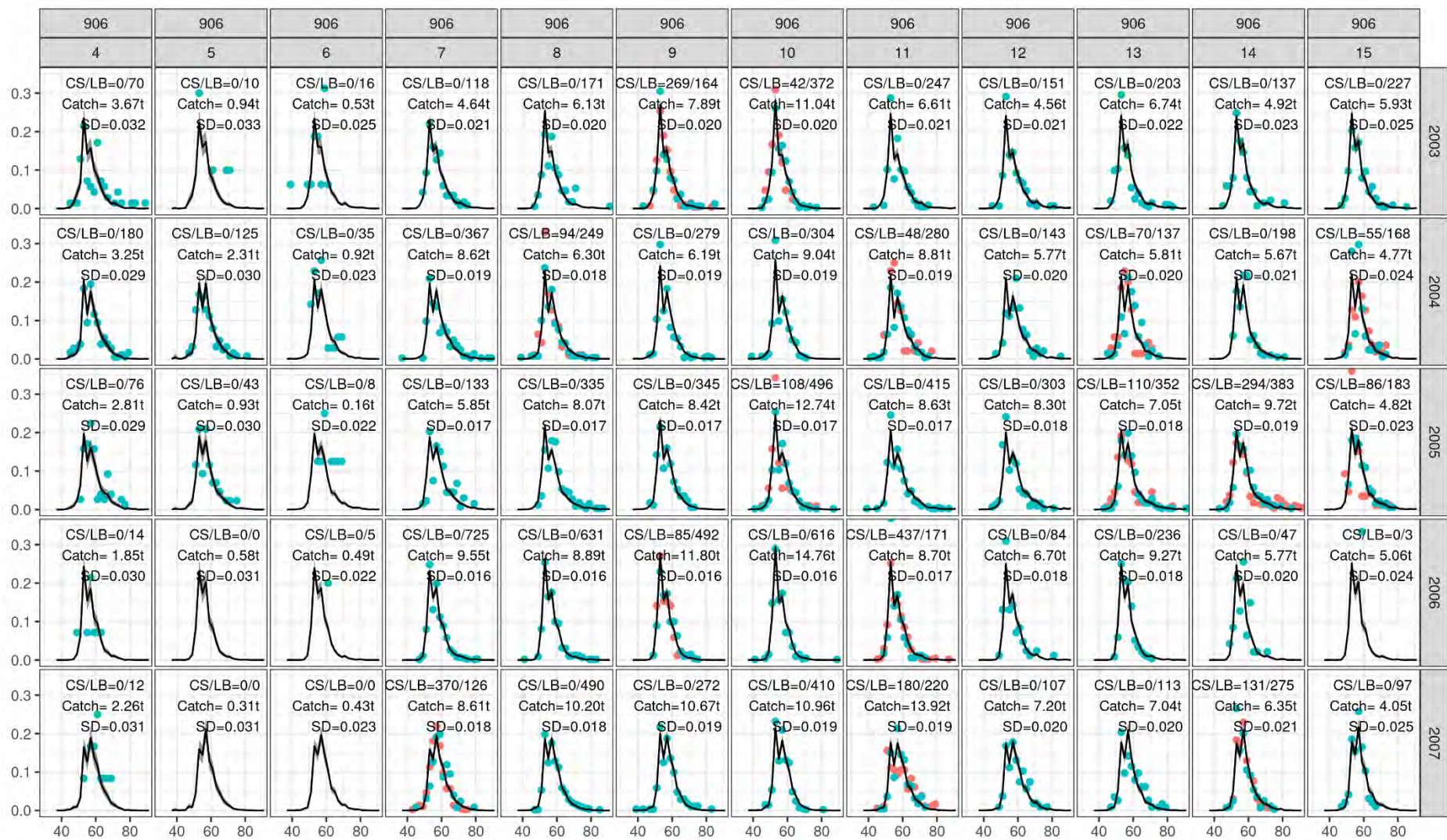


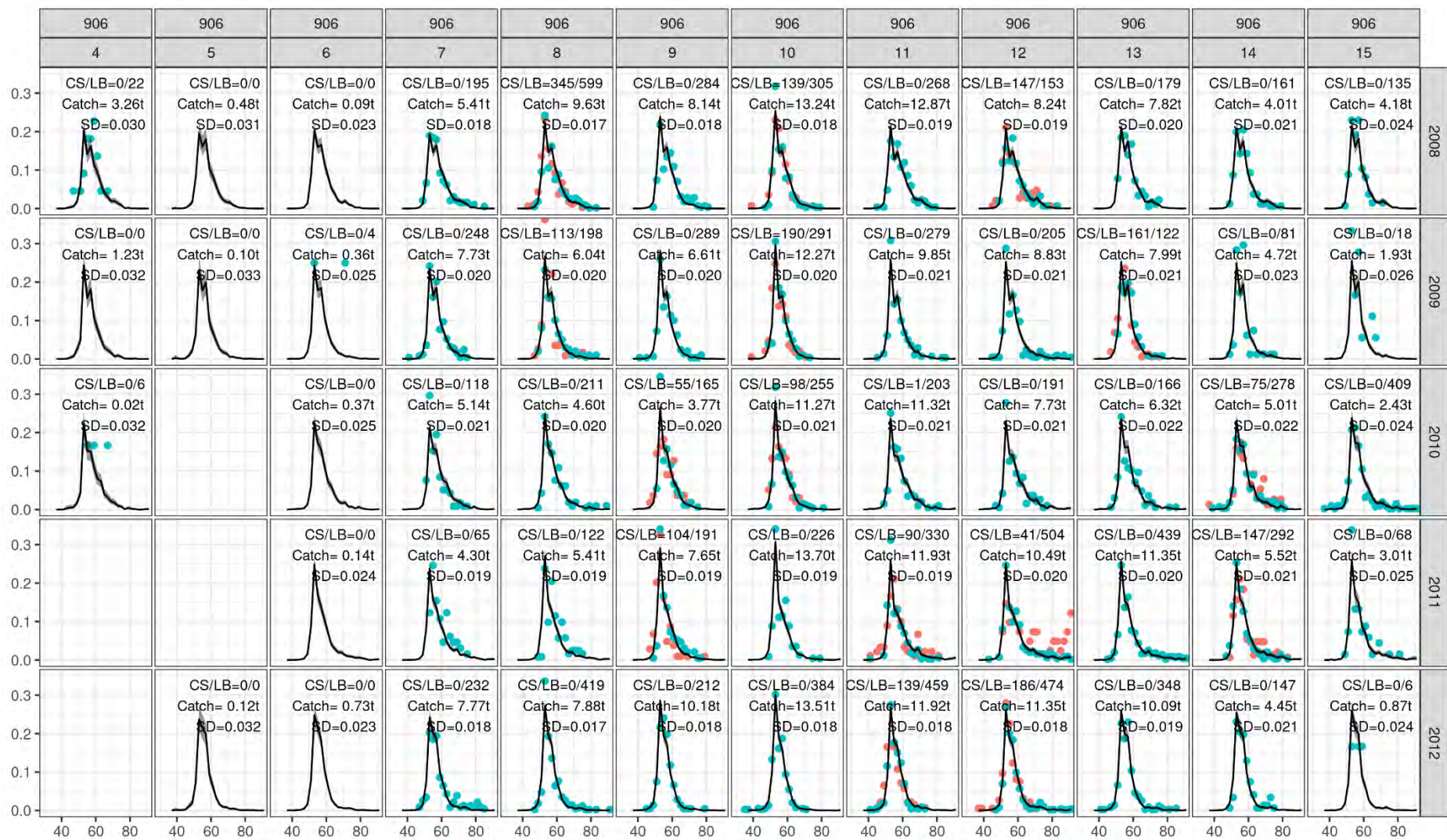


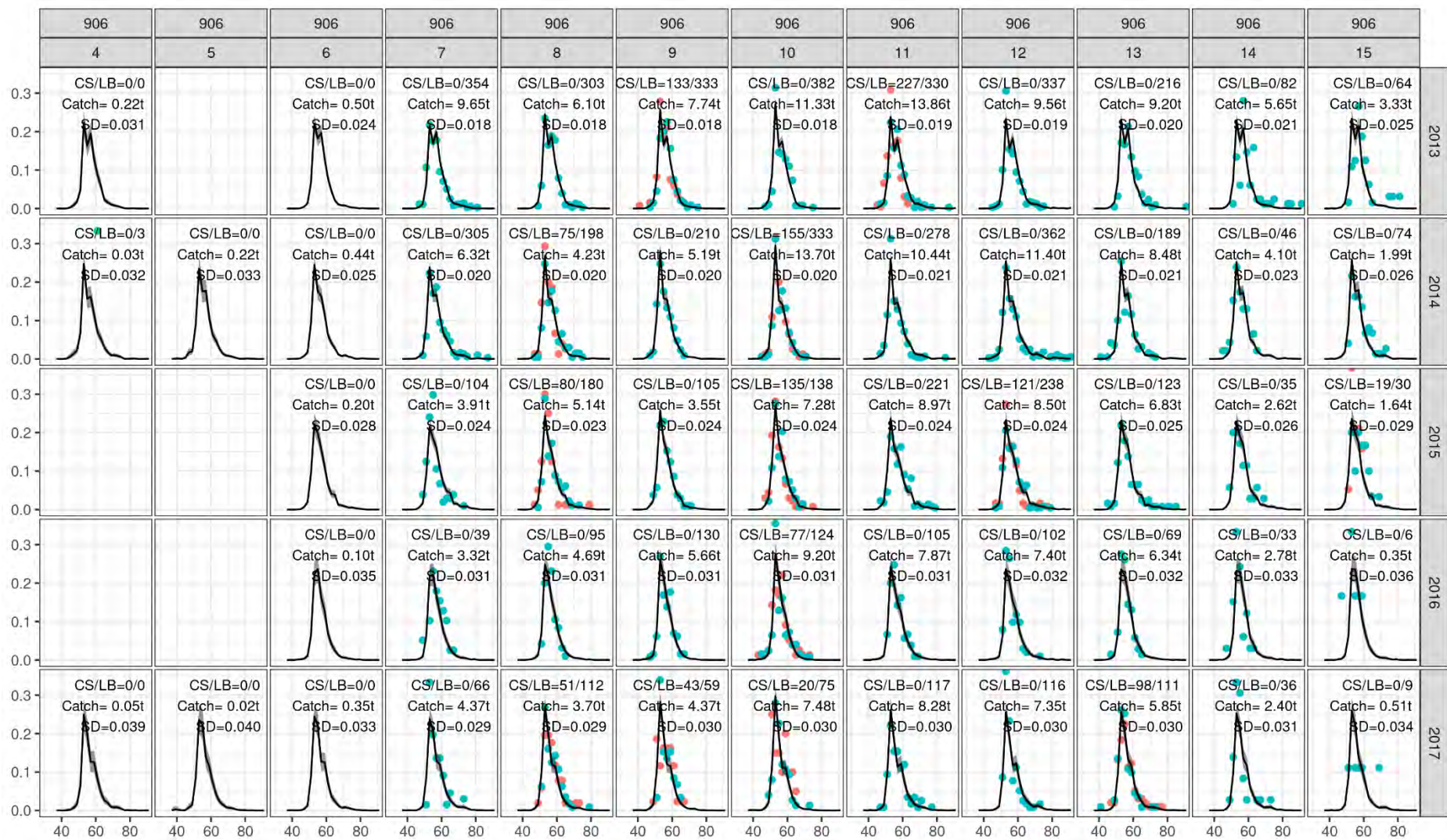


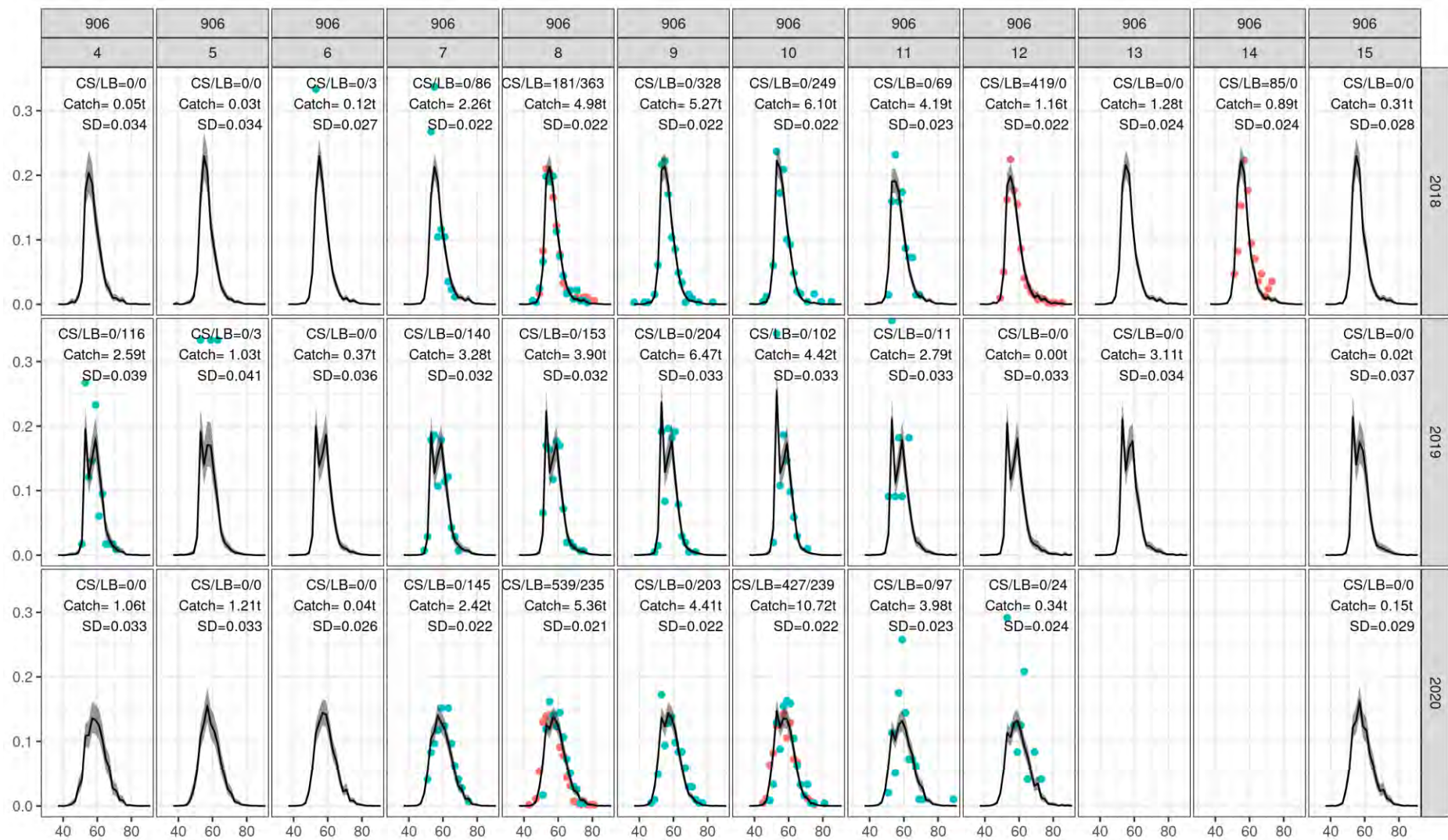


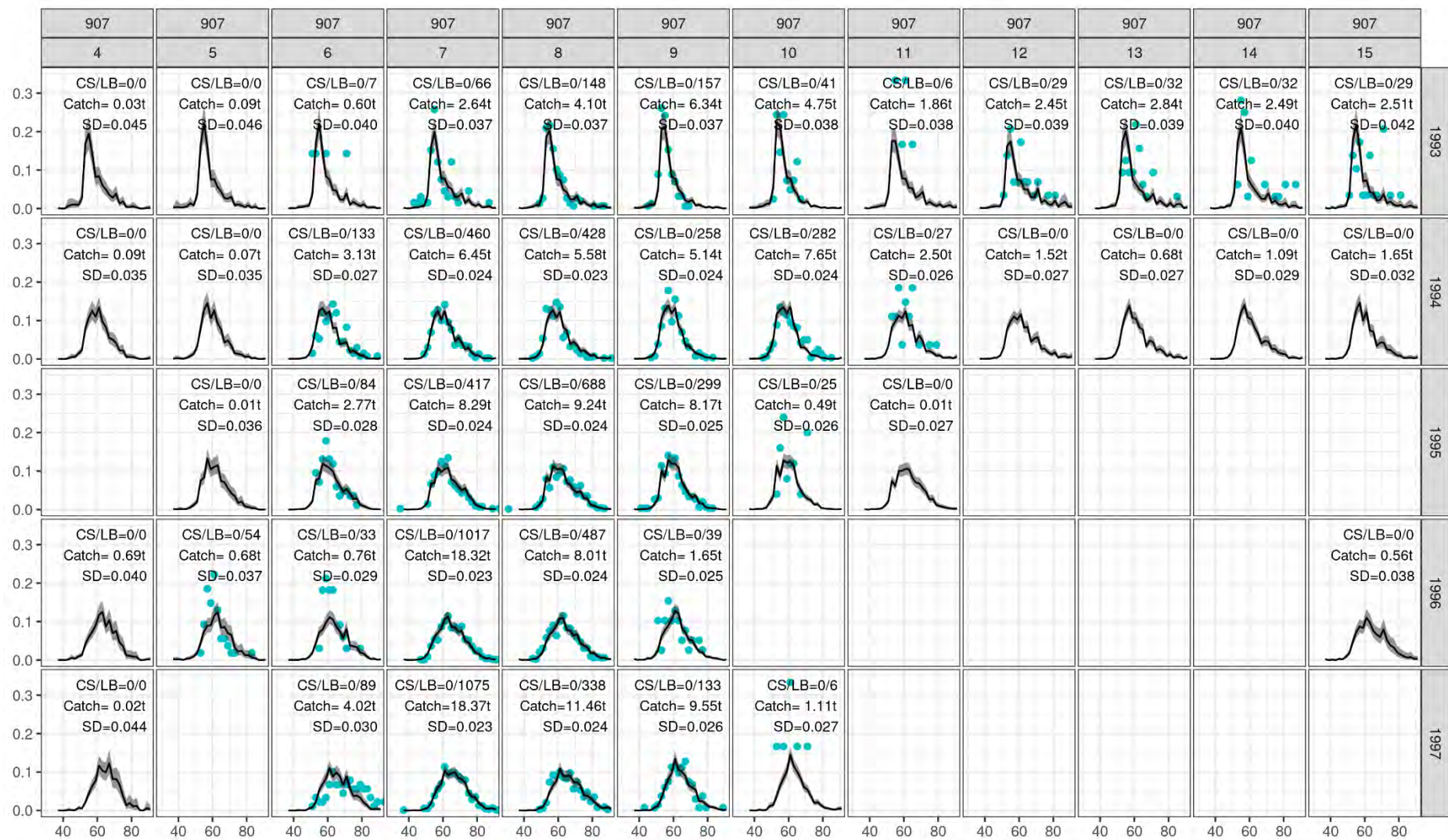


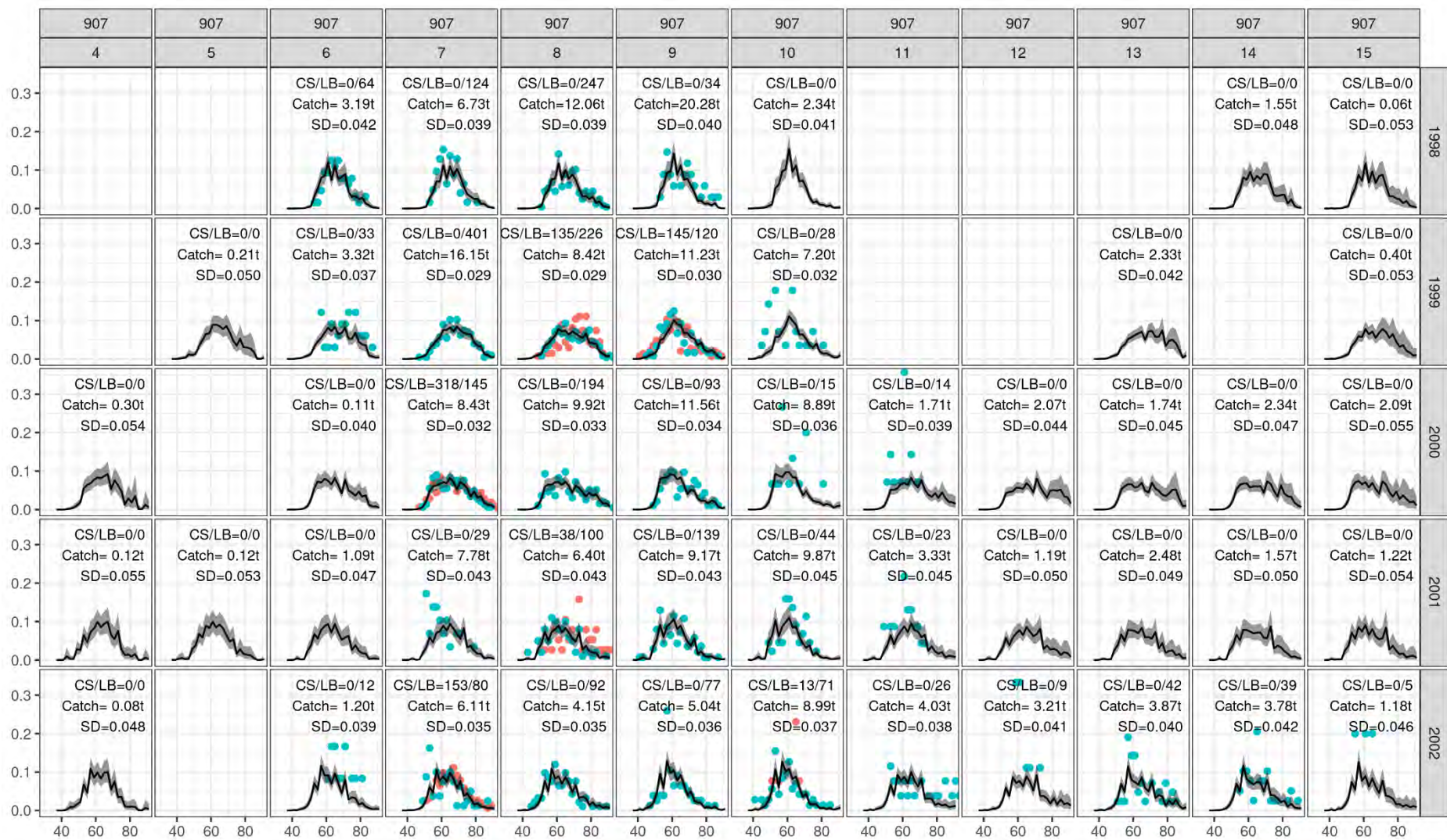


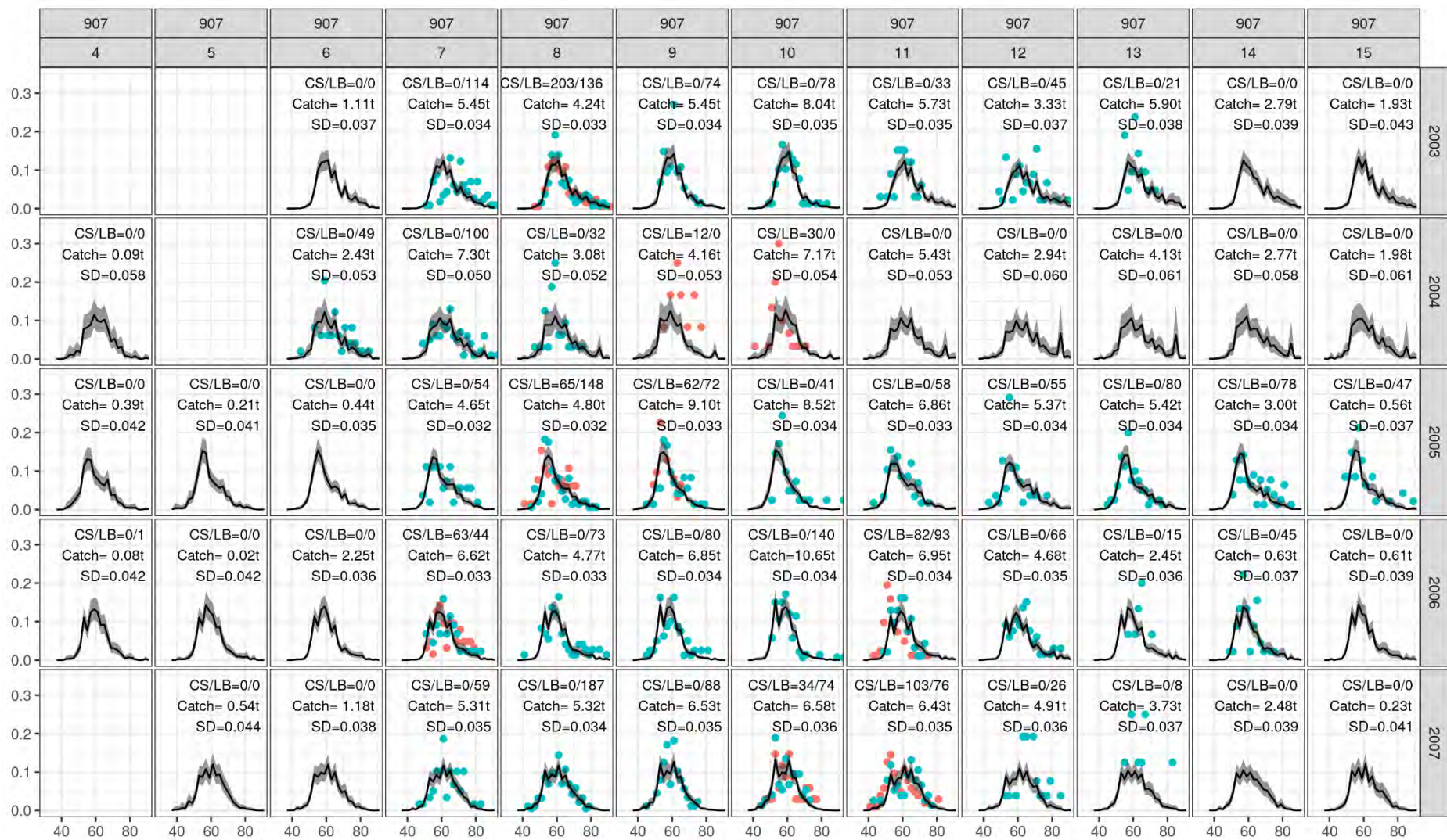


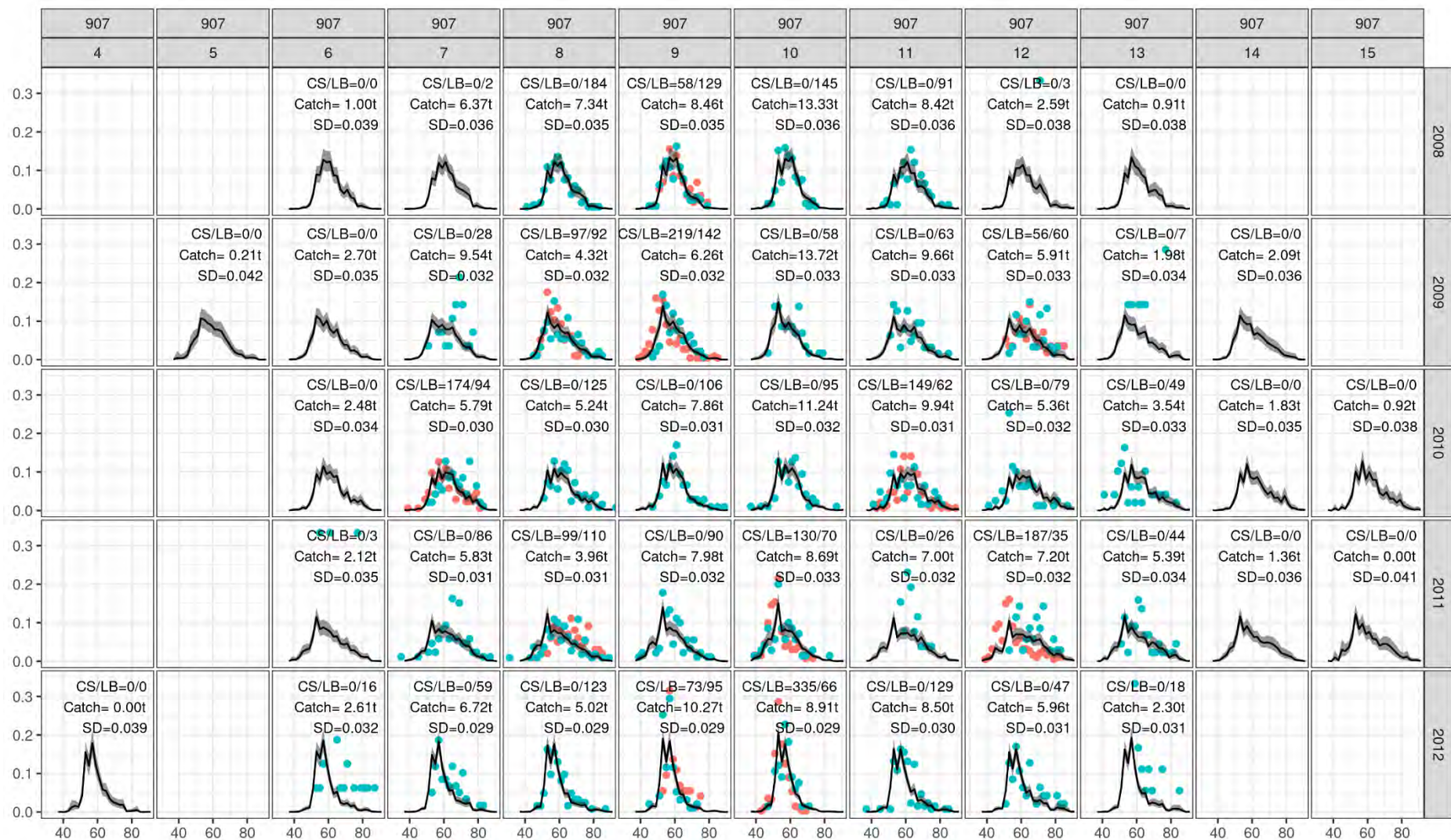


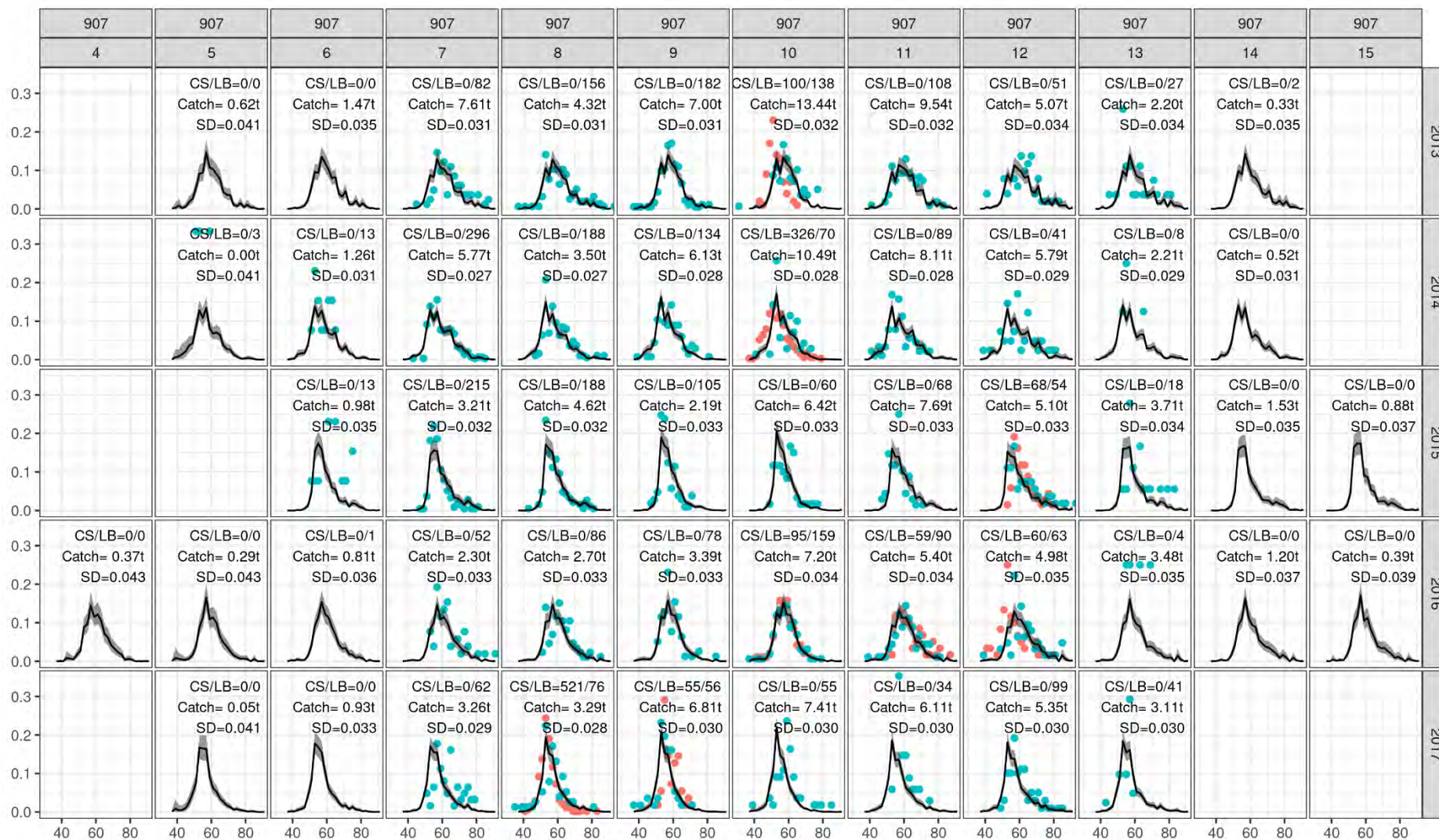


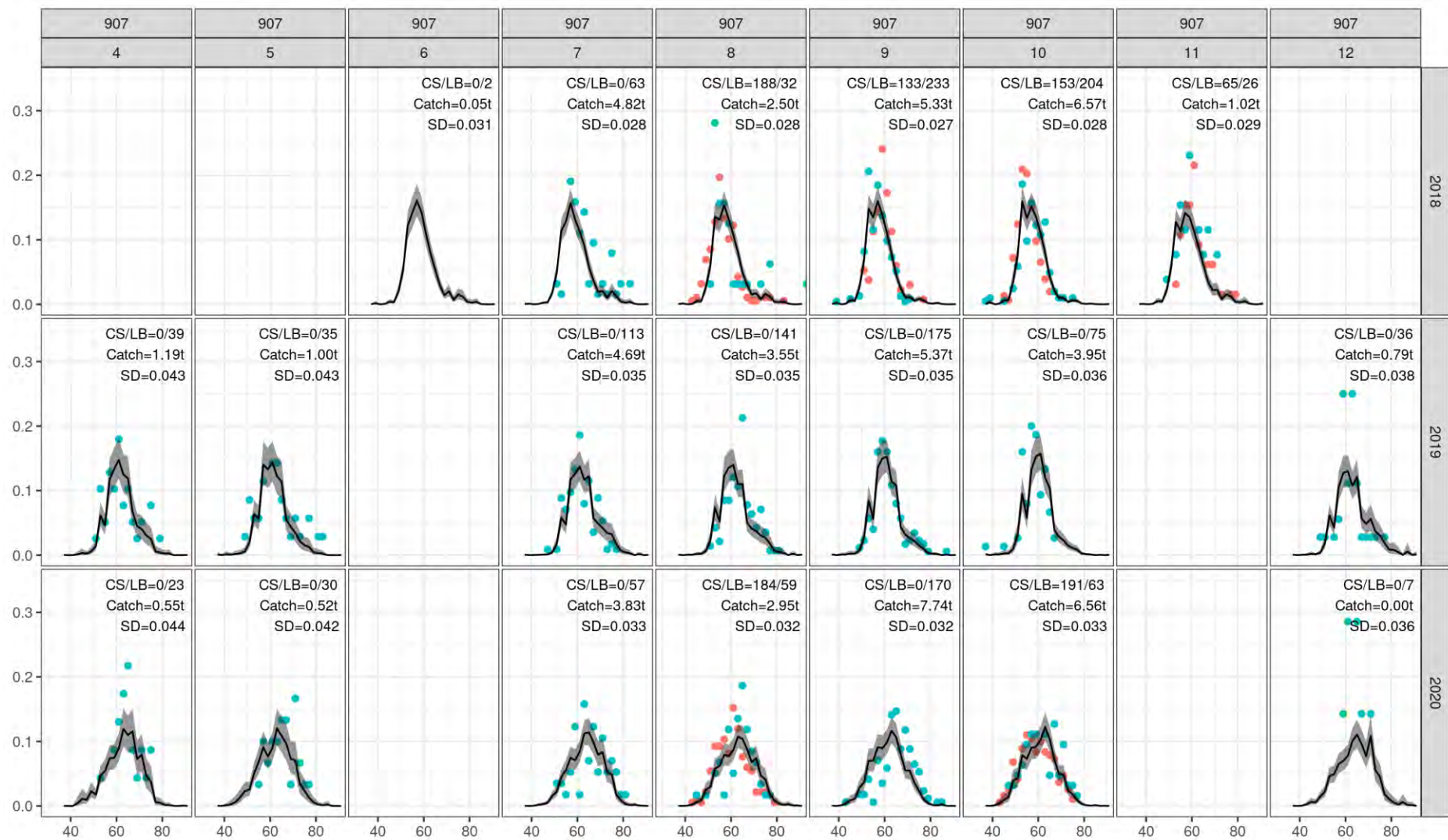


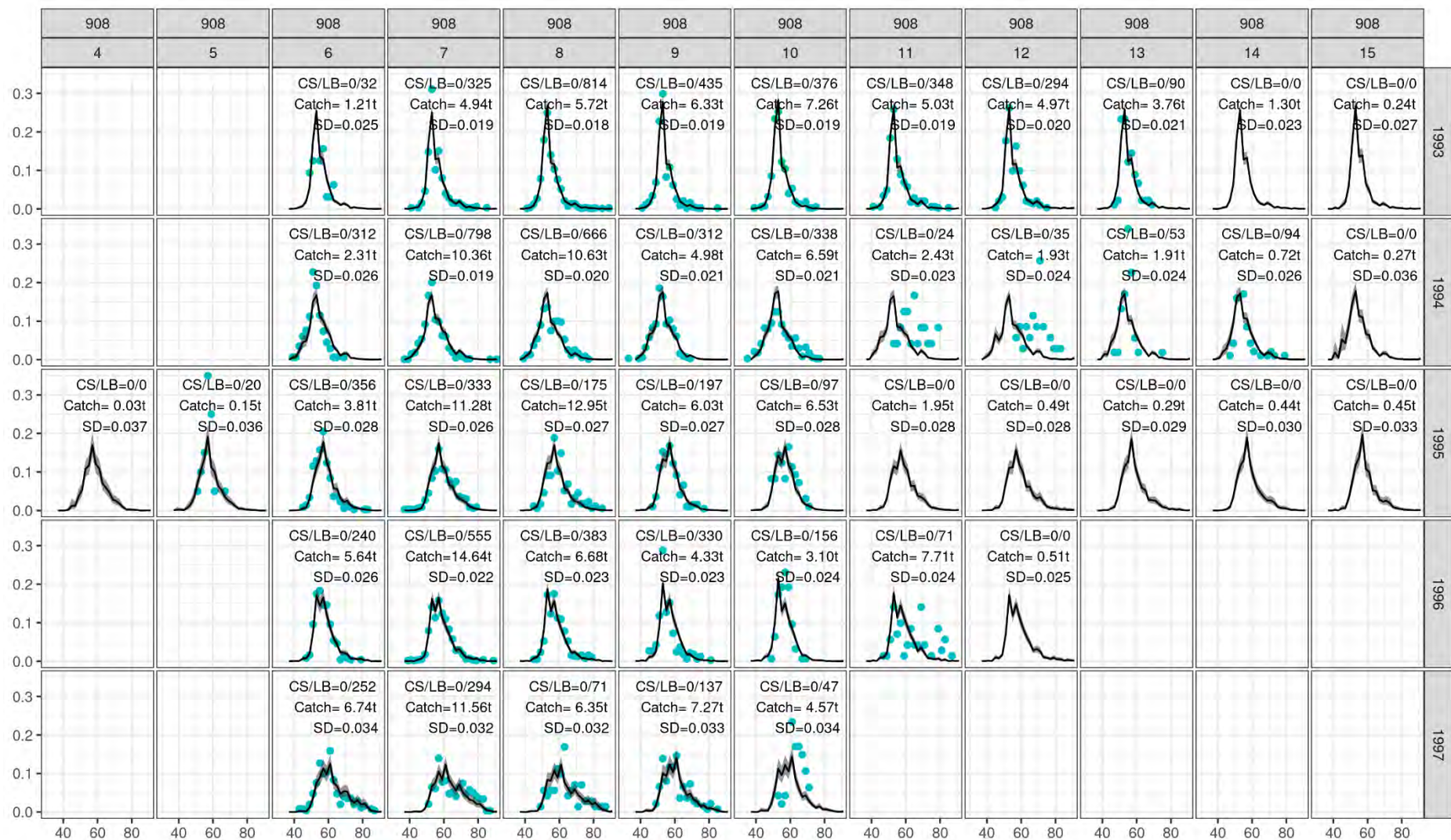


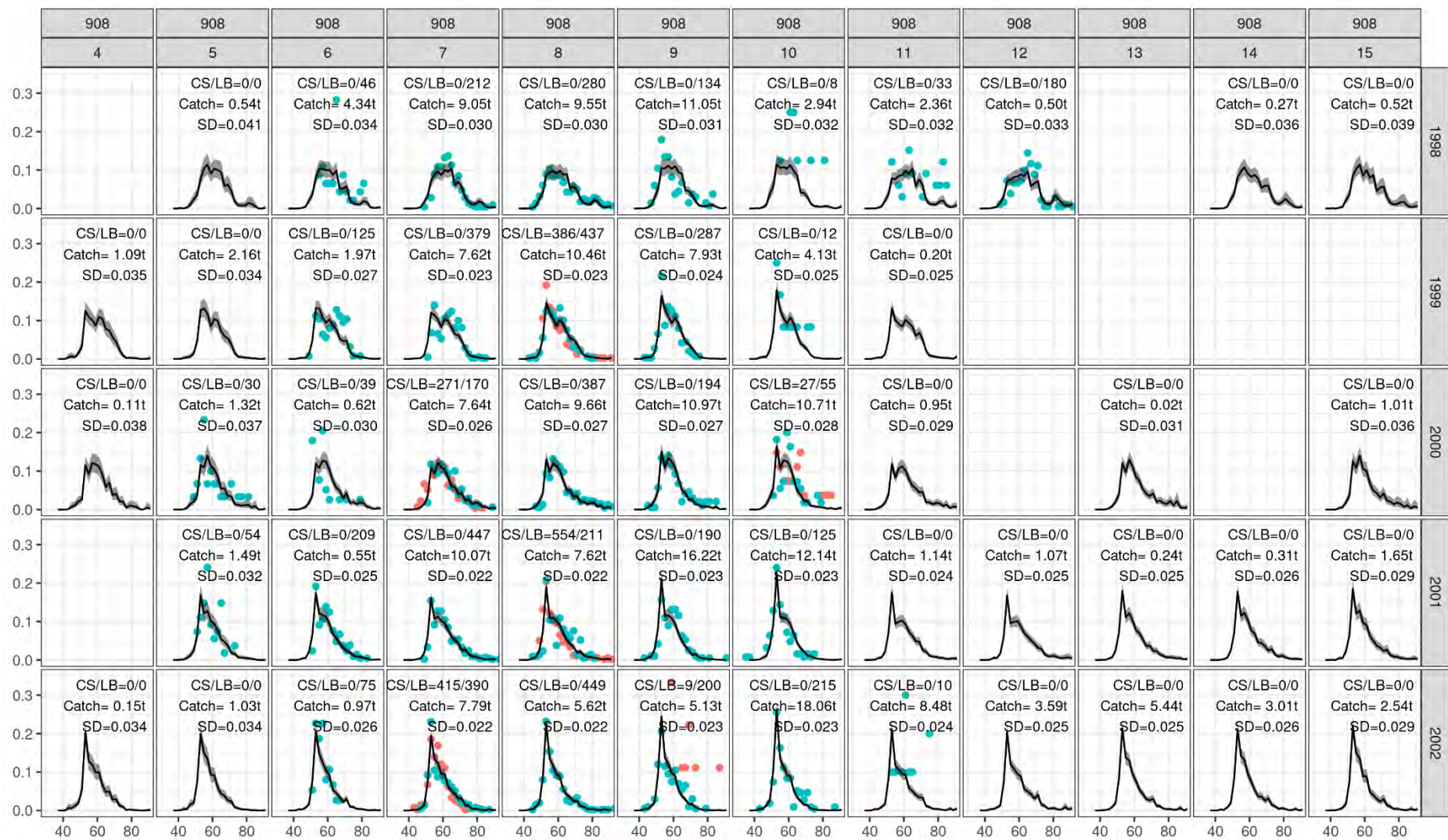


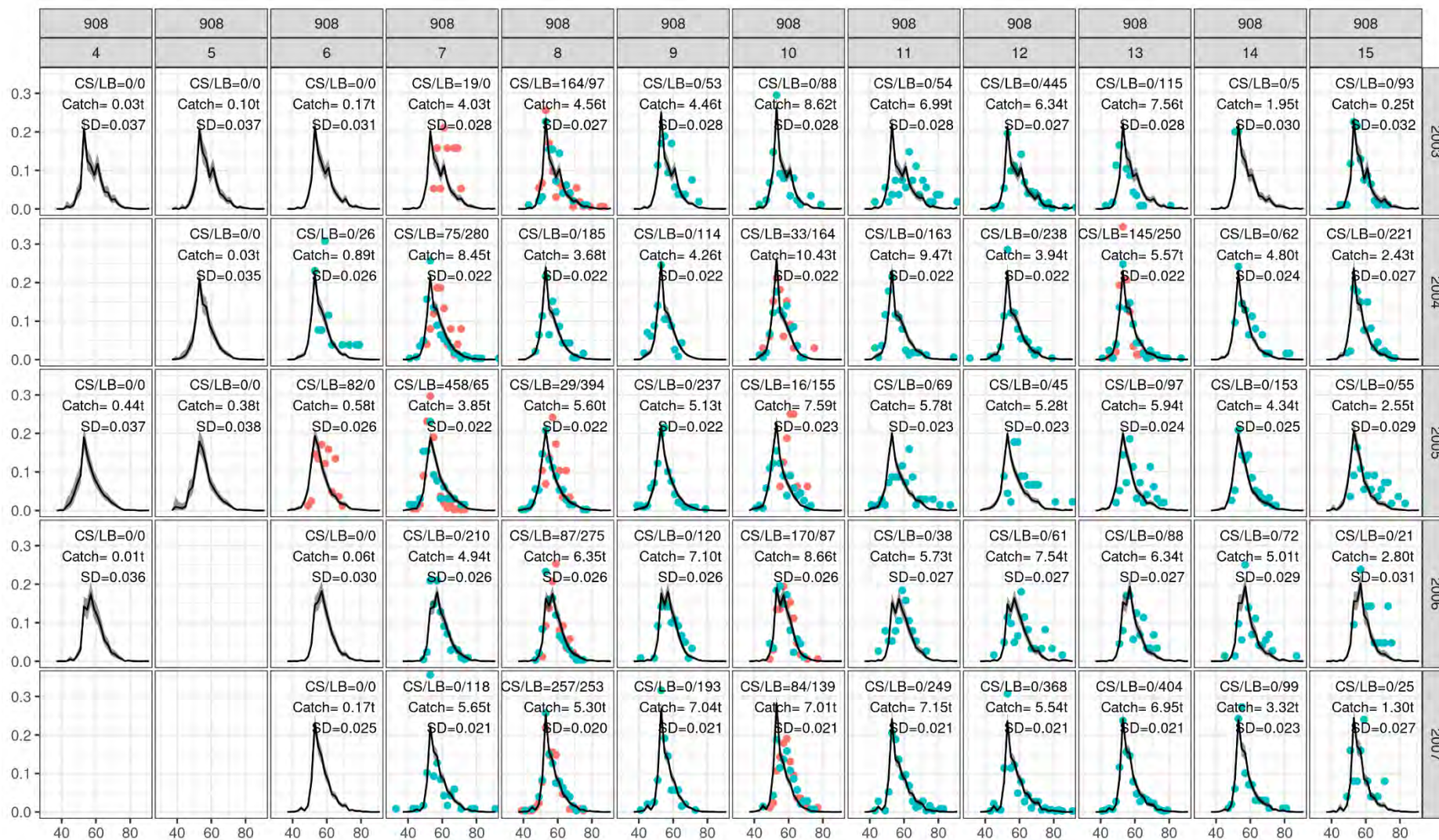


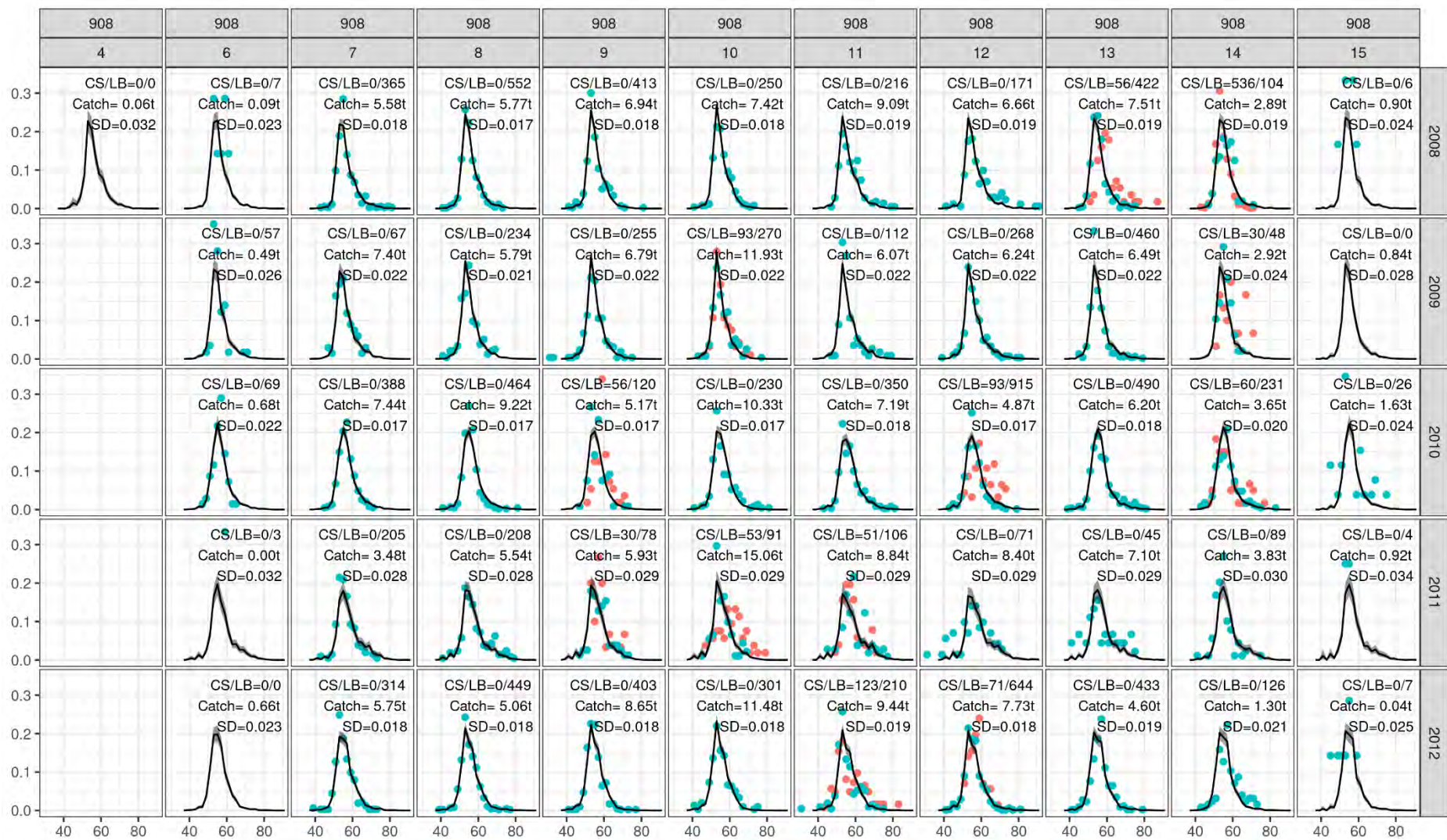


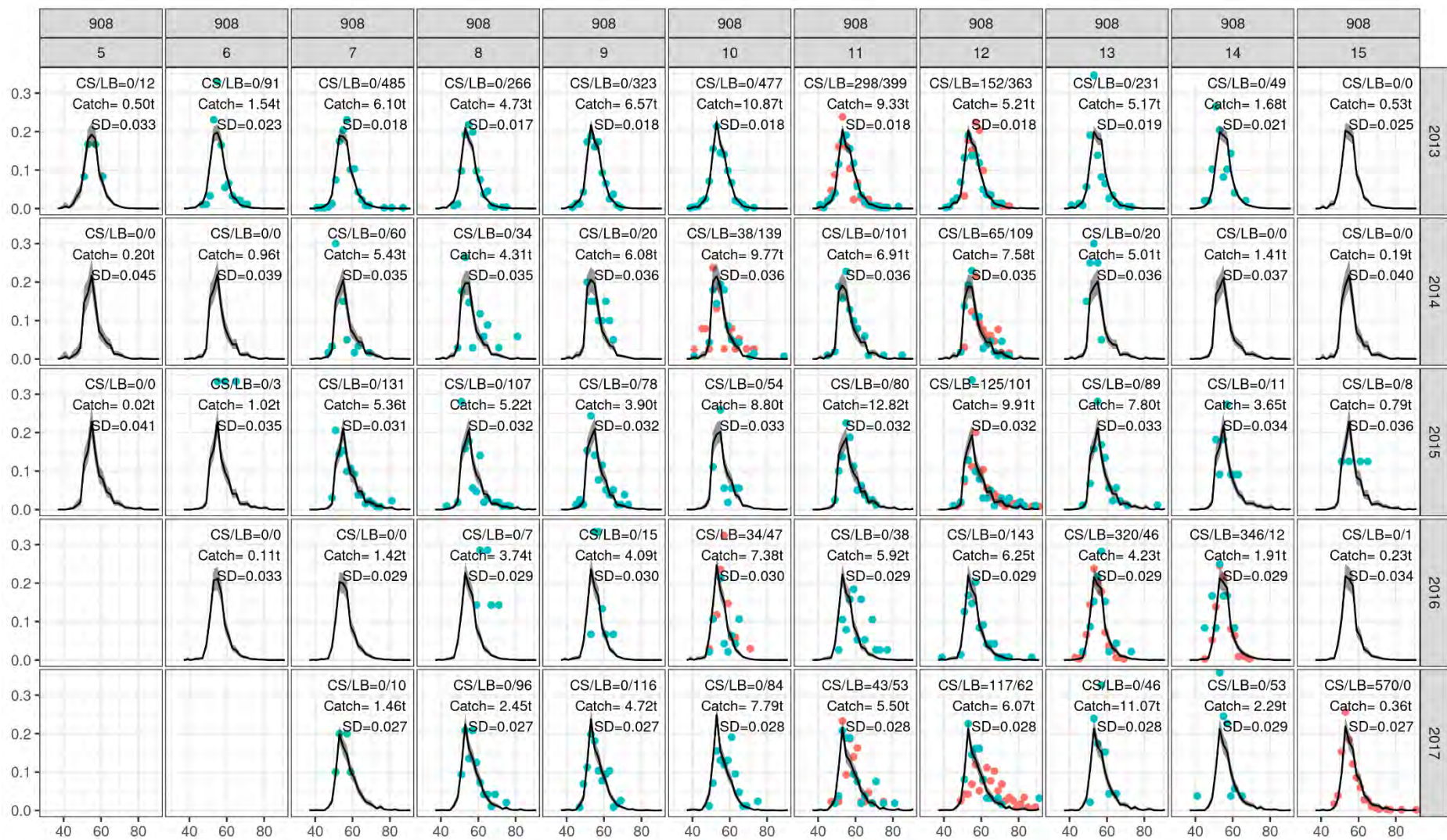


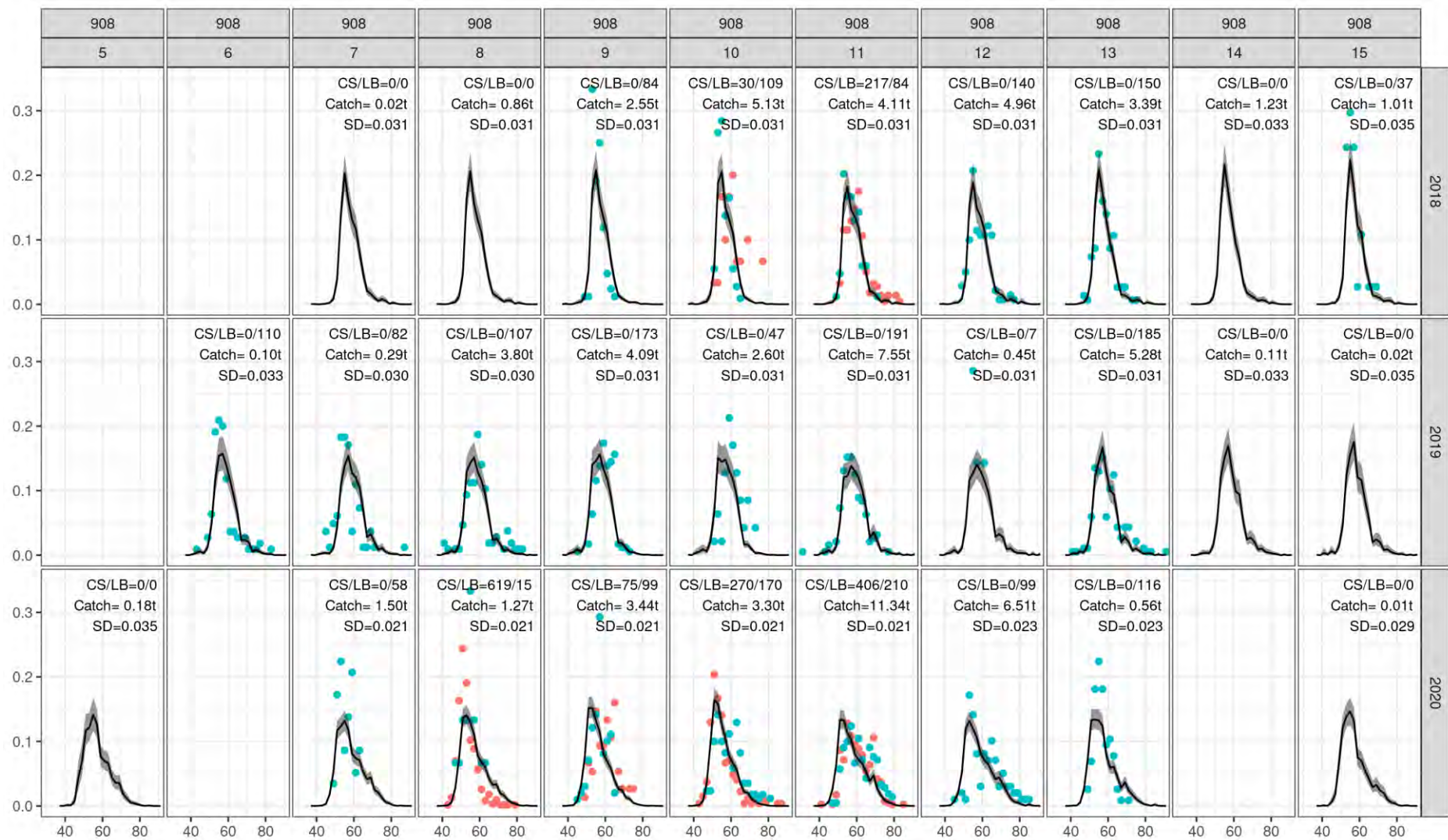






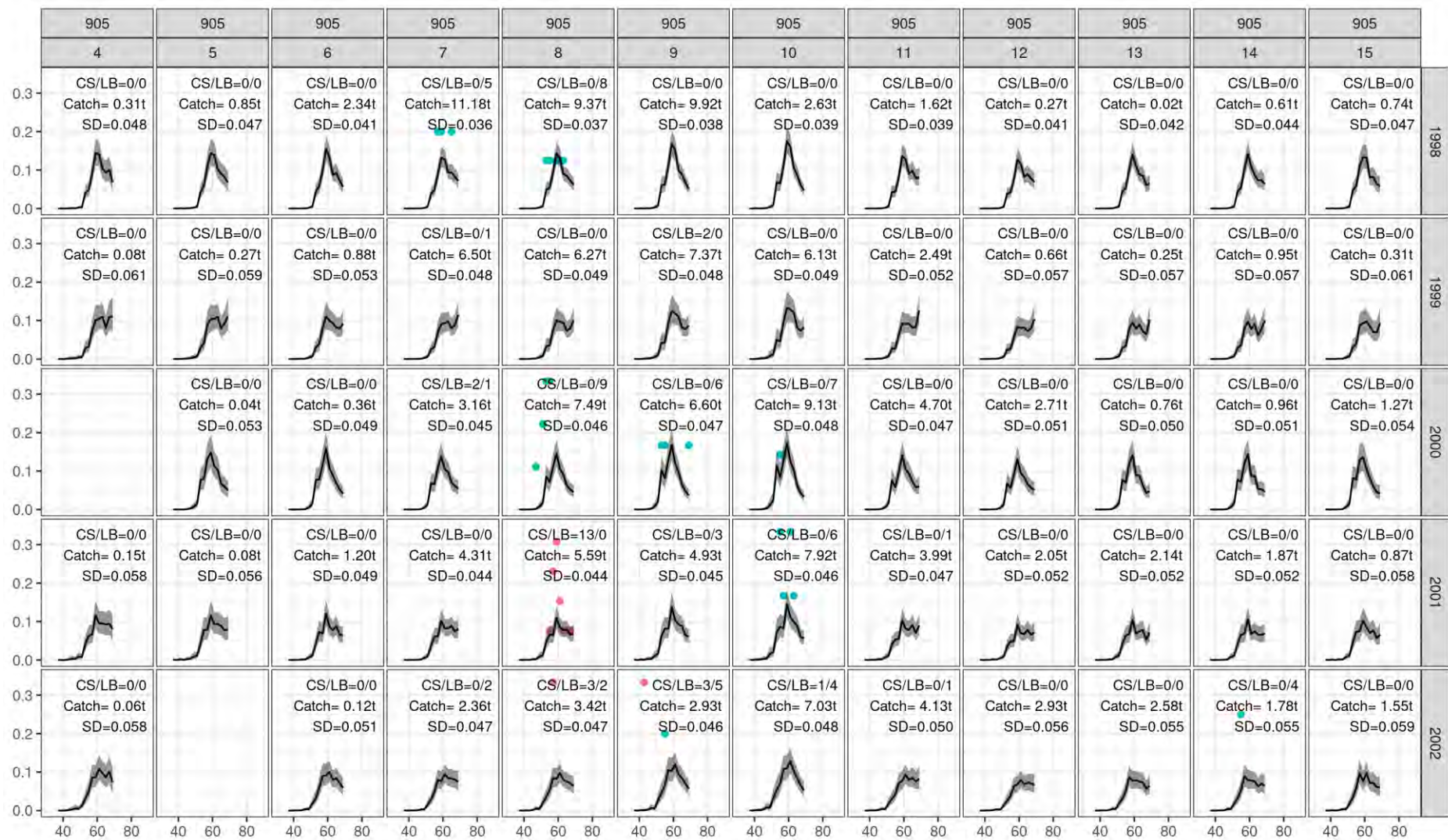




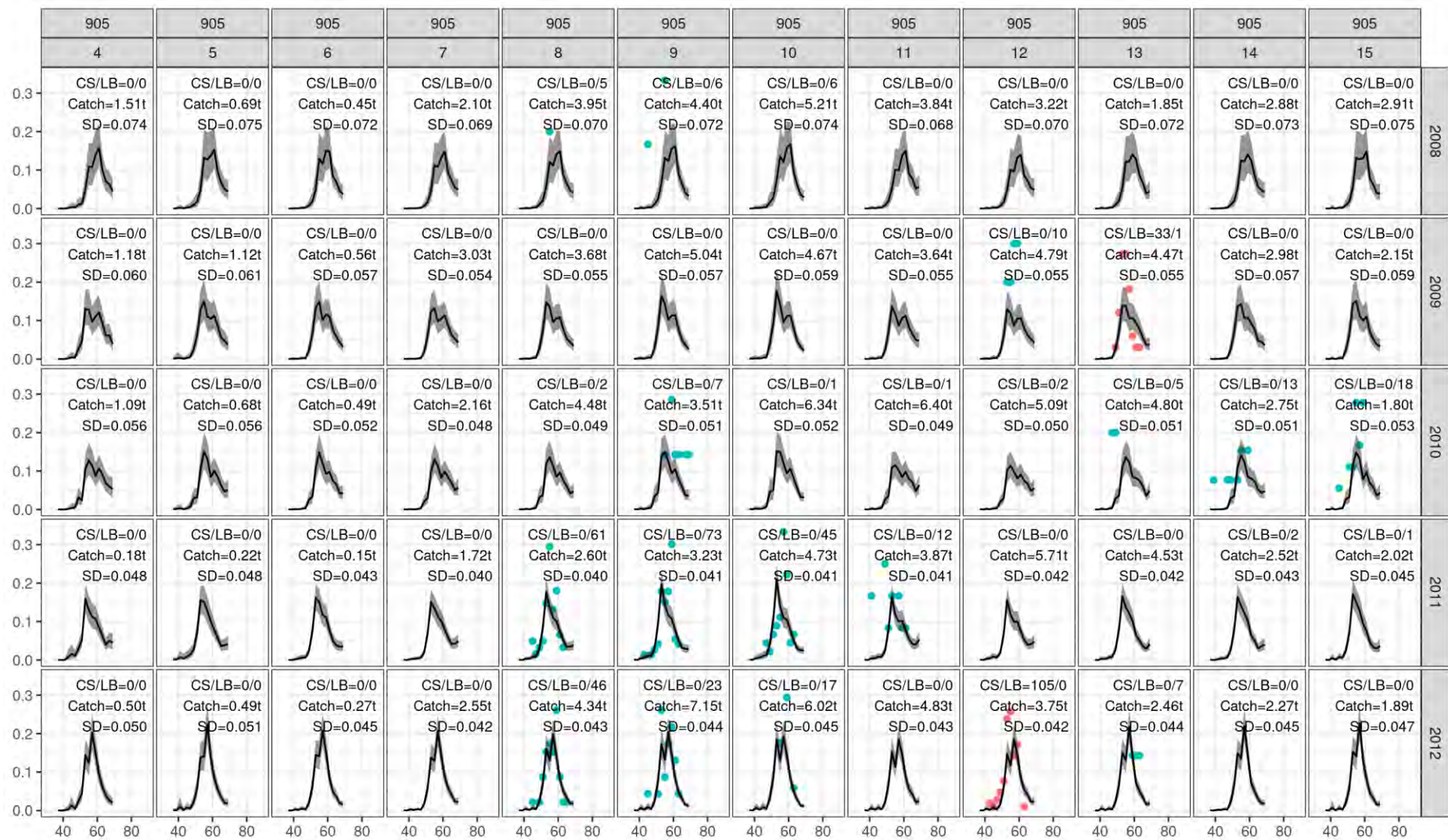


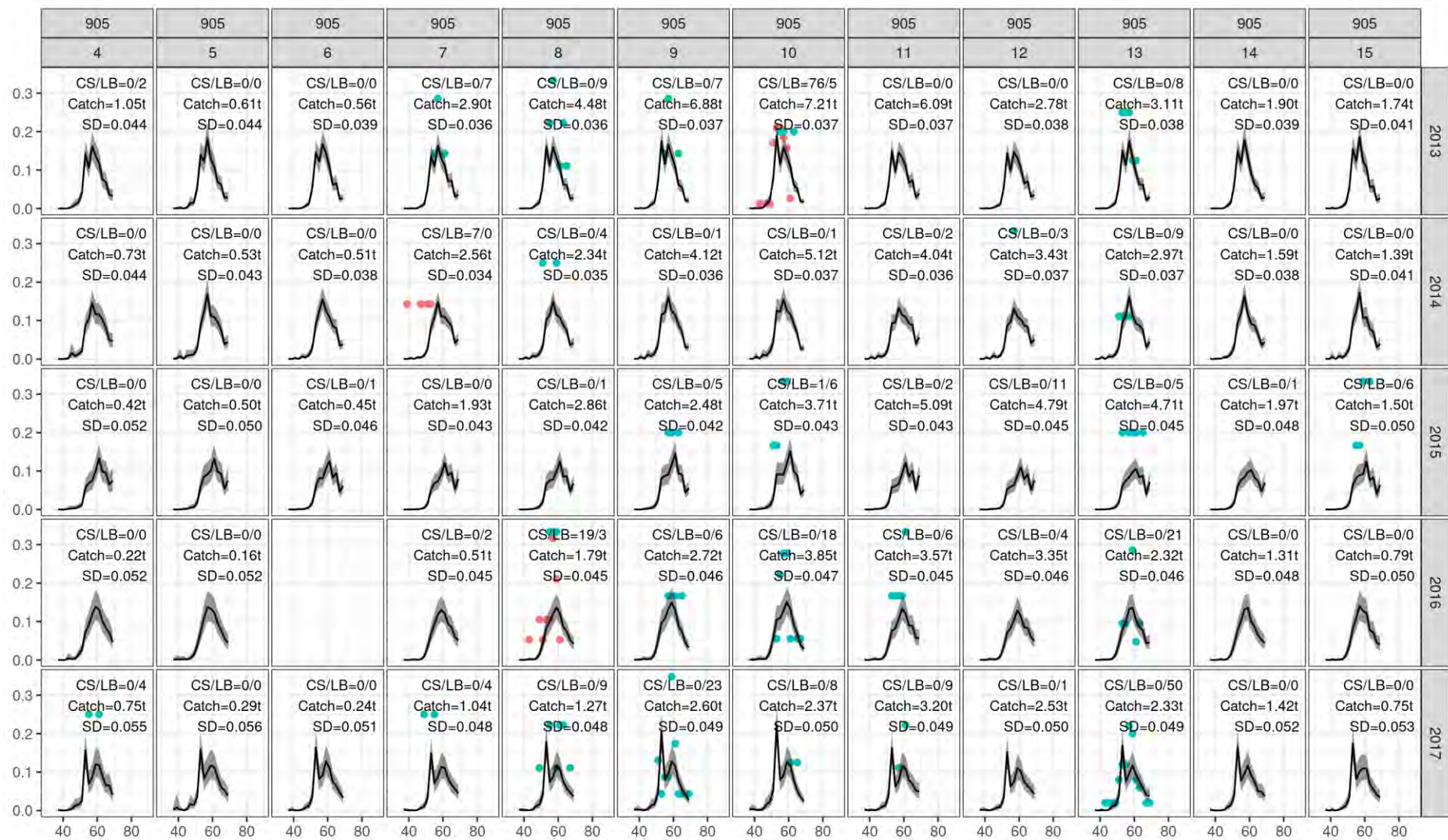
11.2 CRA 2 immature females

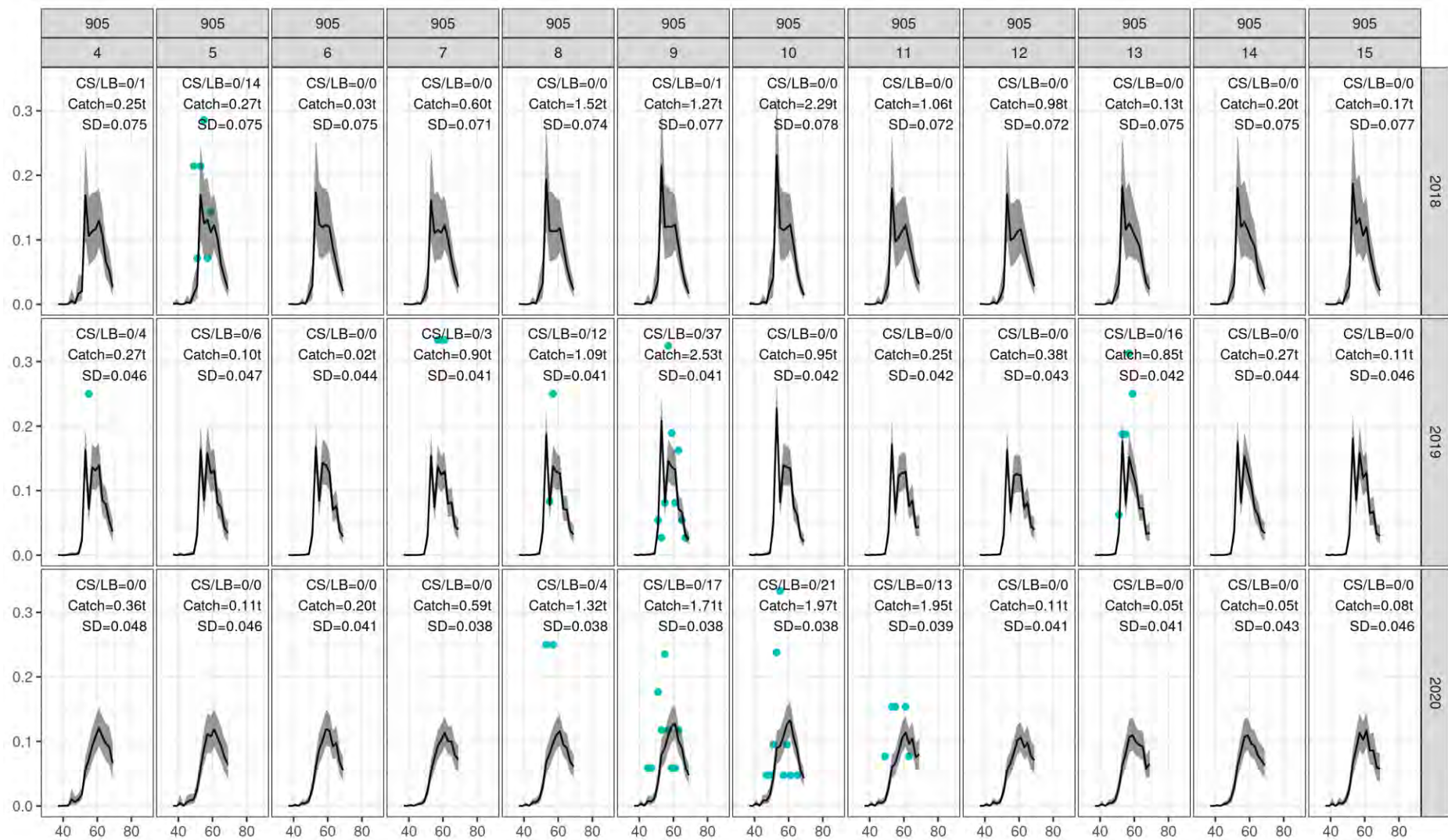


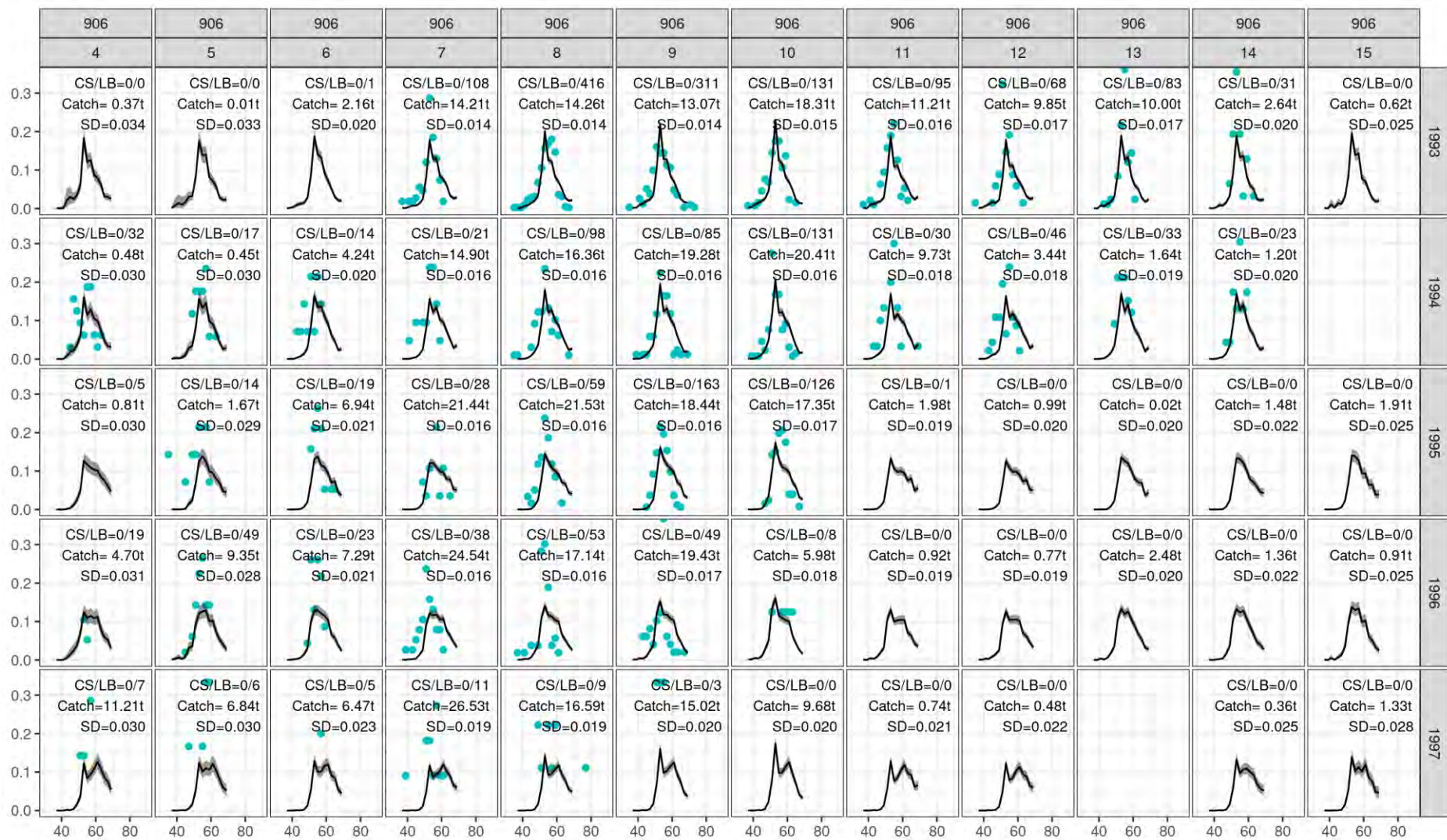




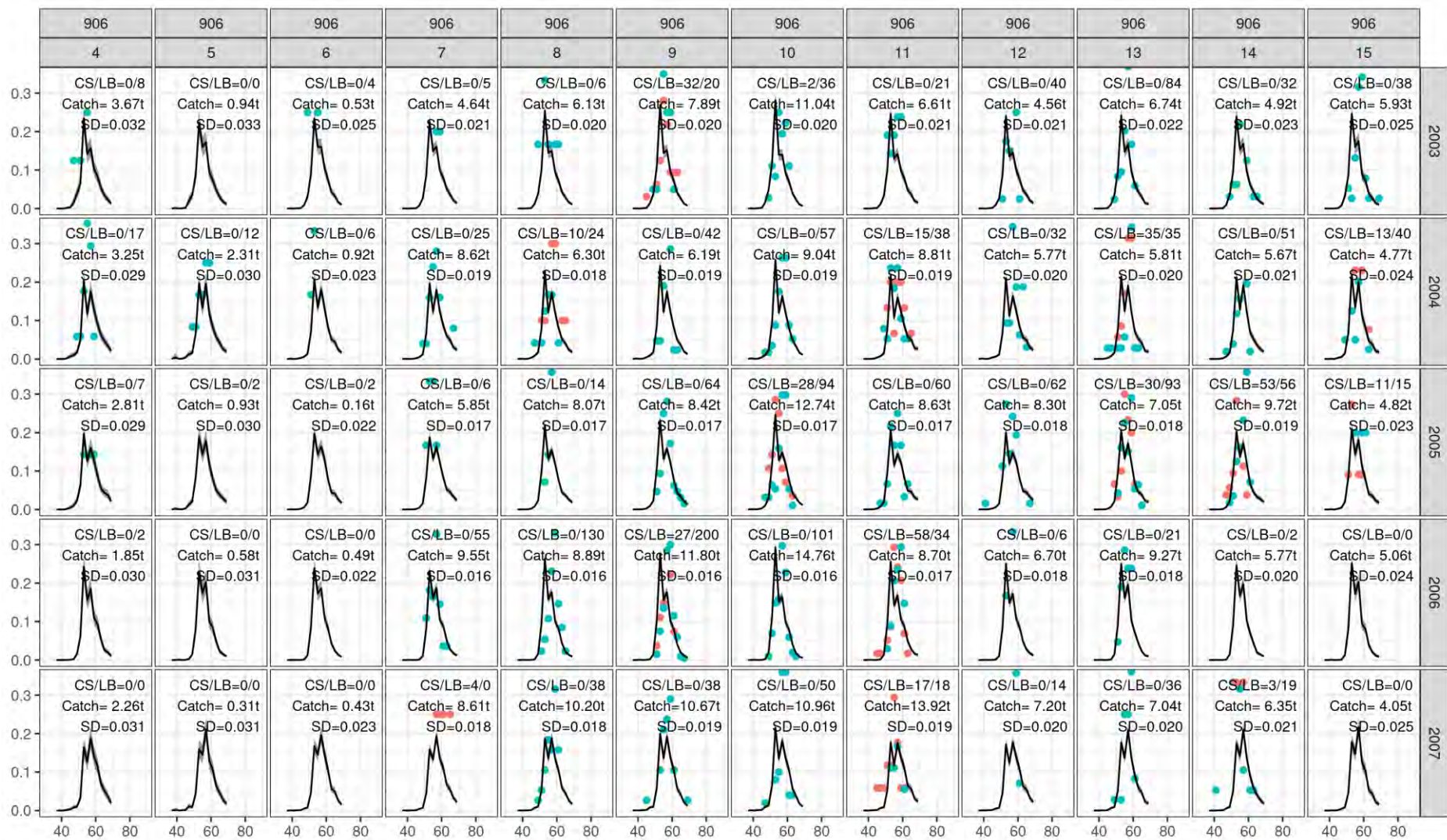


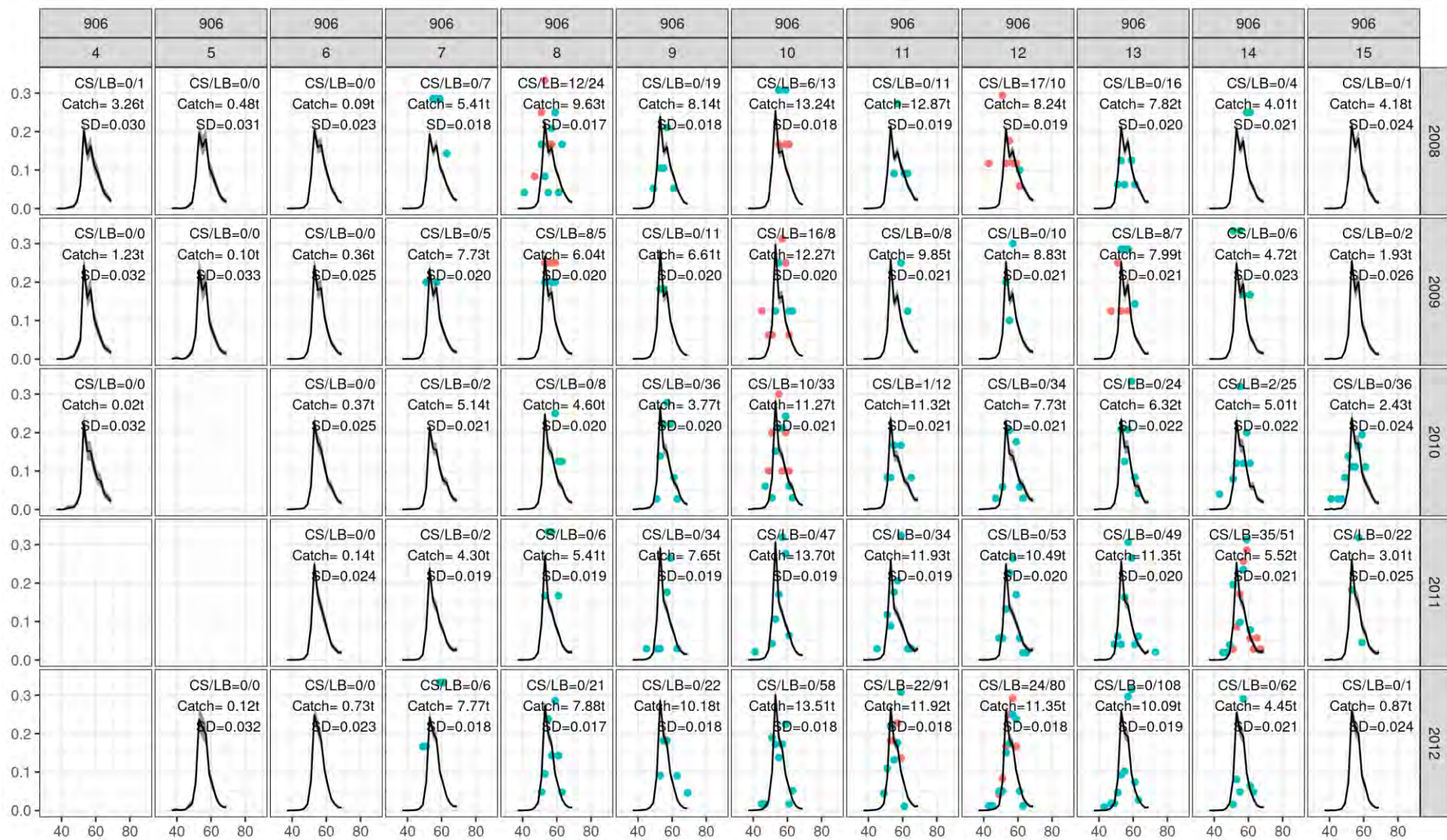


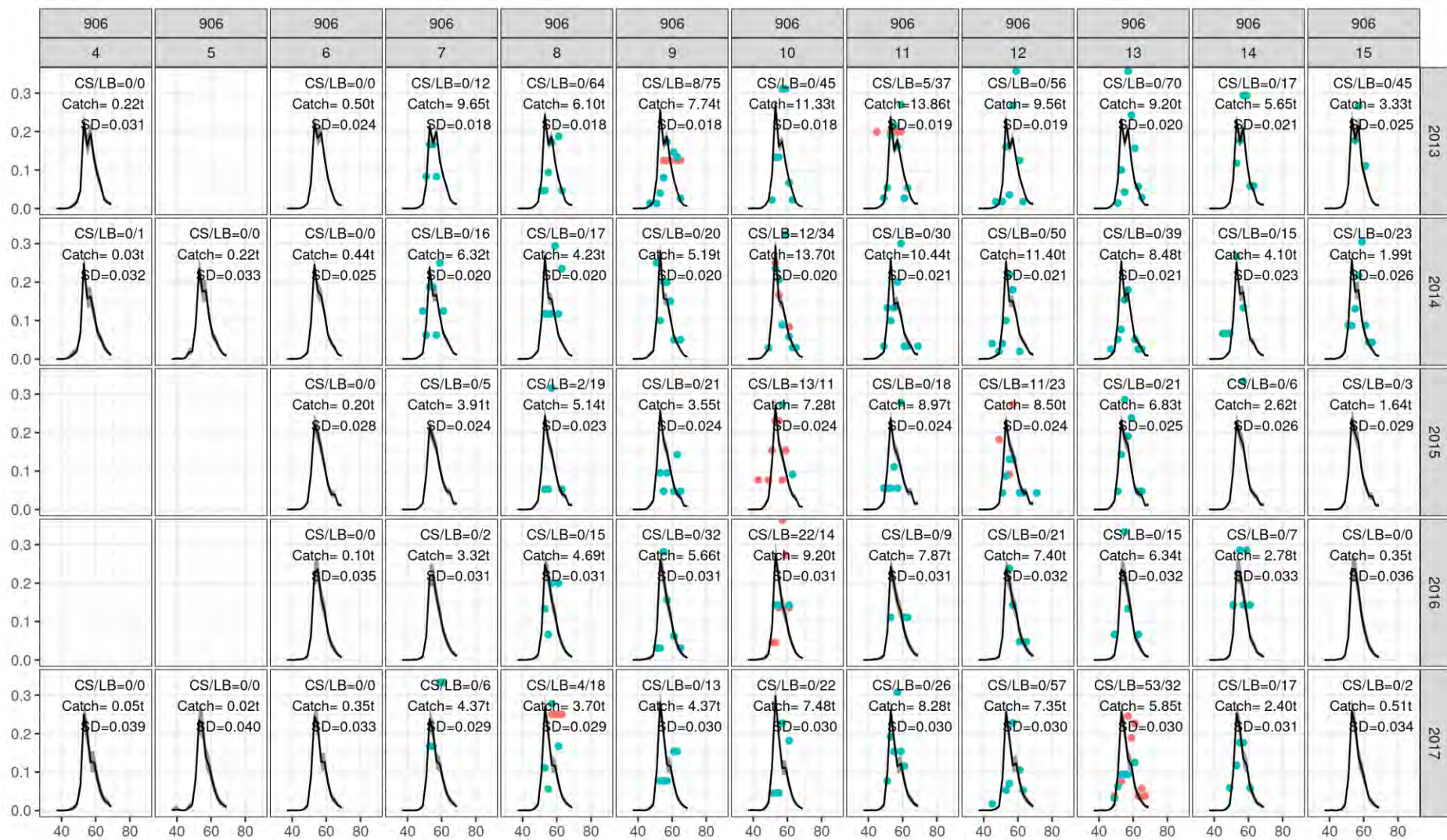


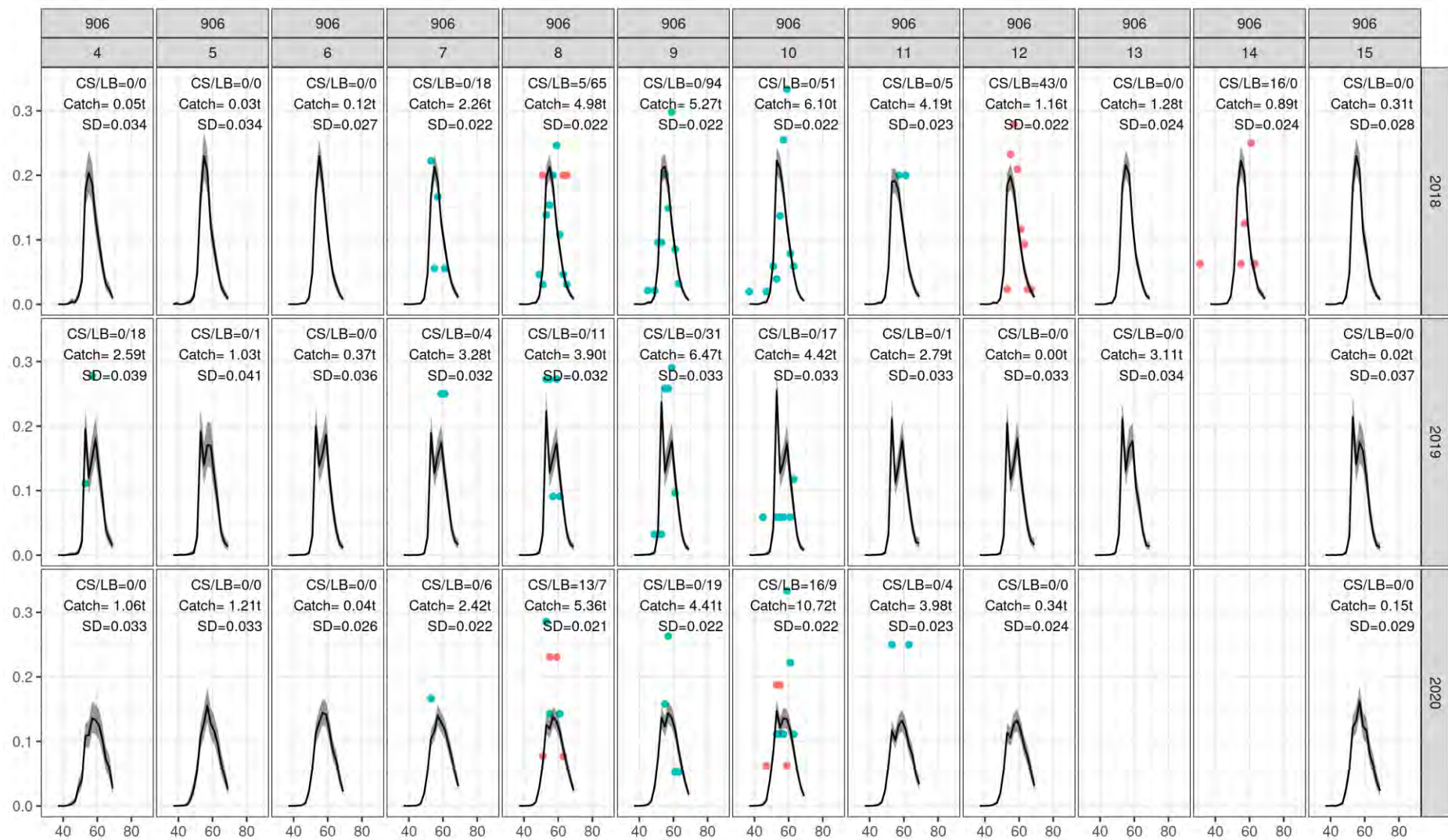


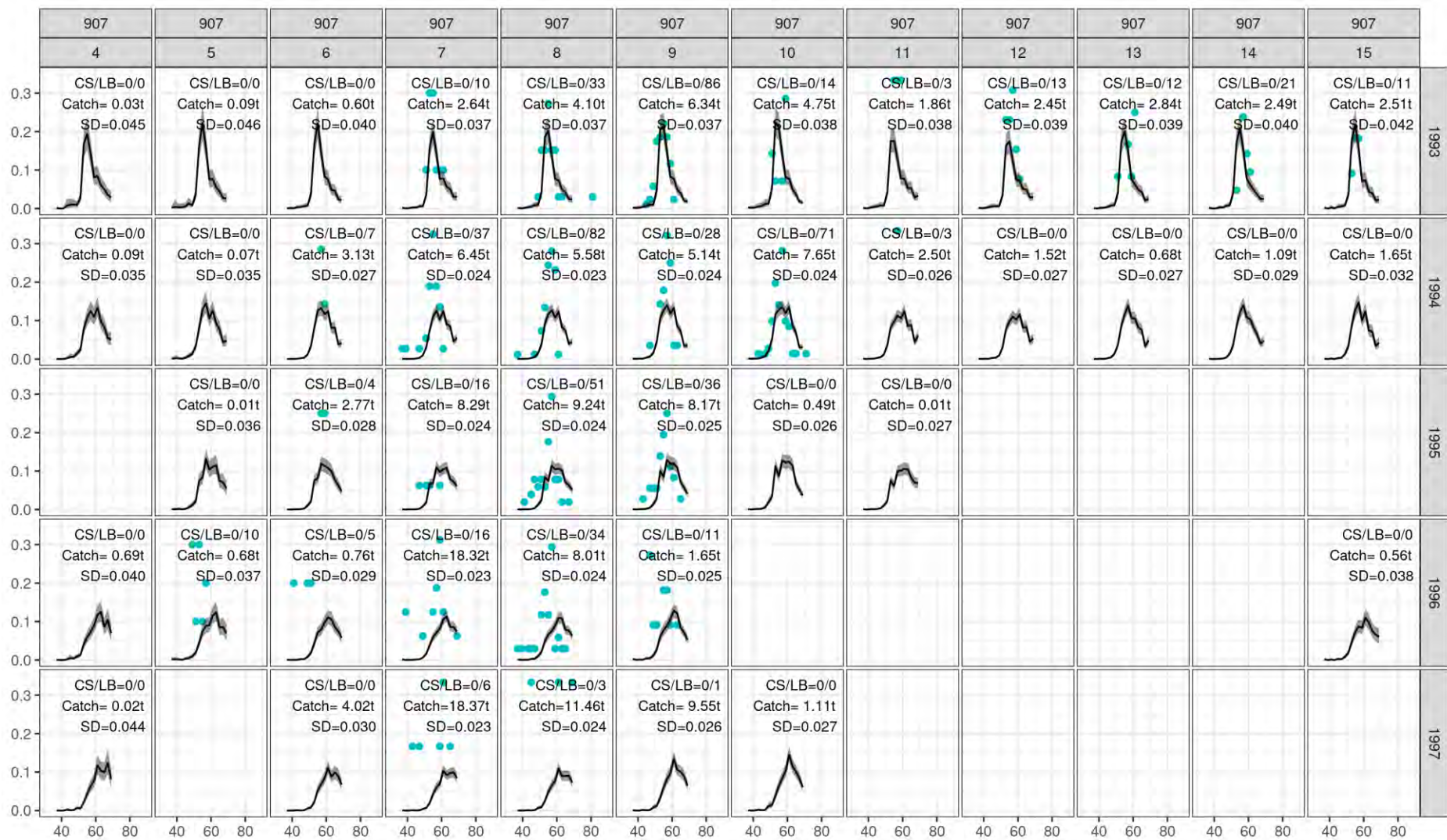


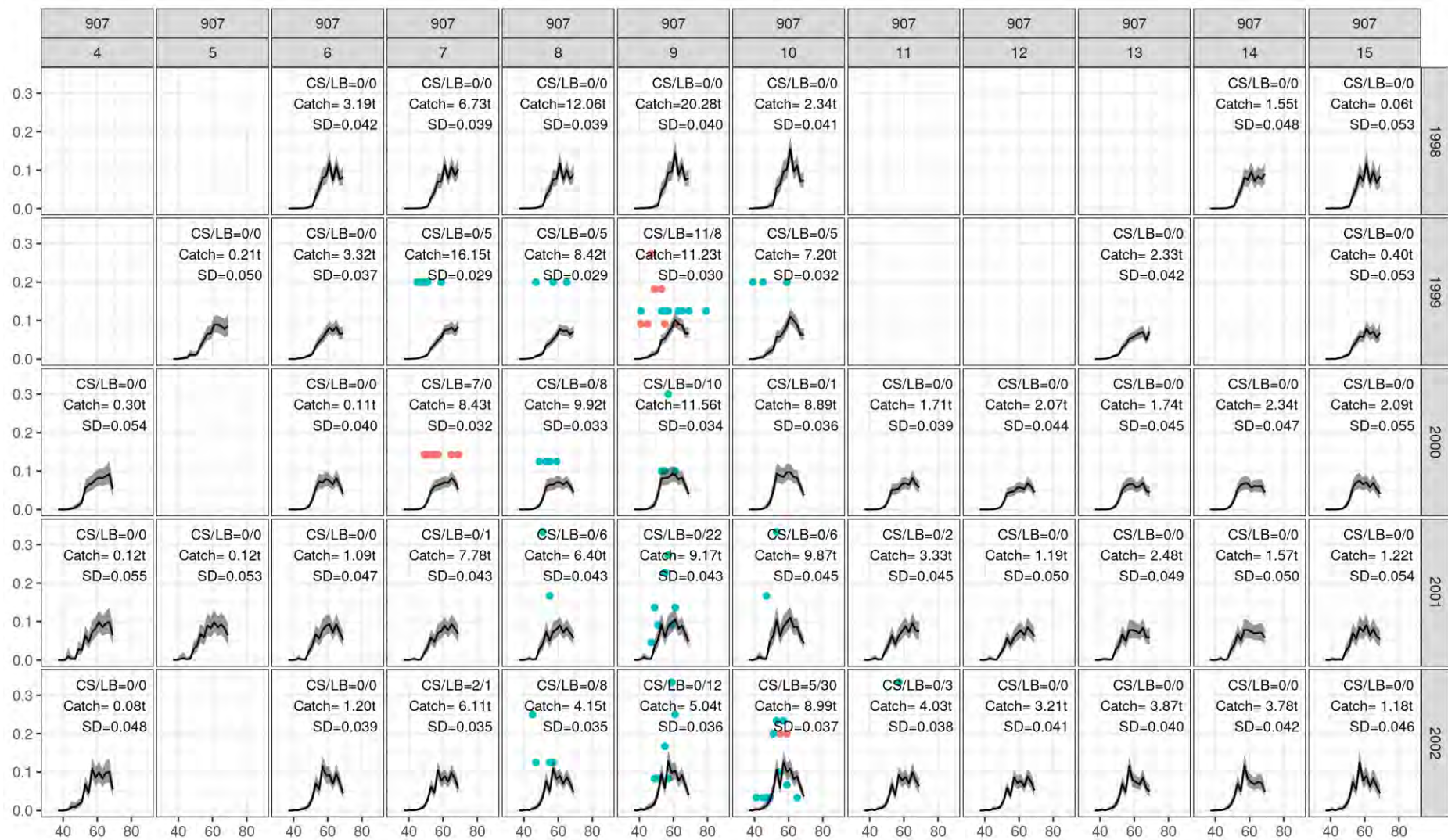






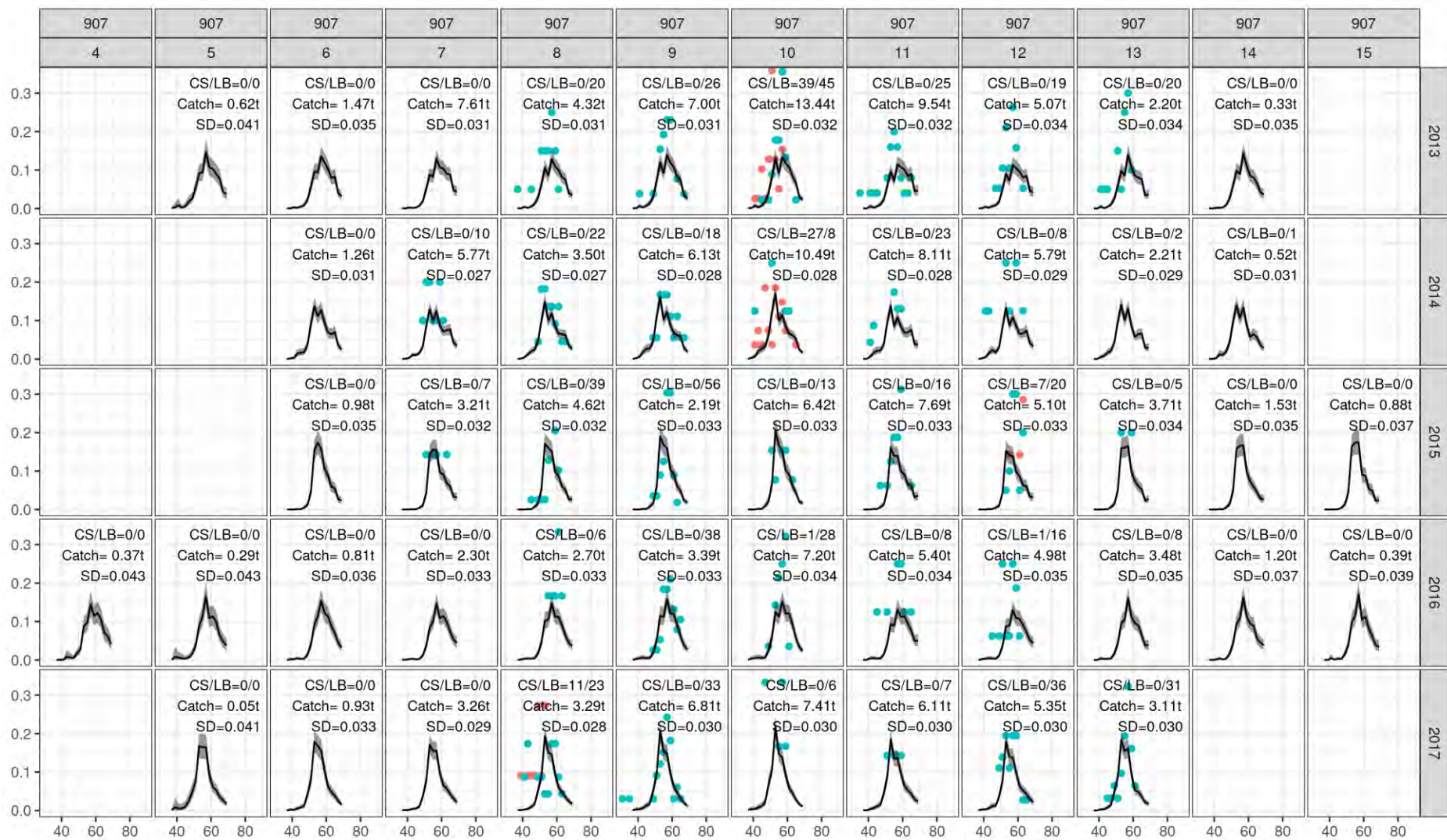


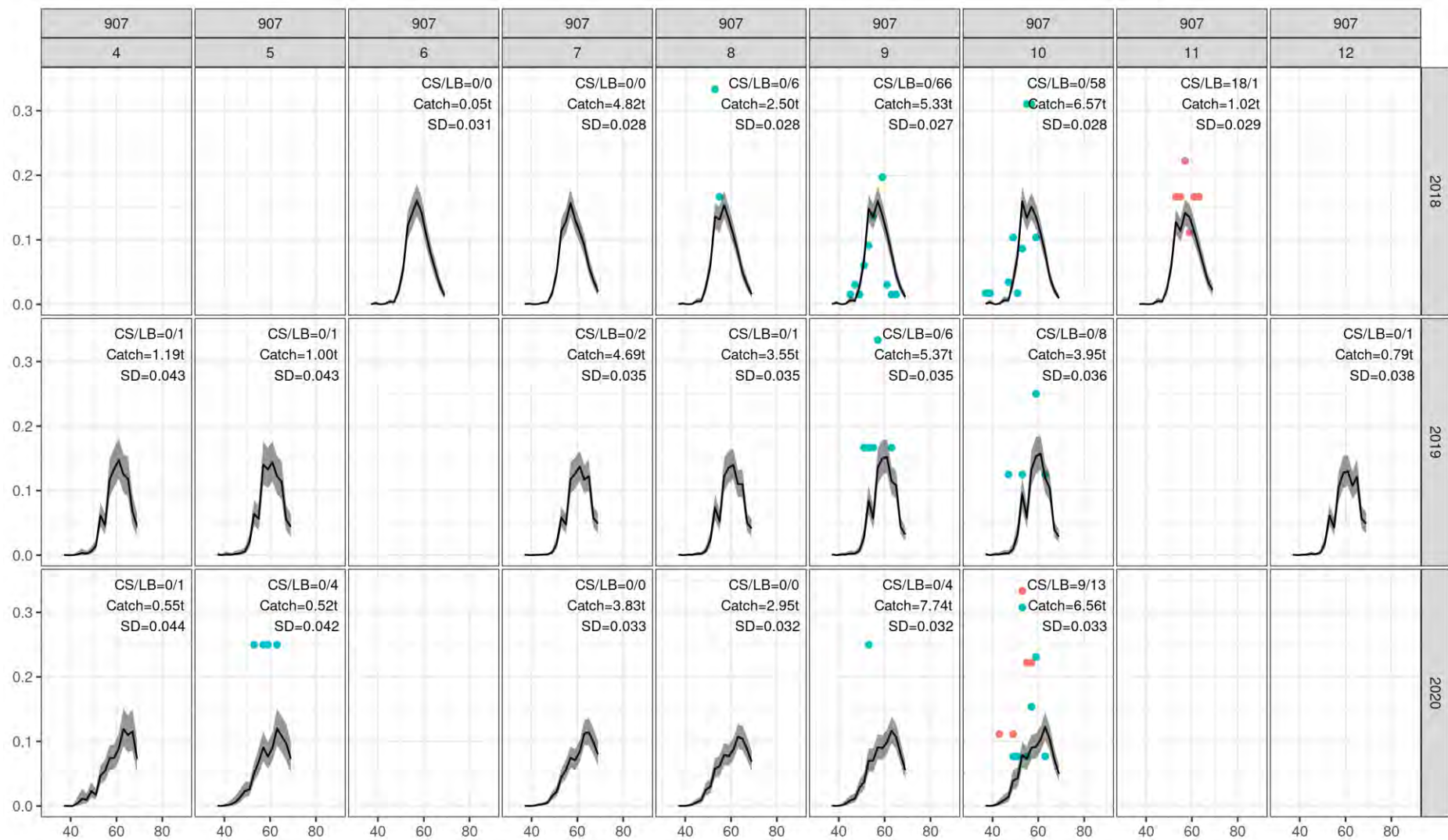


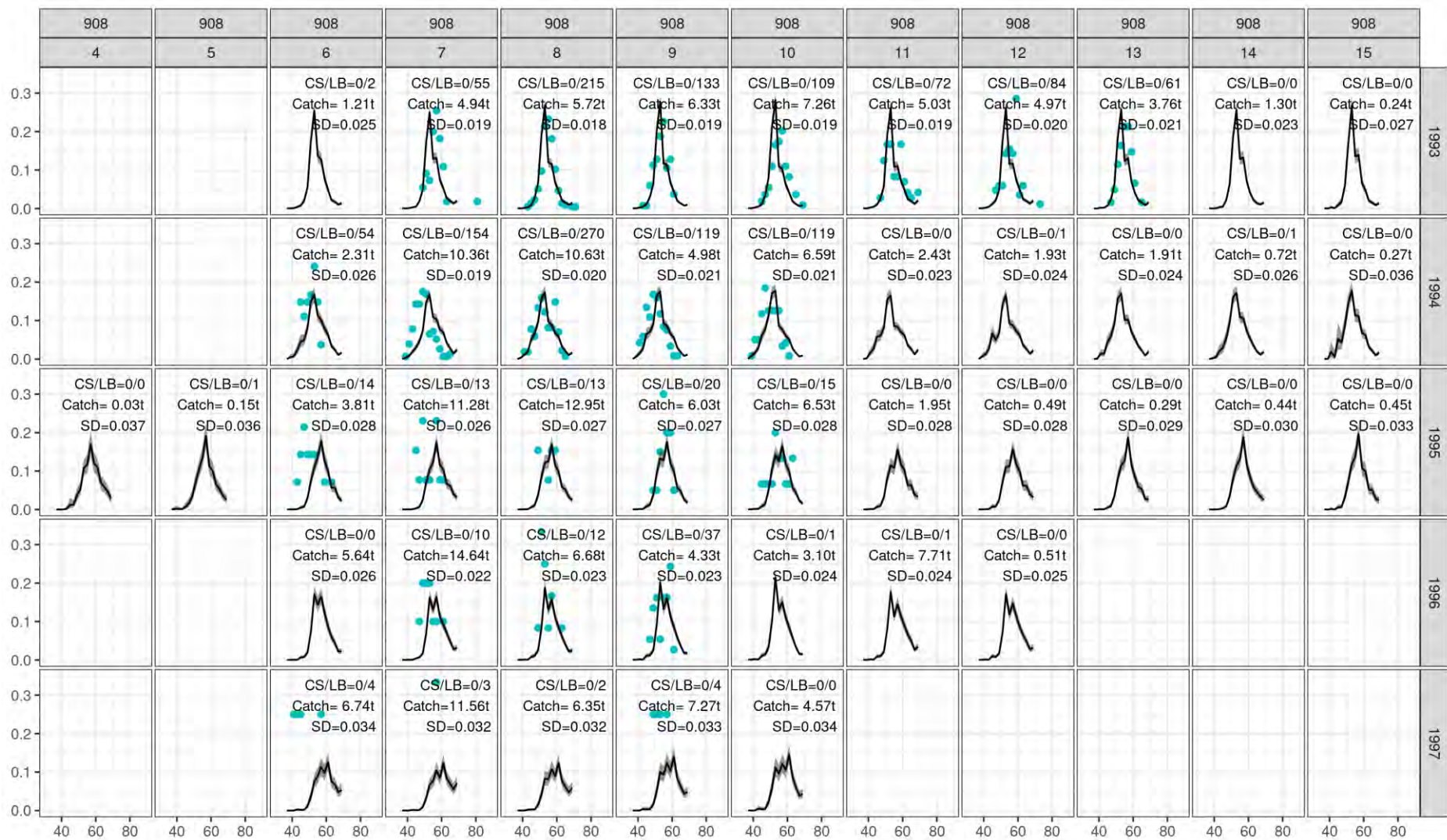


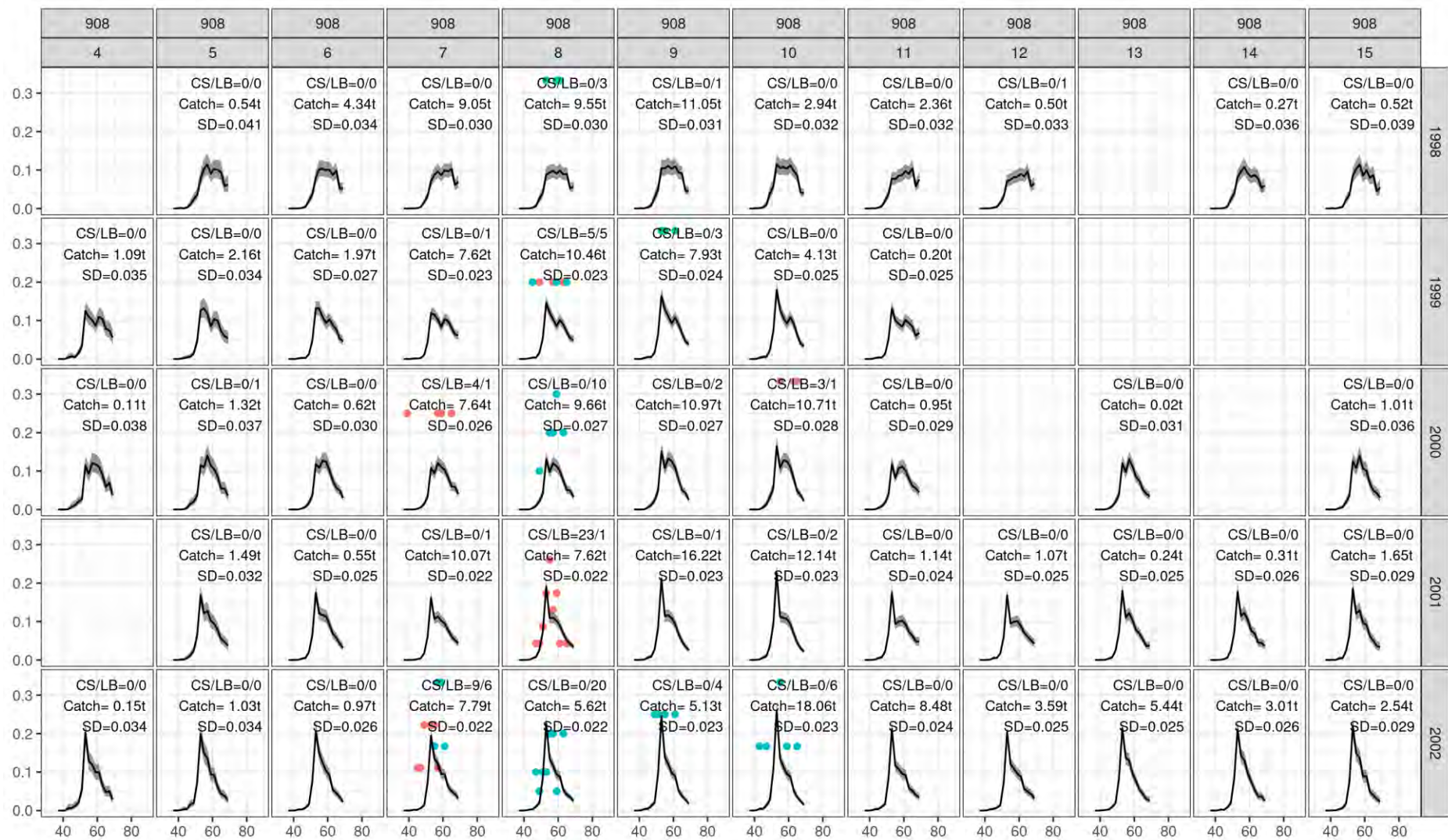


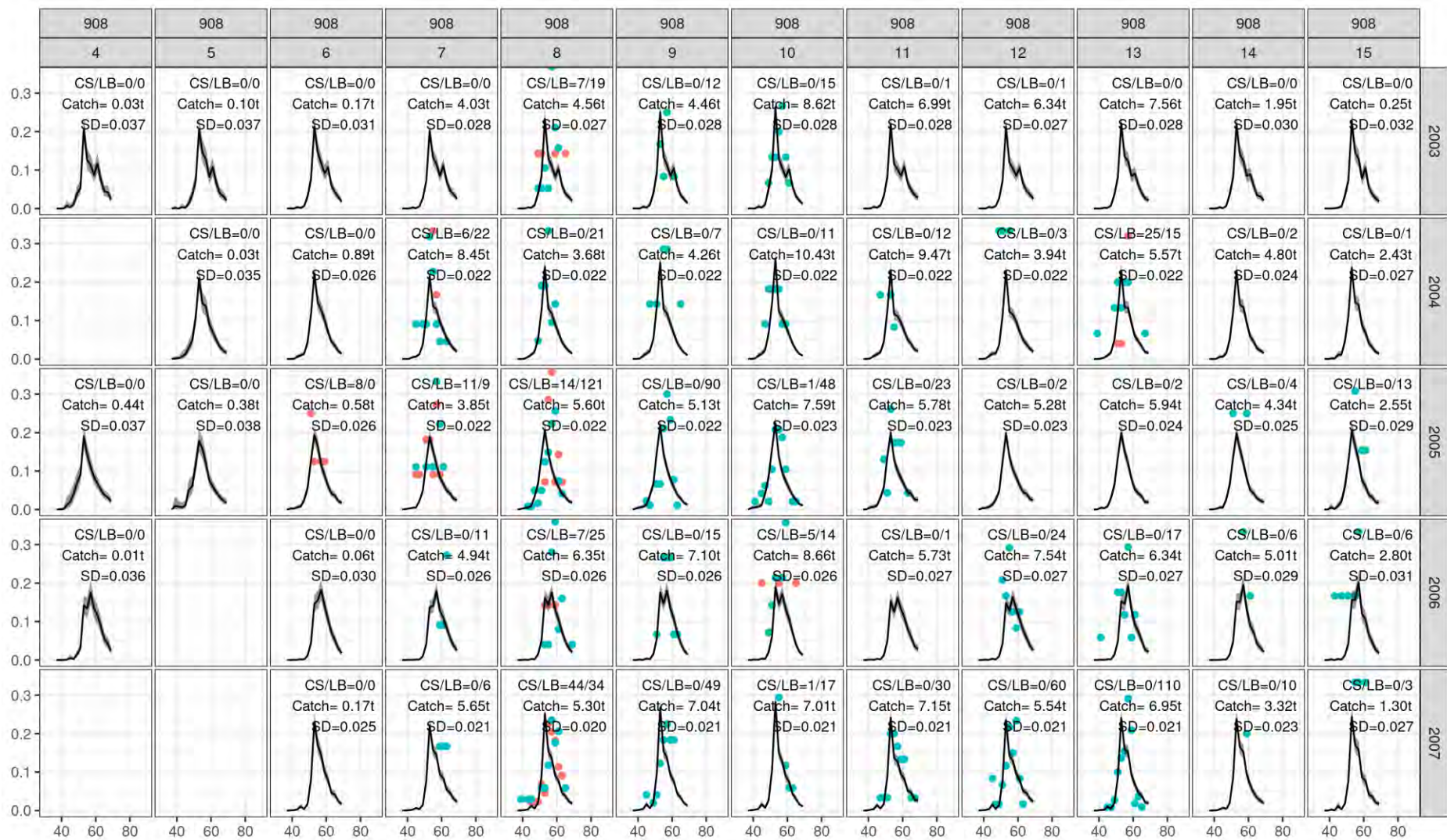


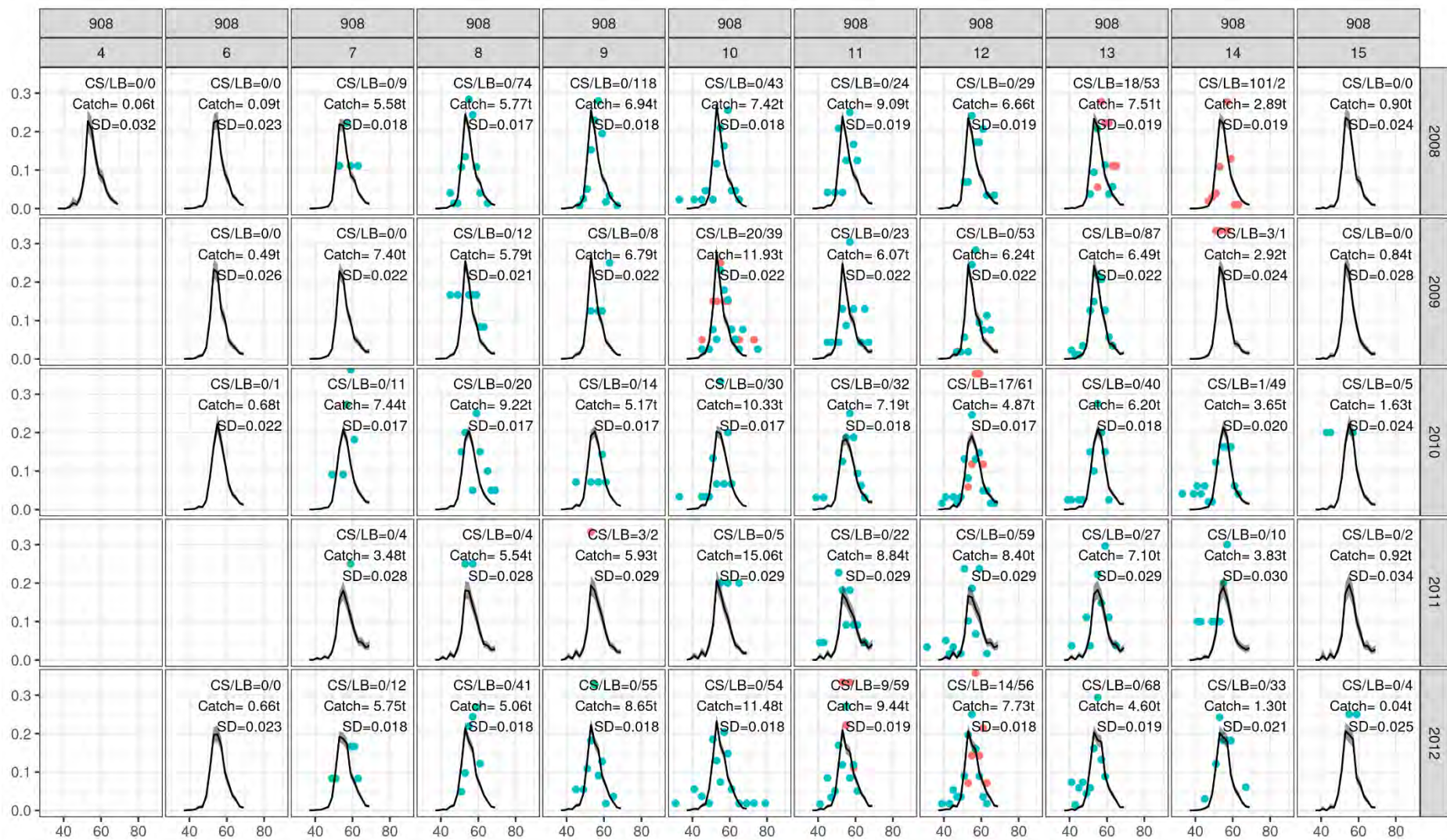


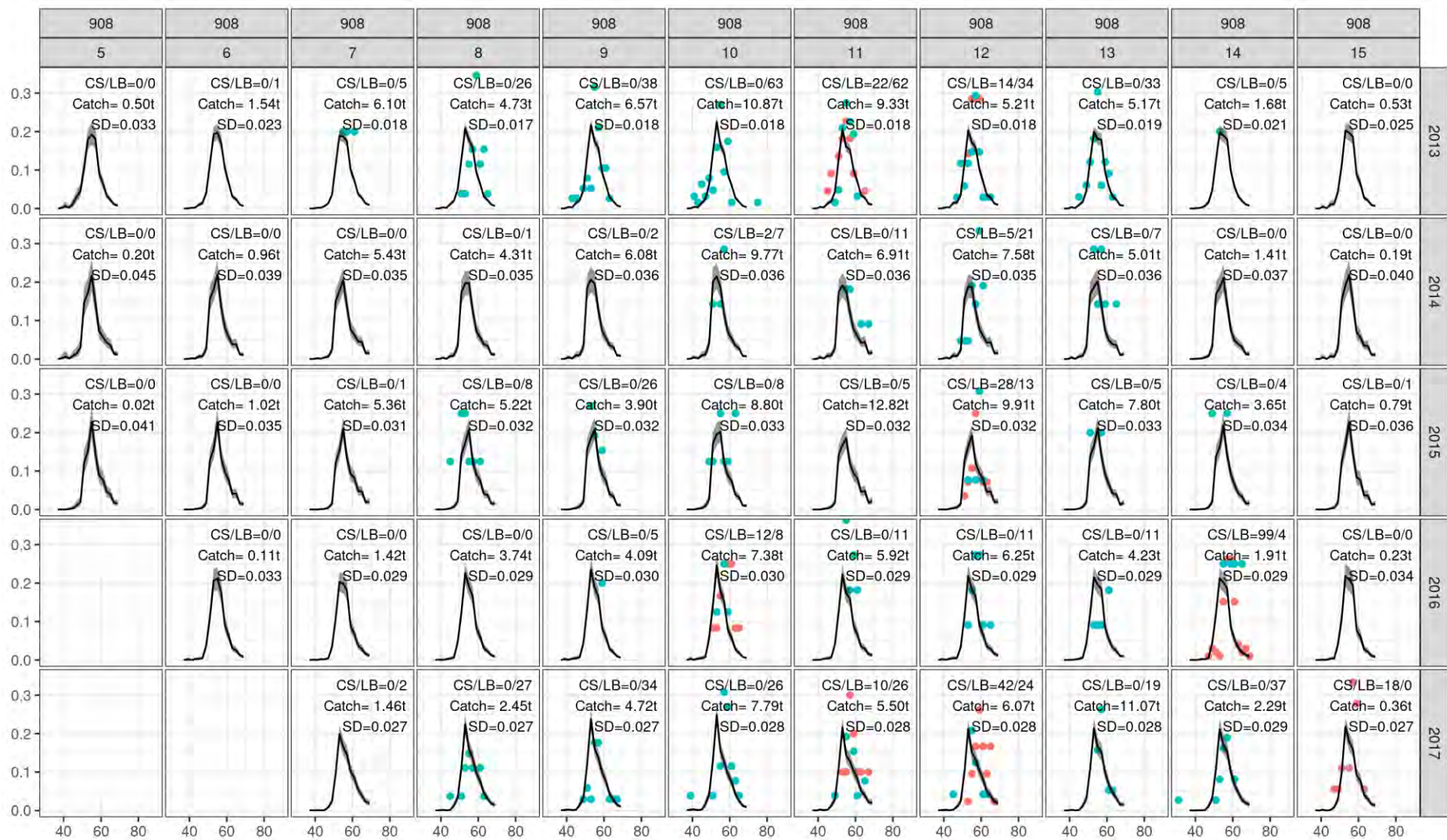


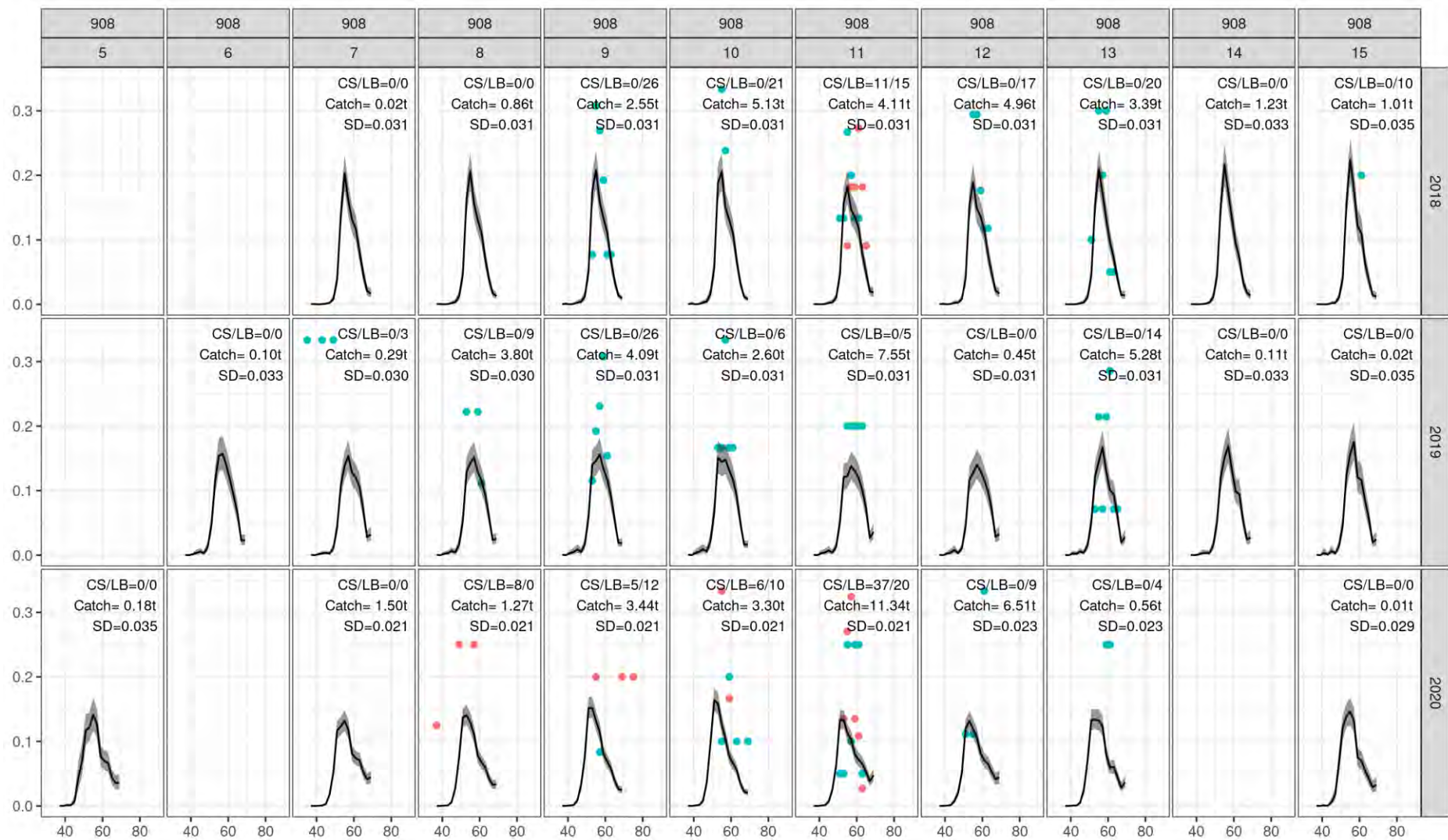




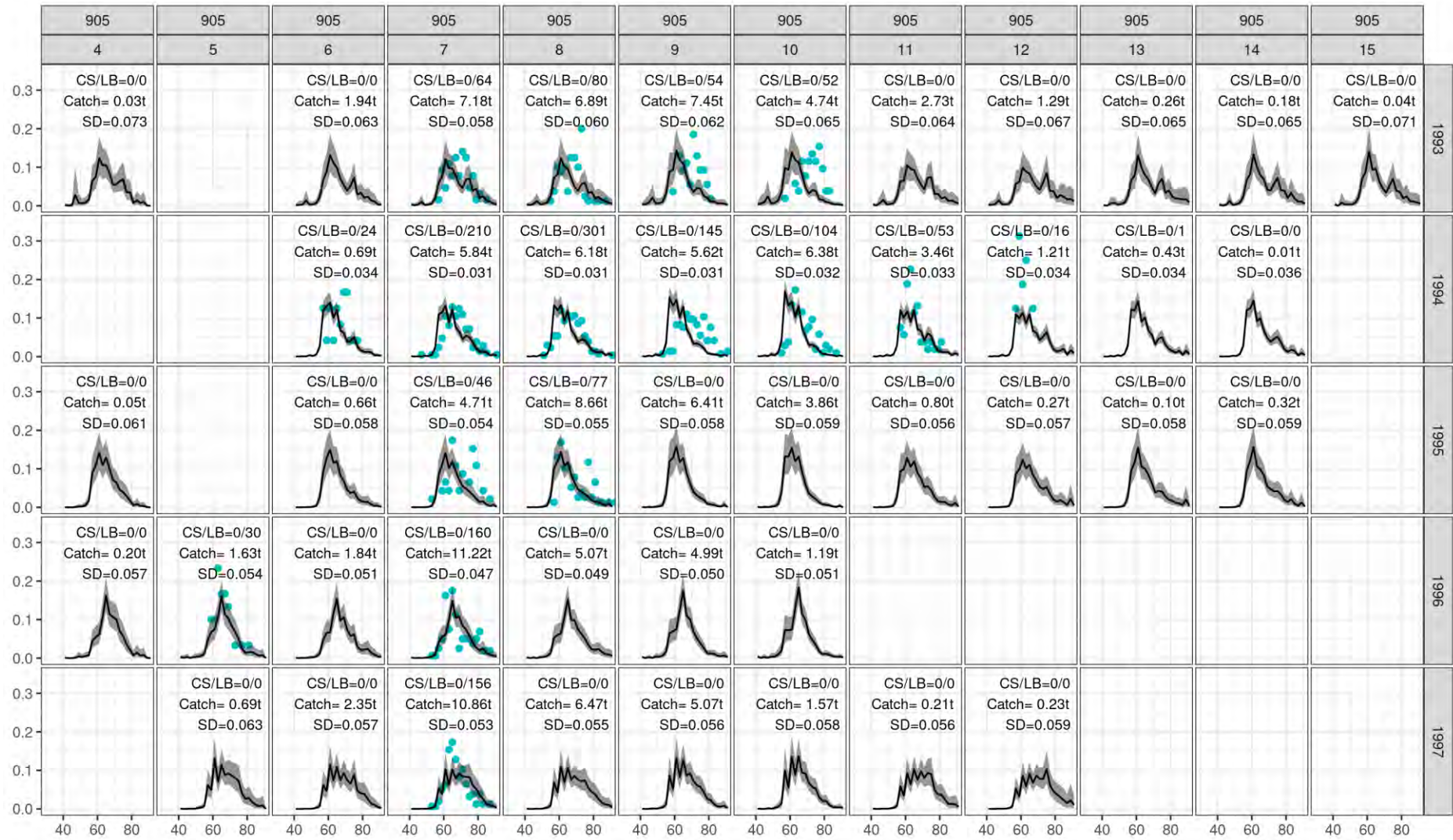


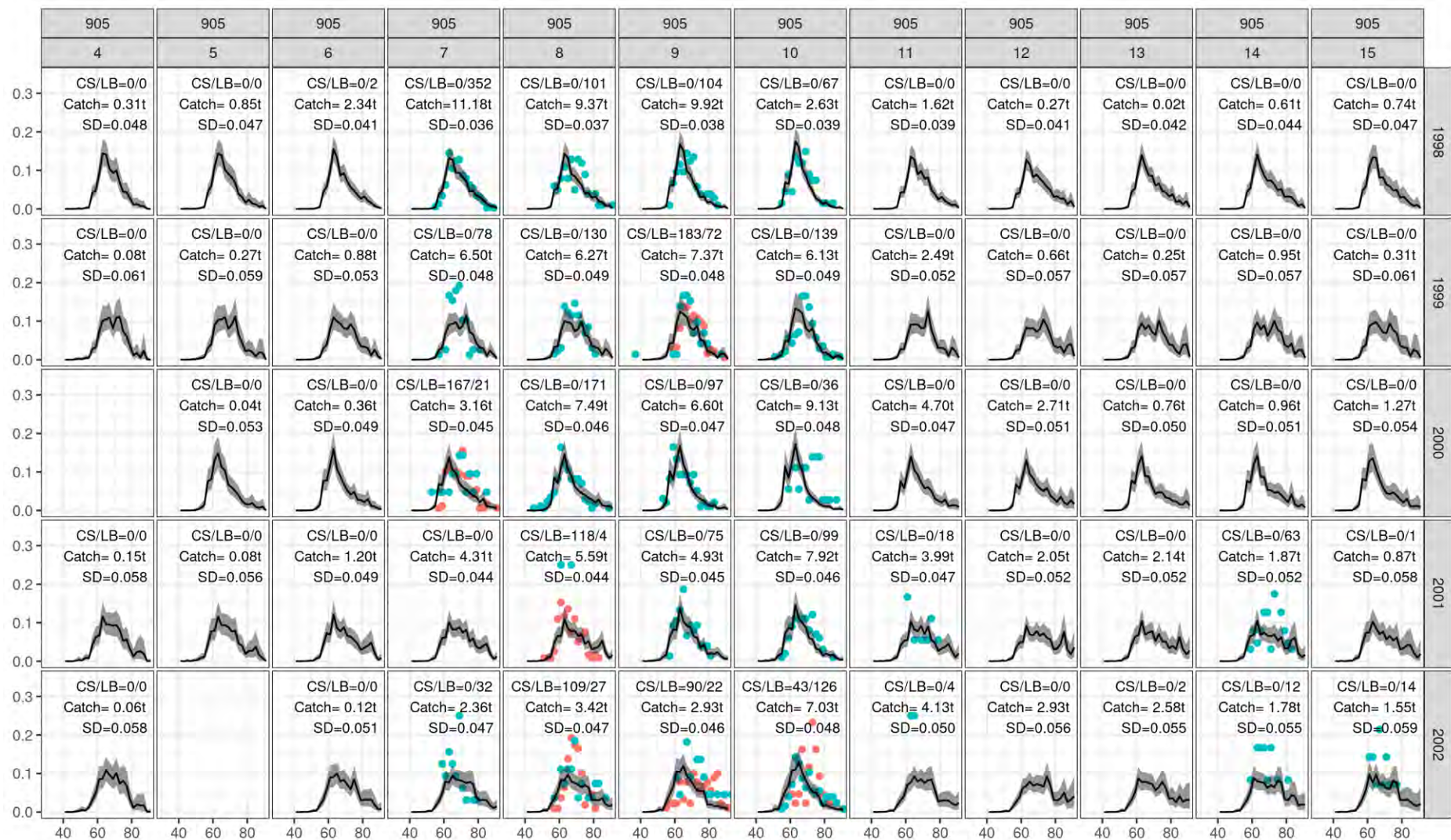


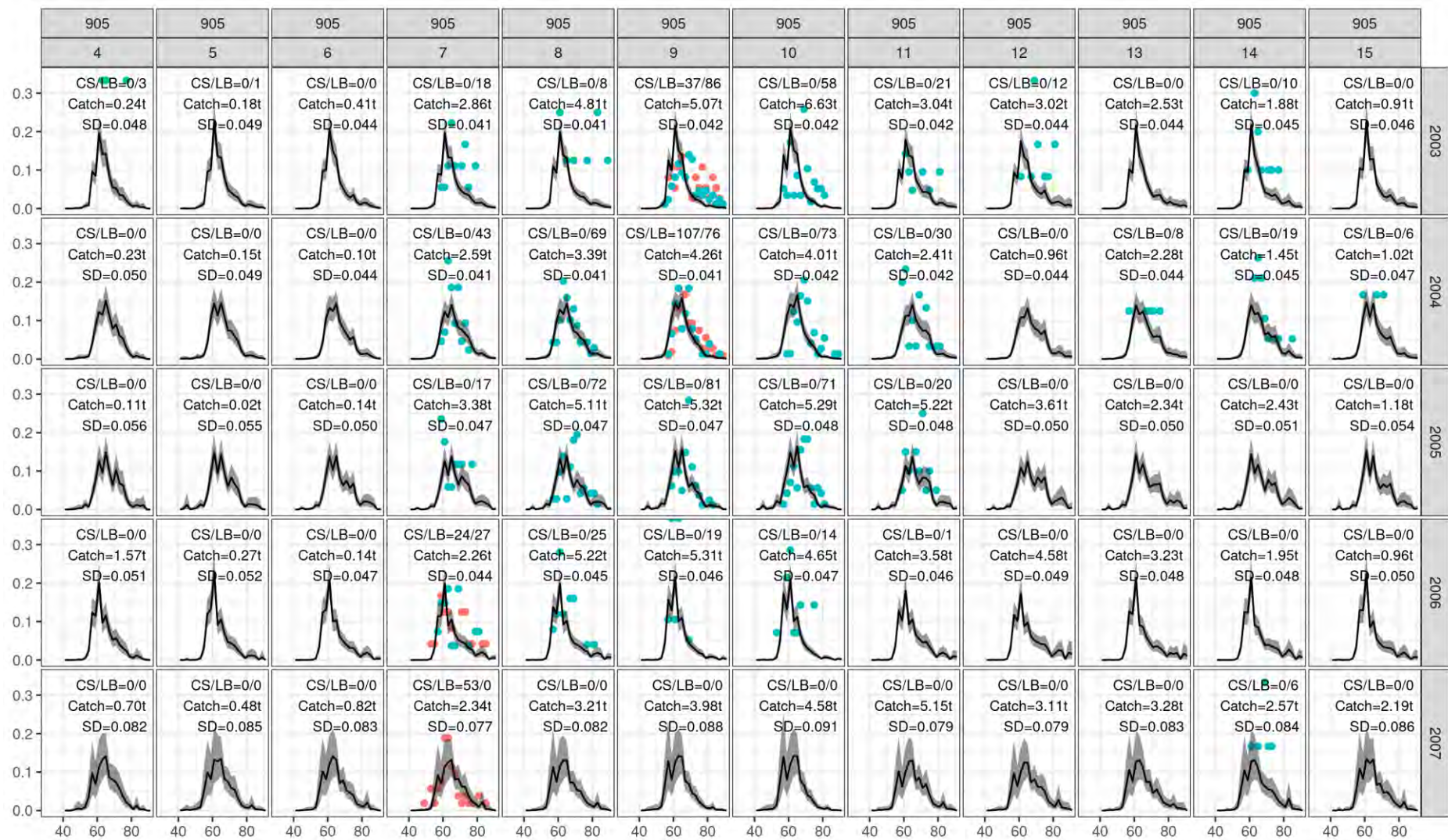


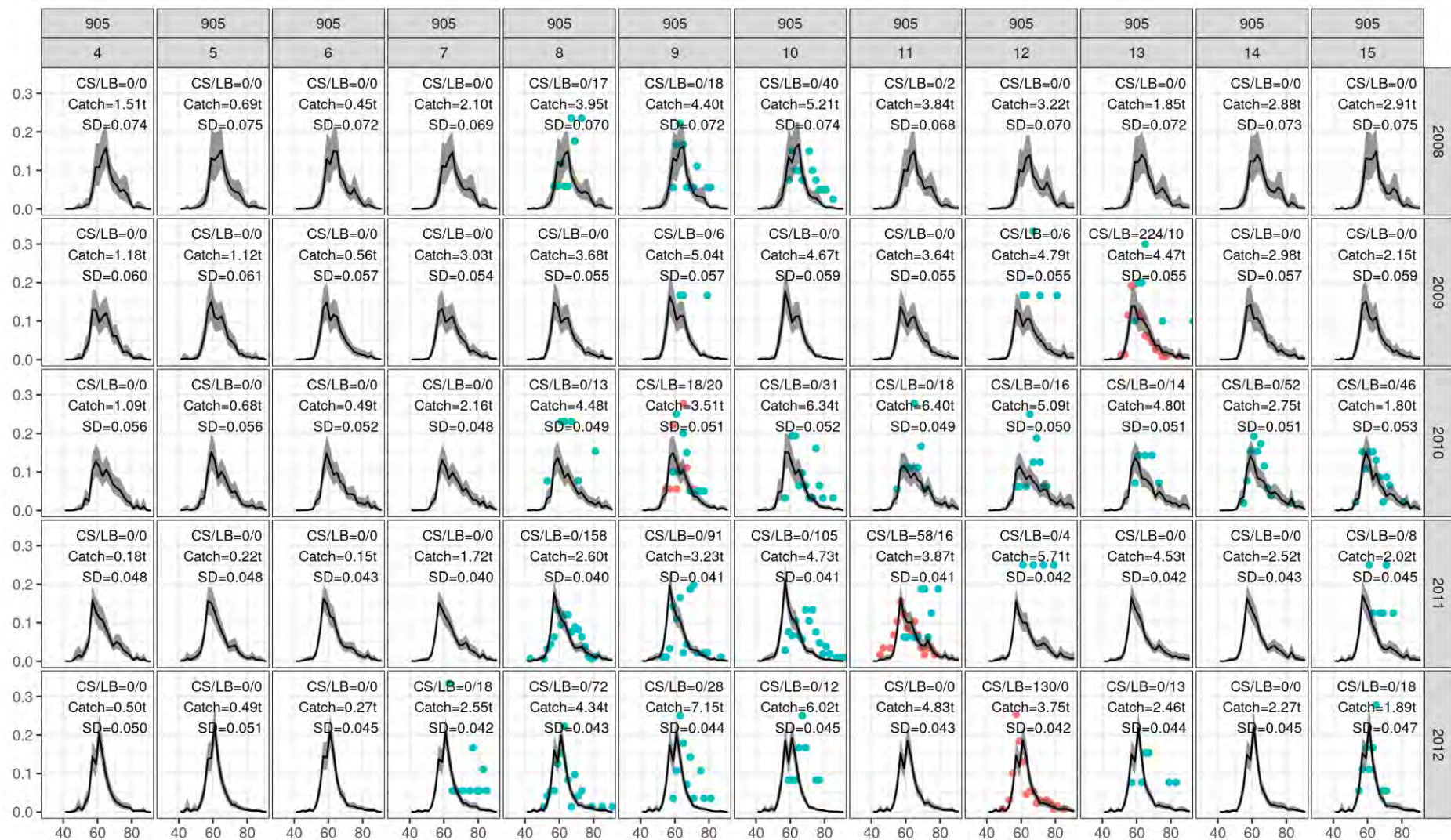


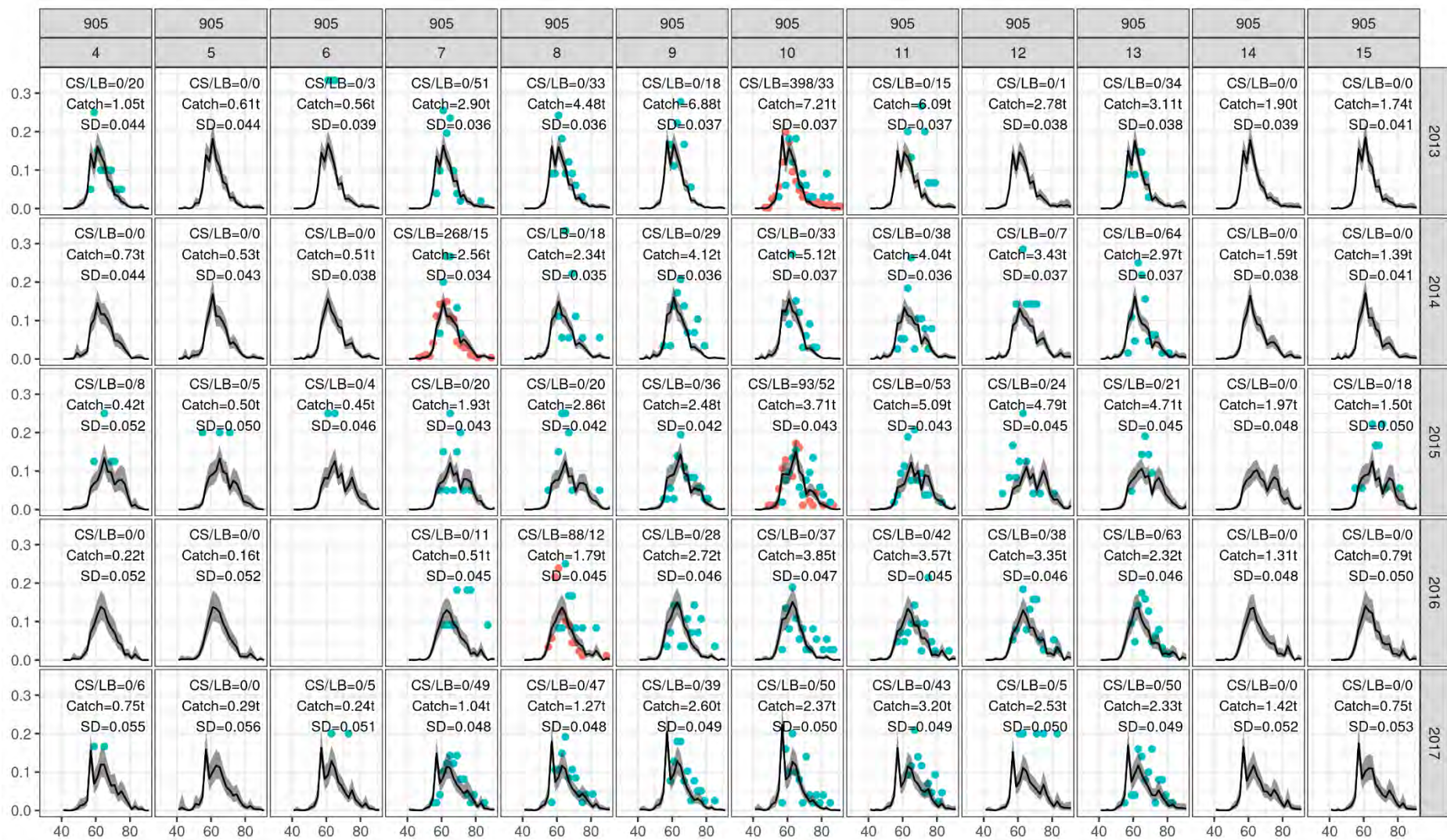
11.3 CRA 2 mature females

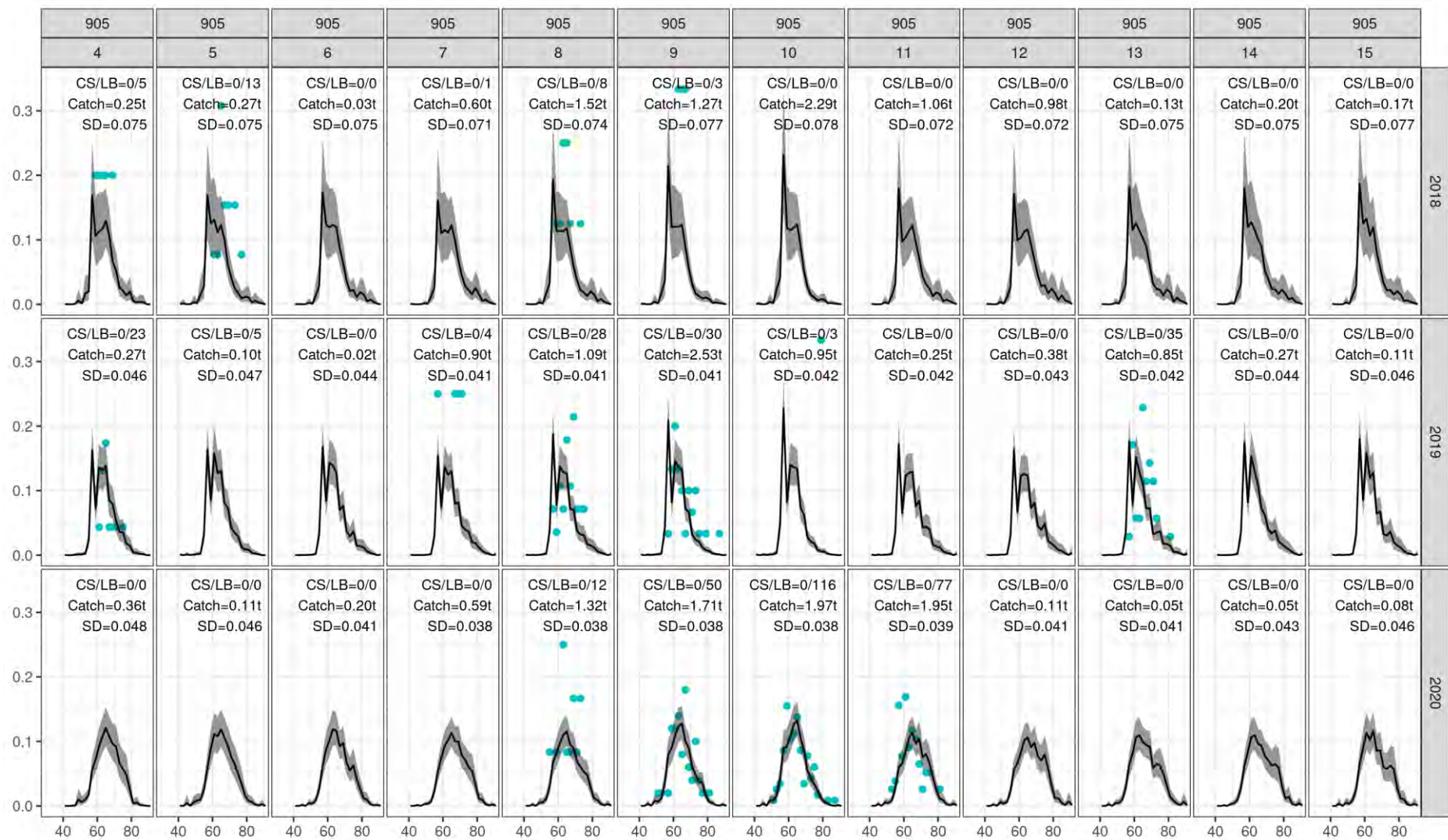


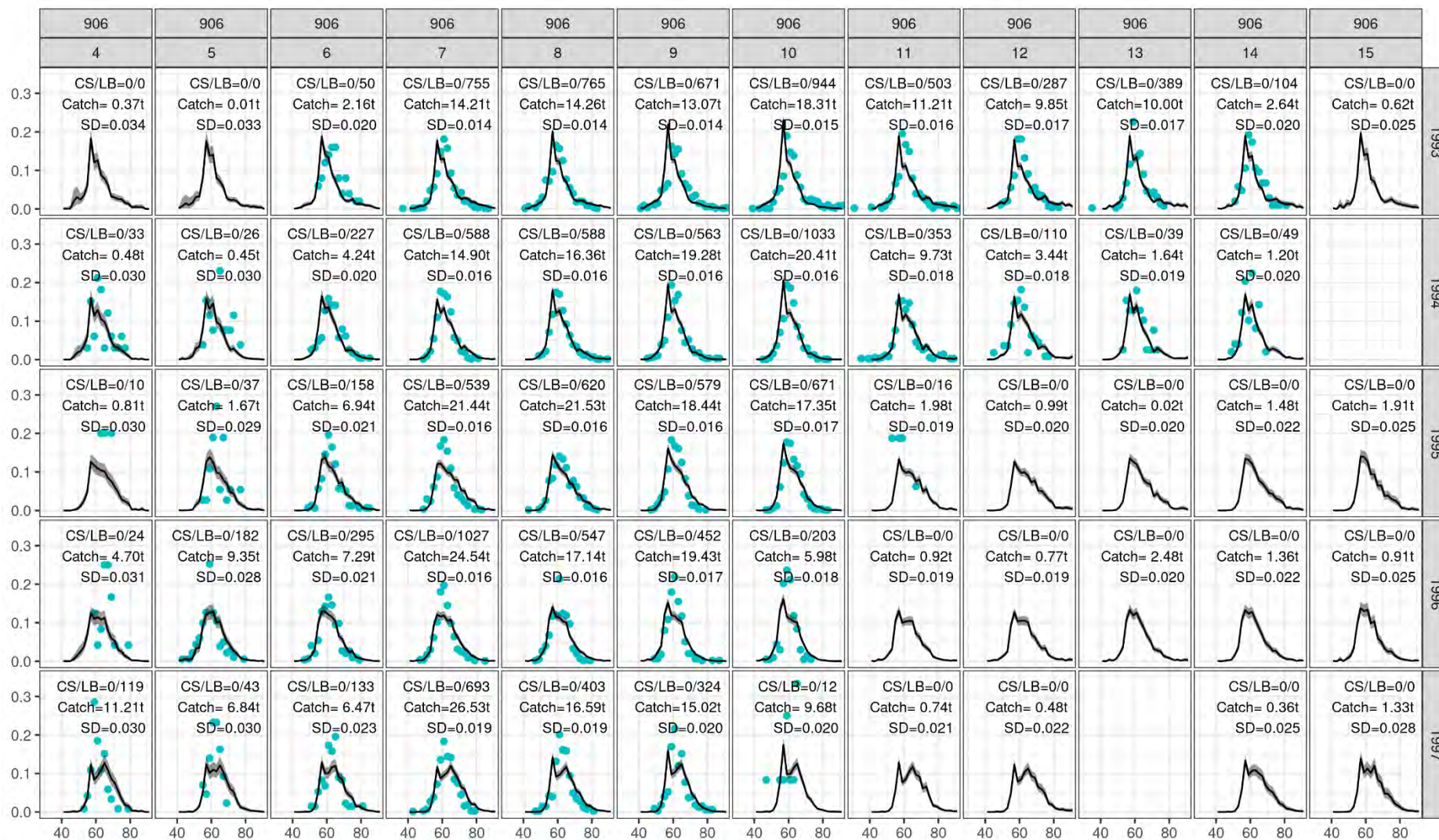


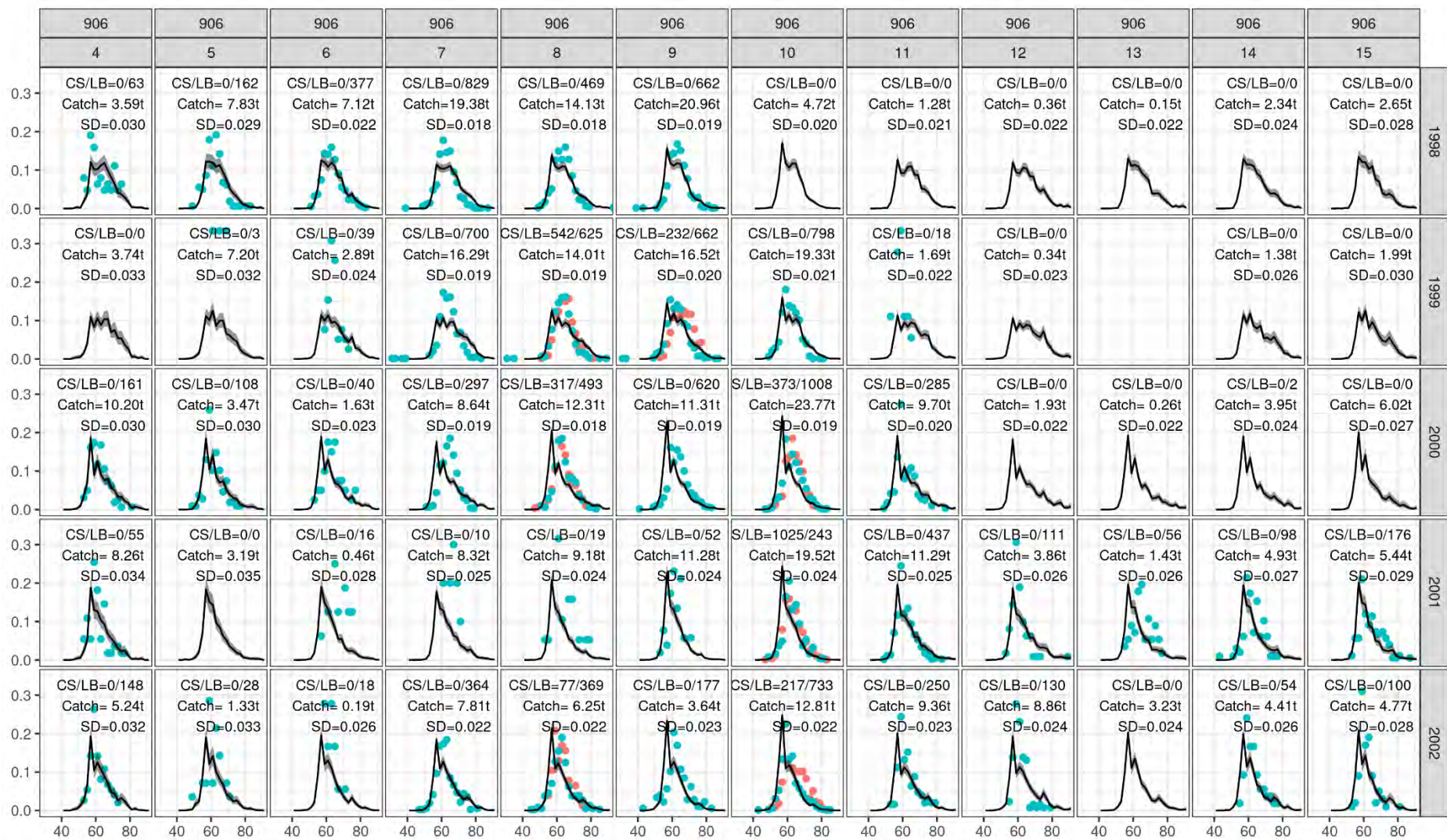


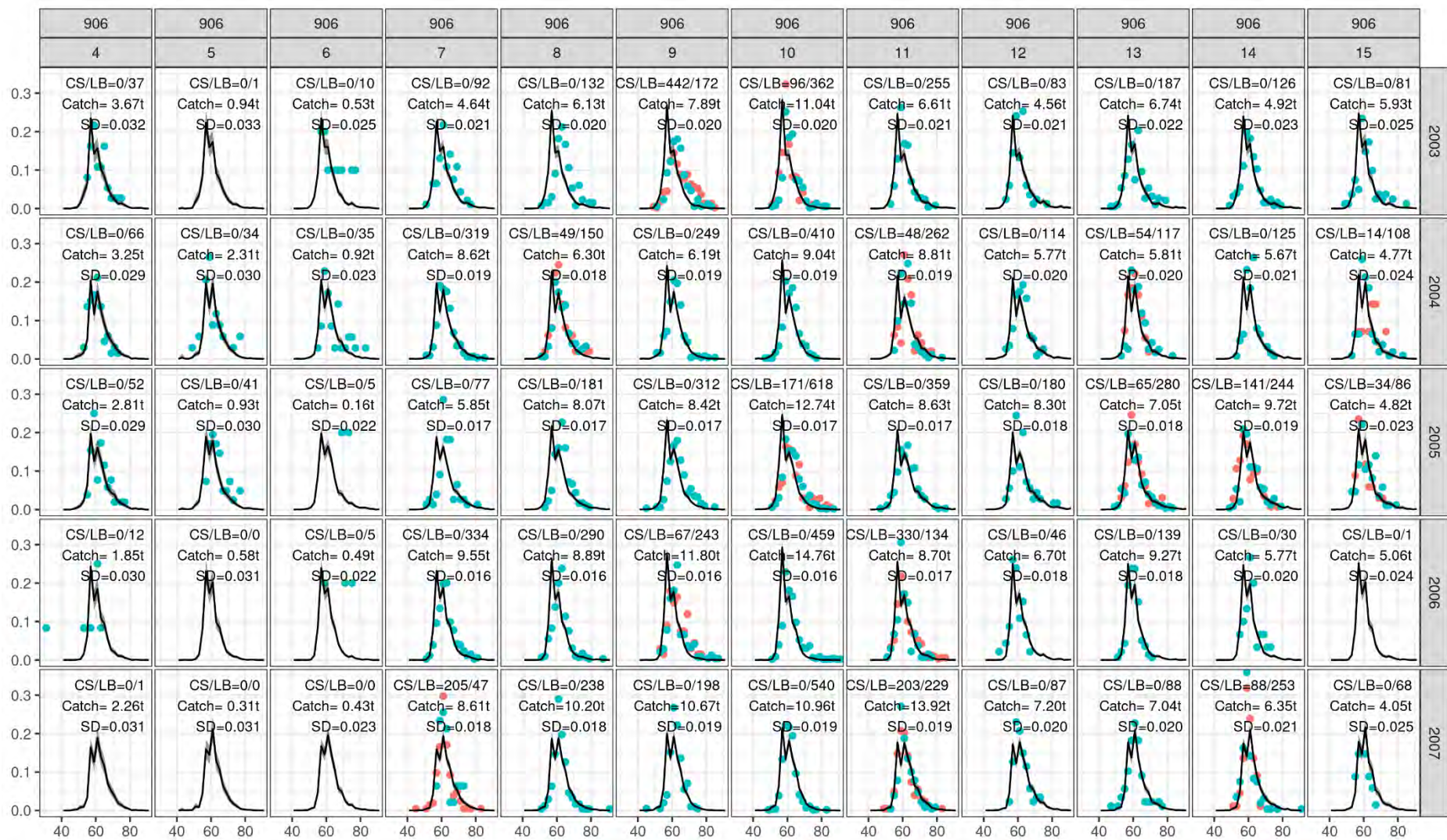


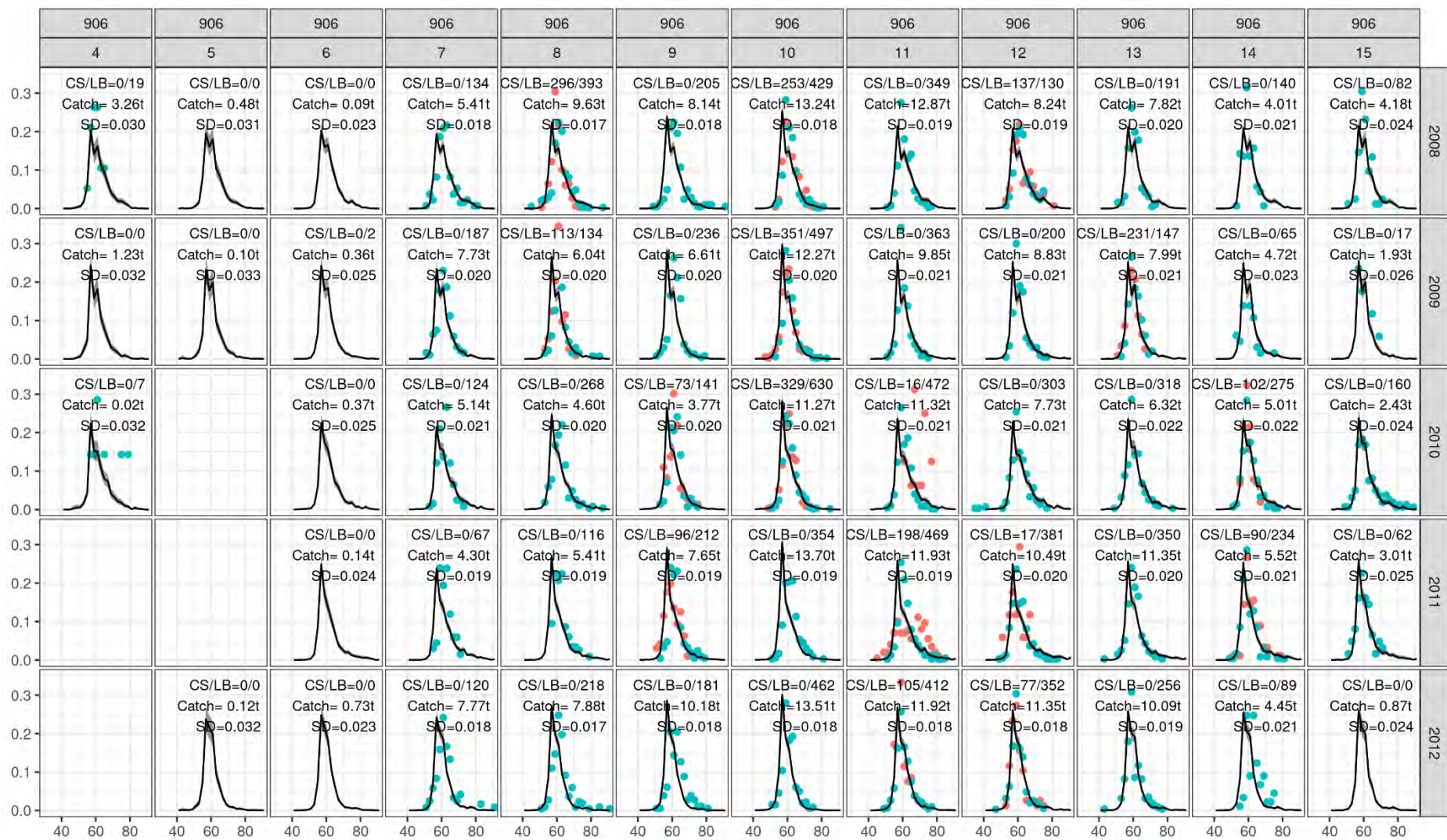


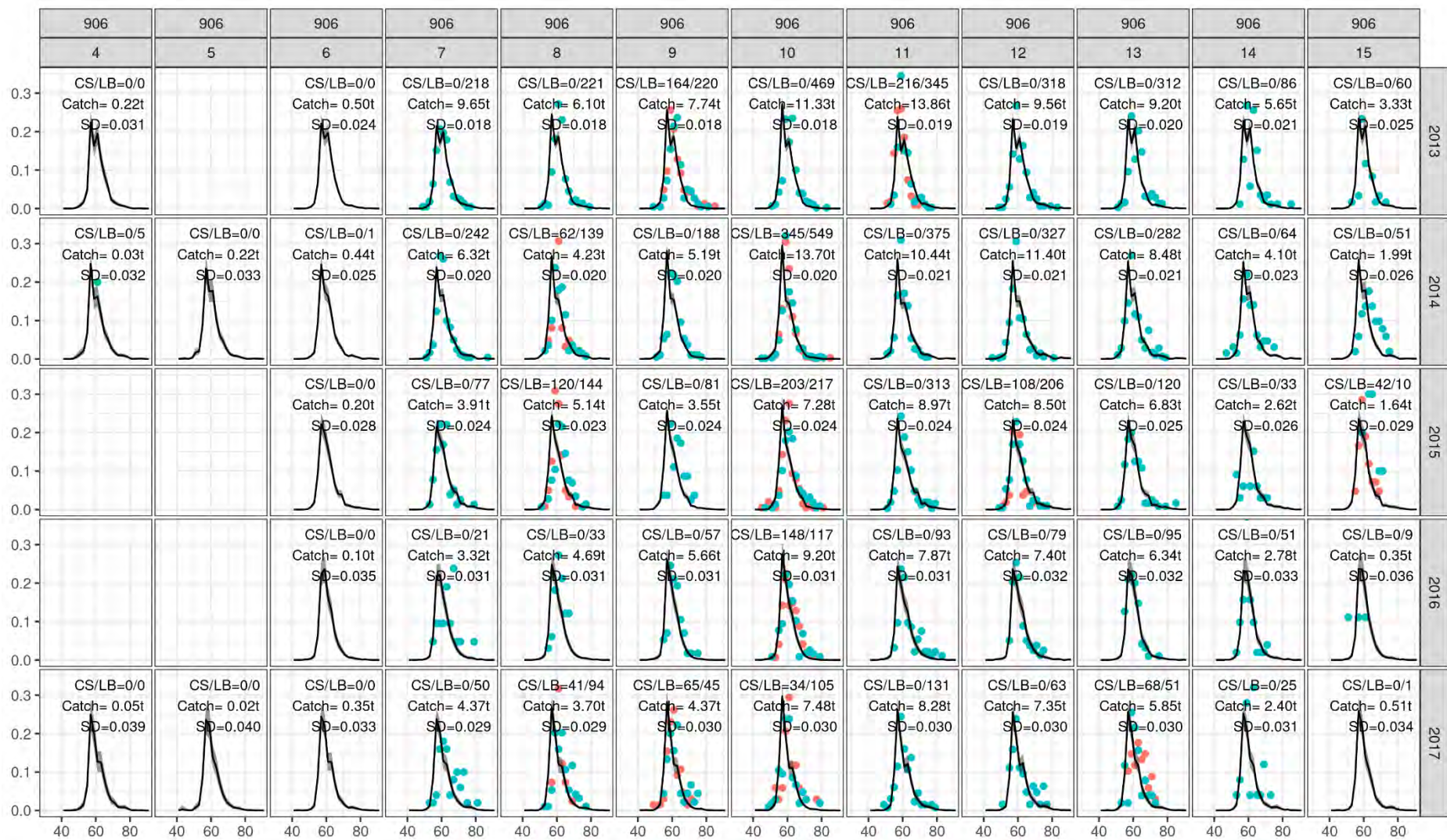


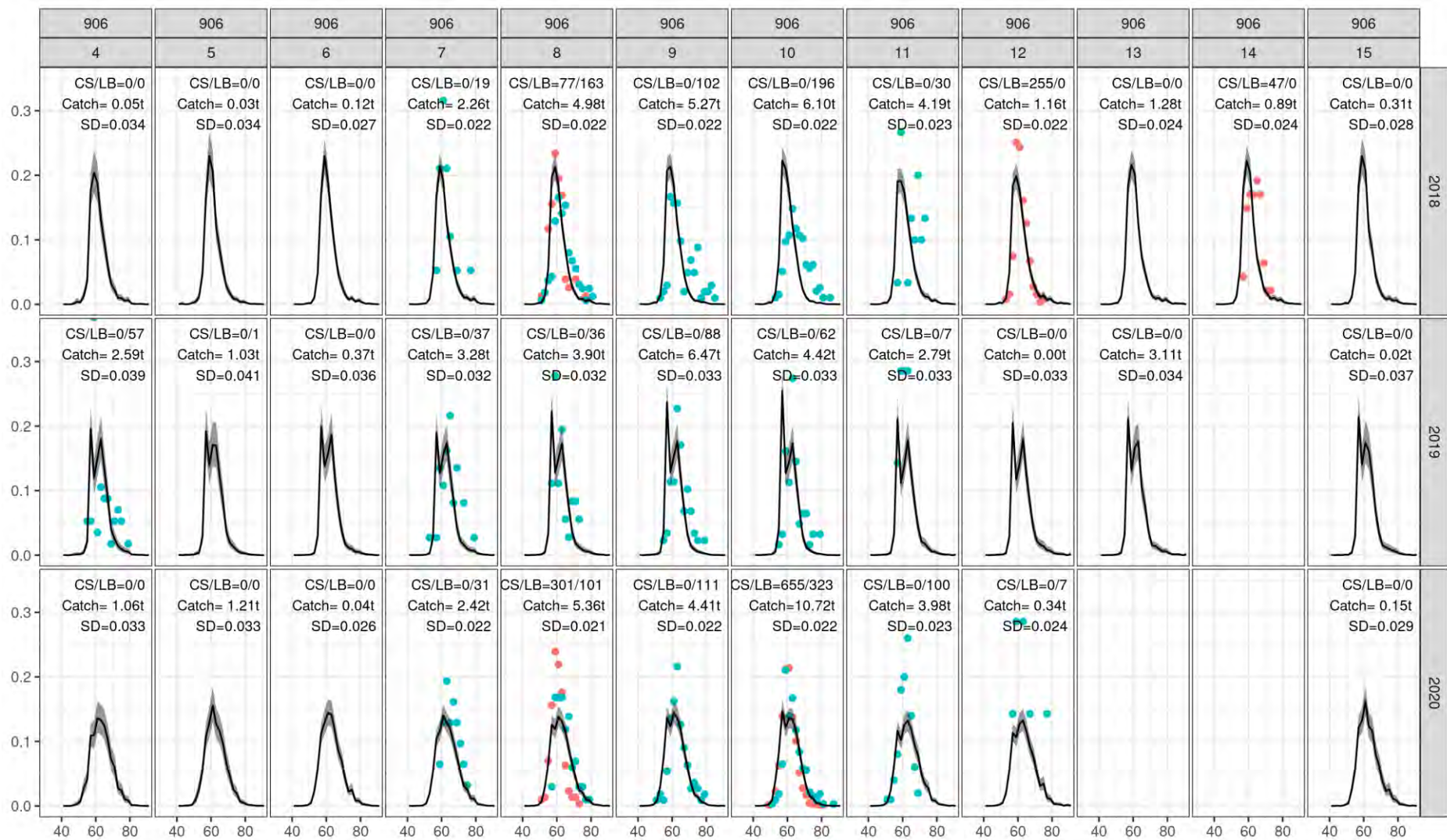


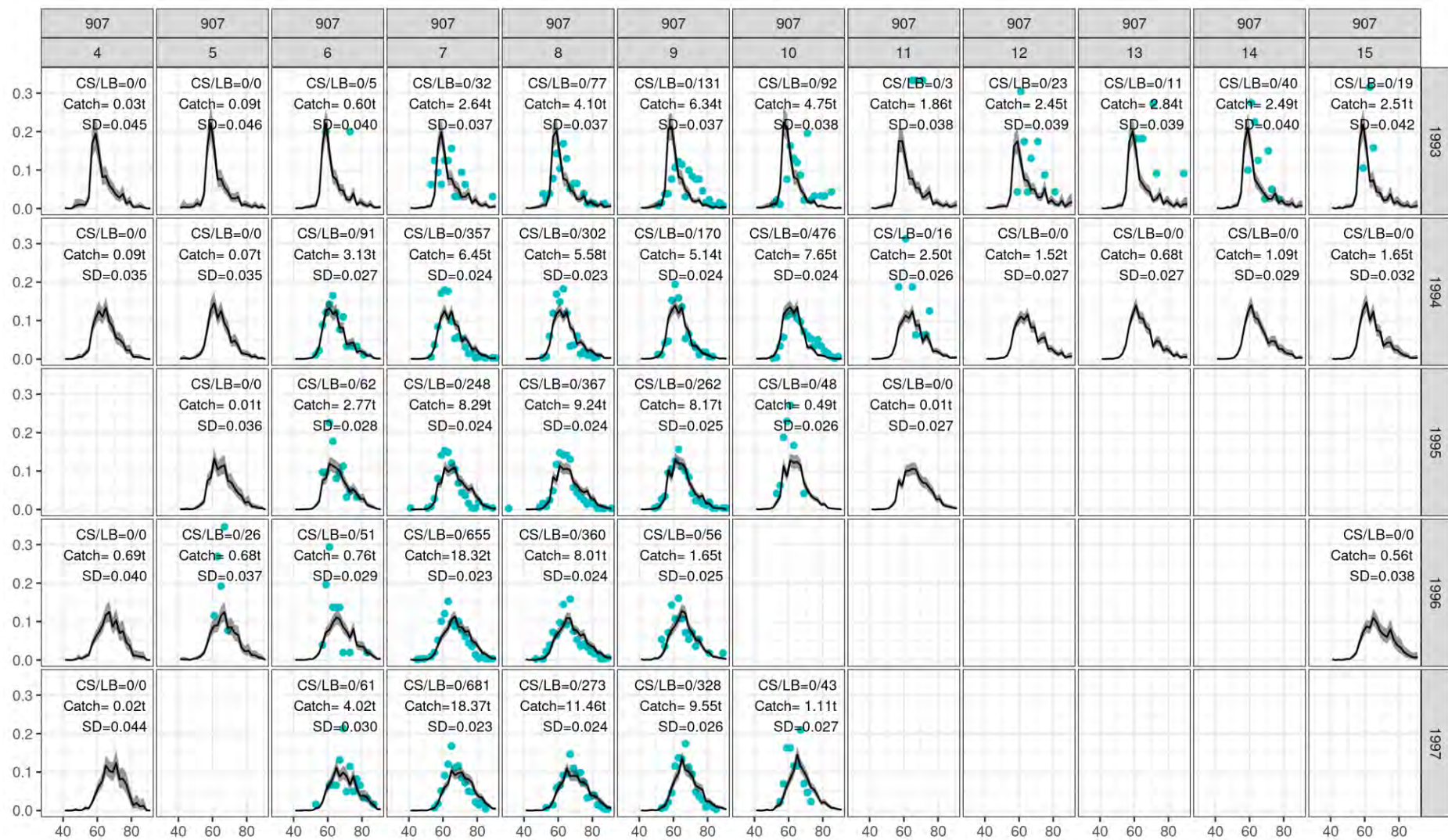


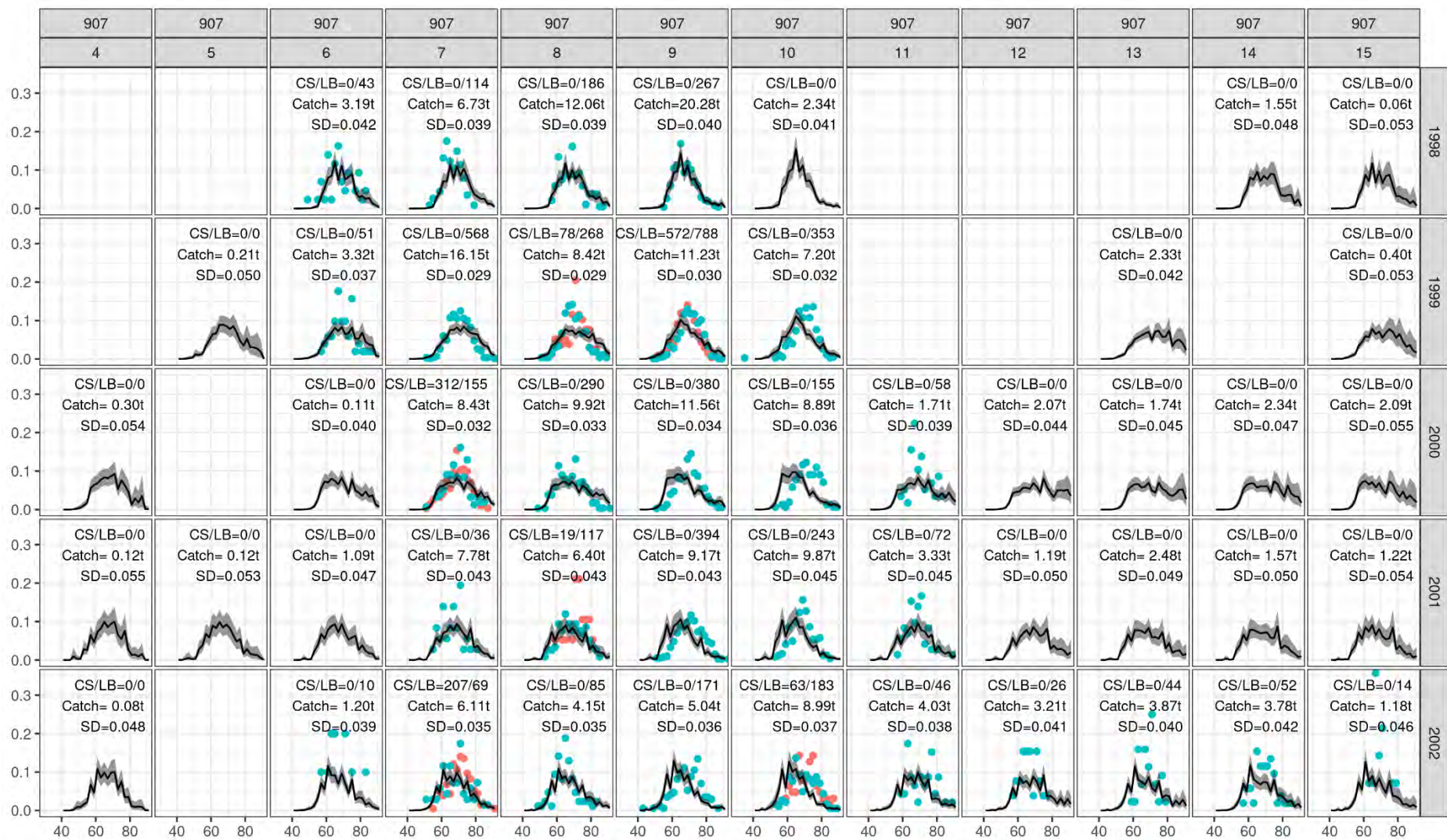


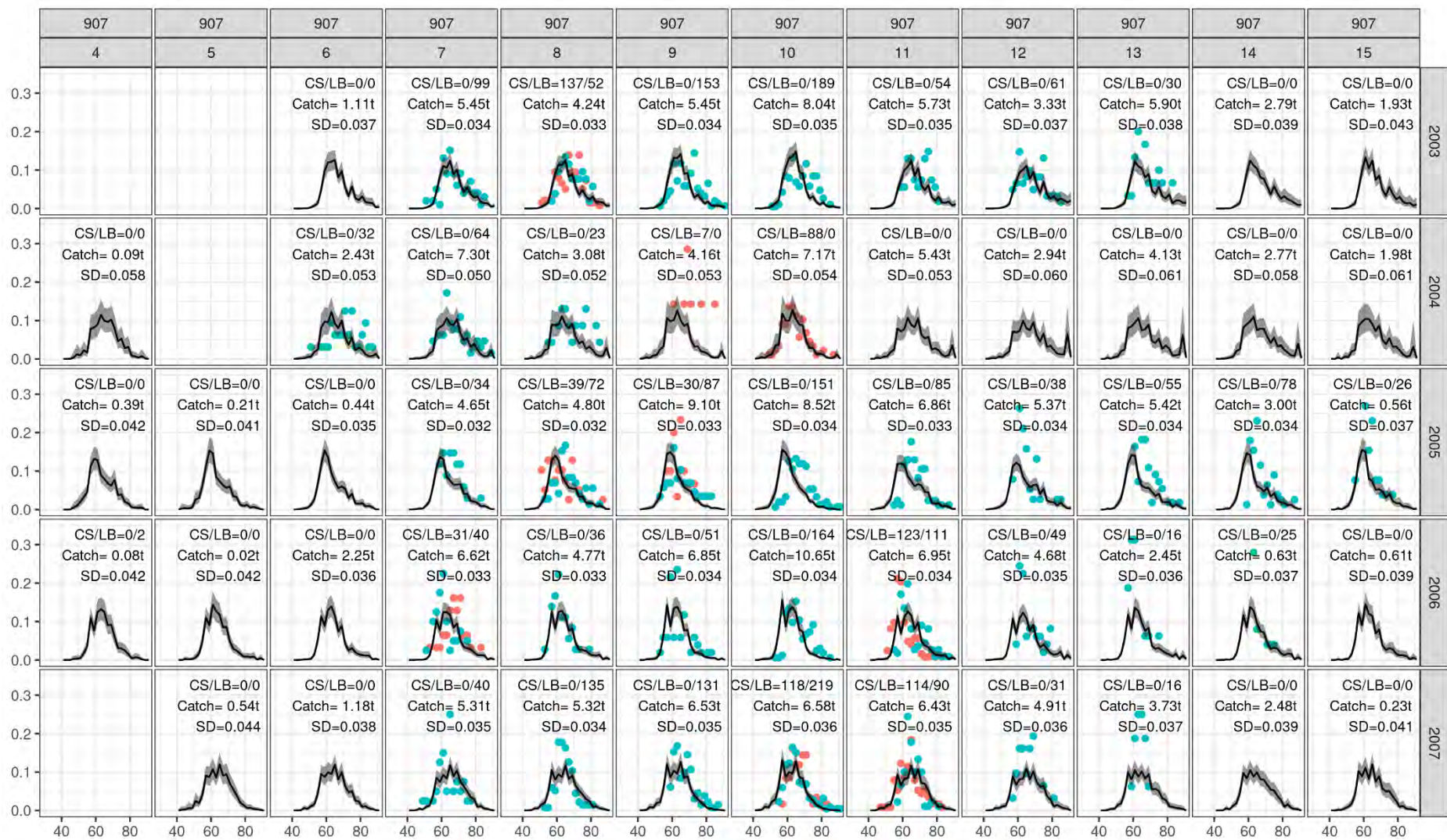


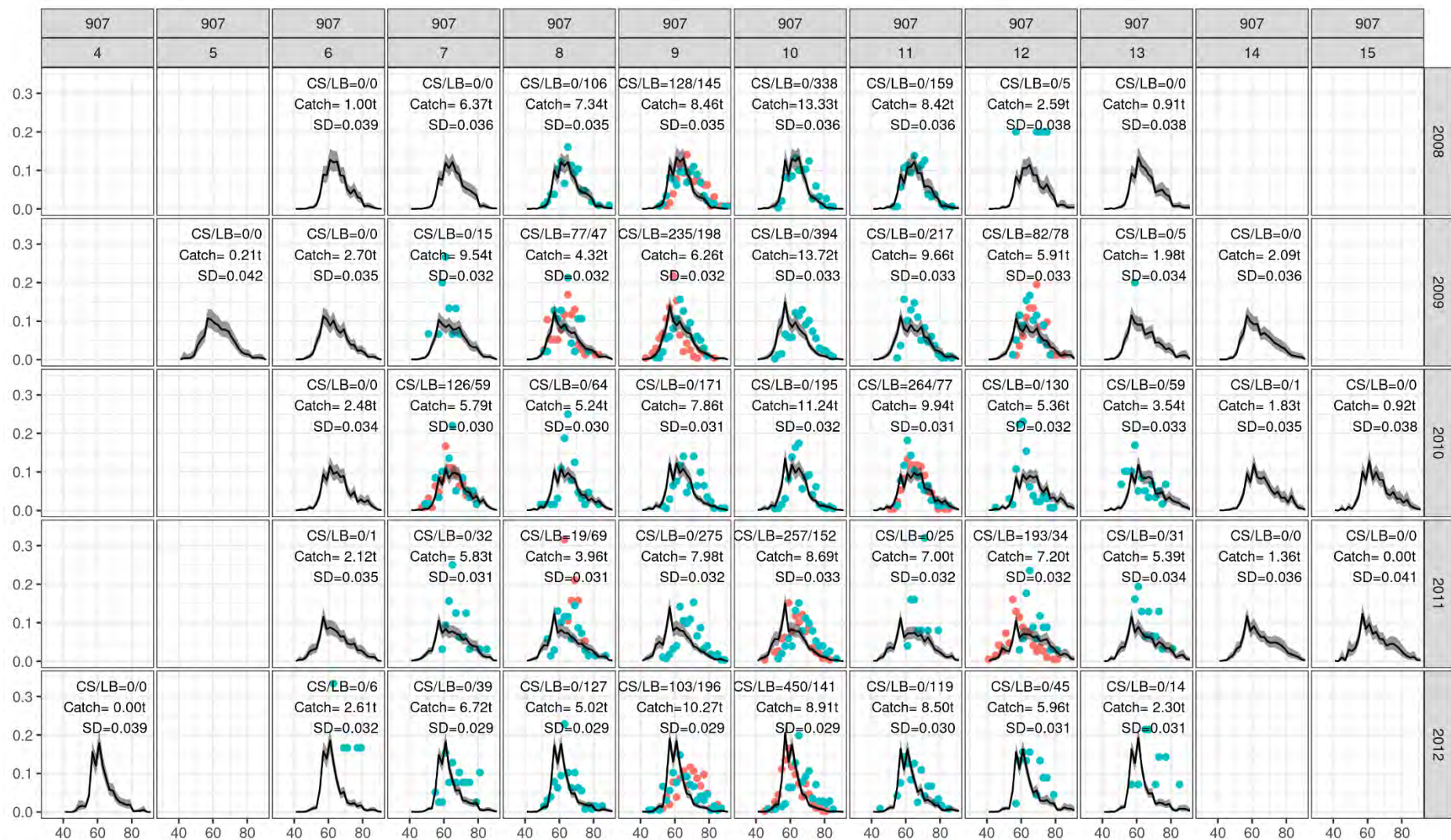


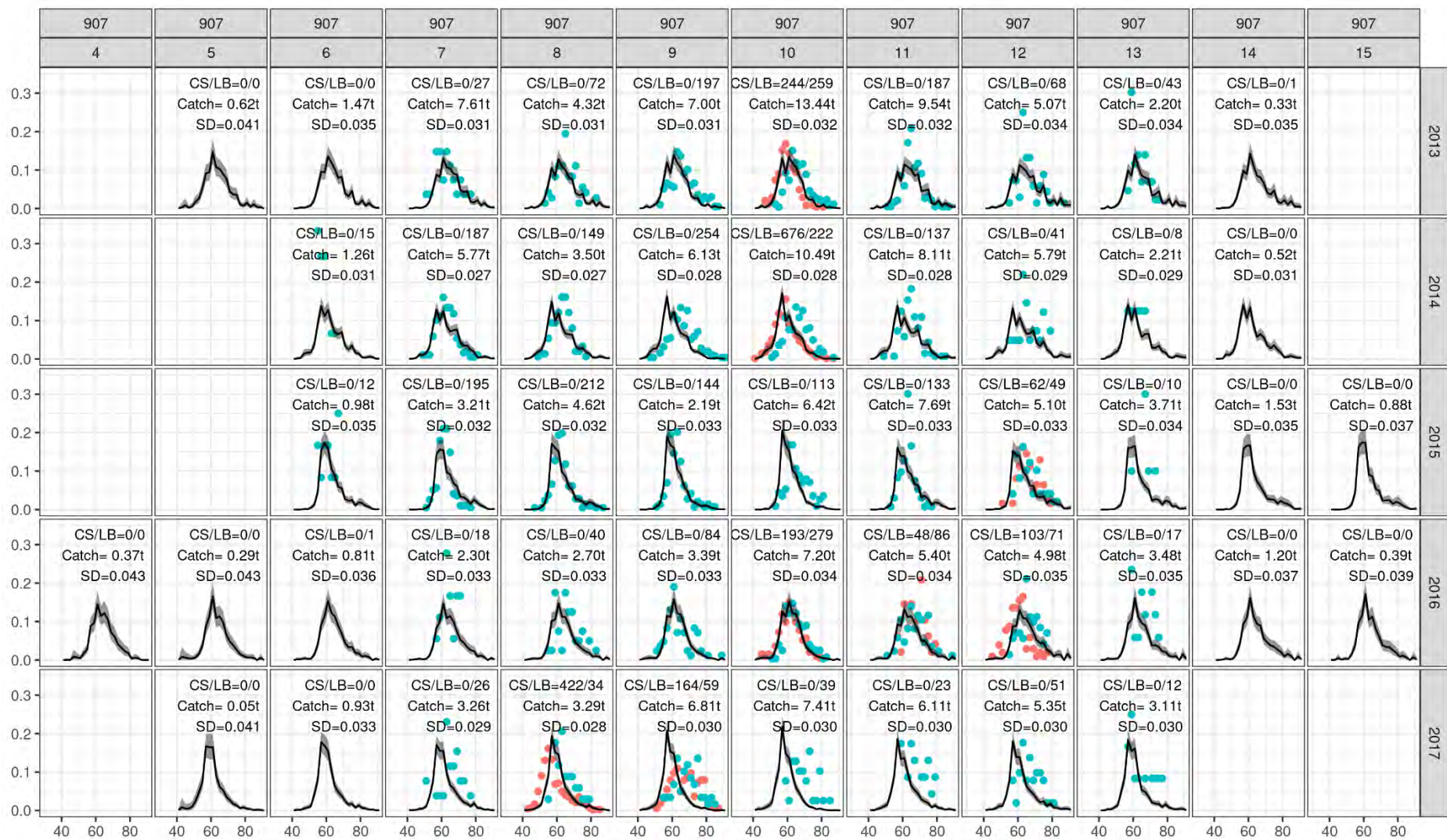


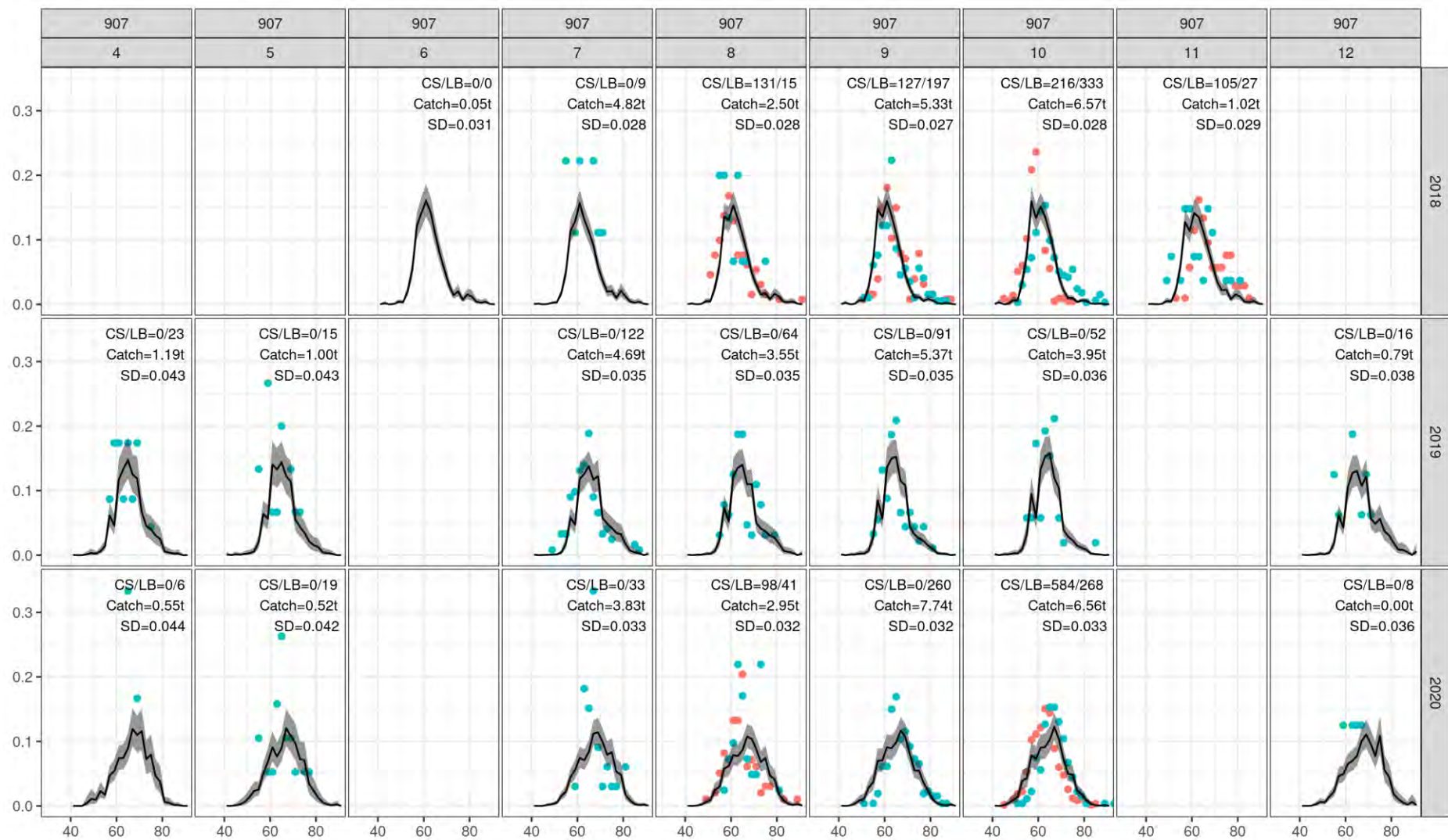


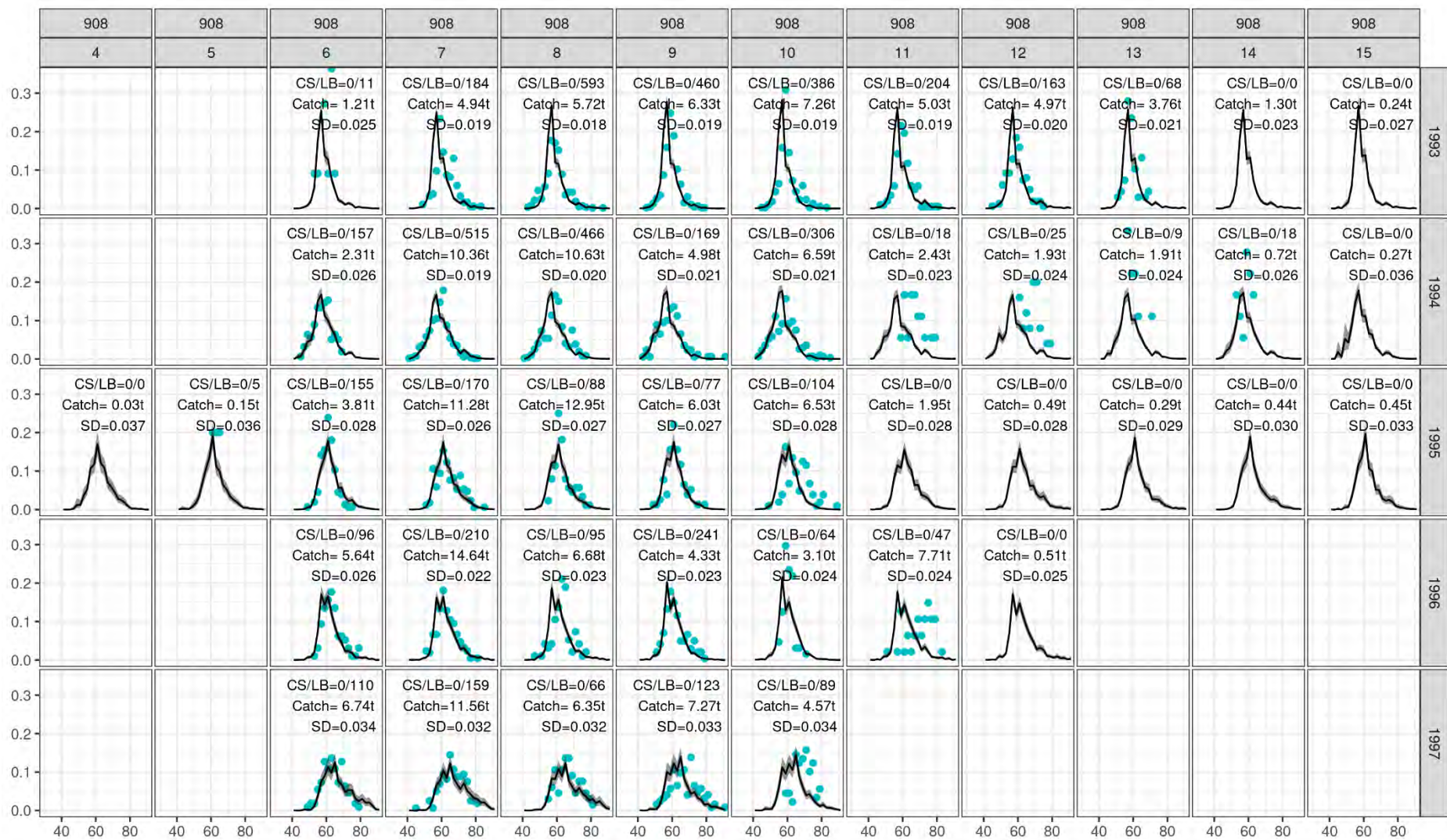


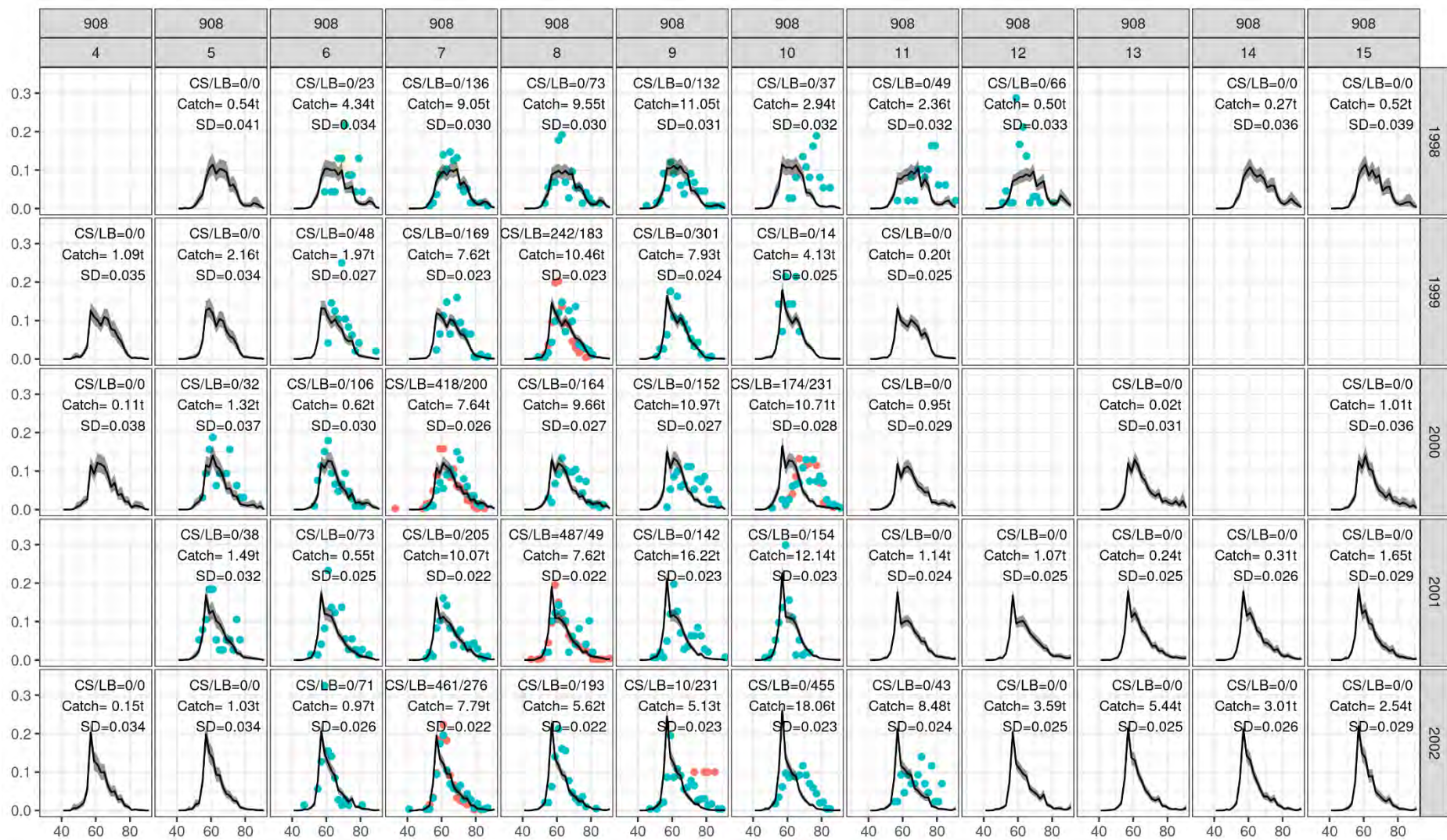


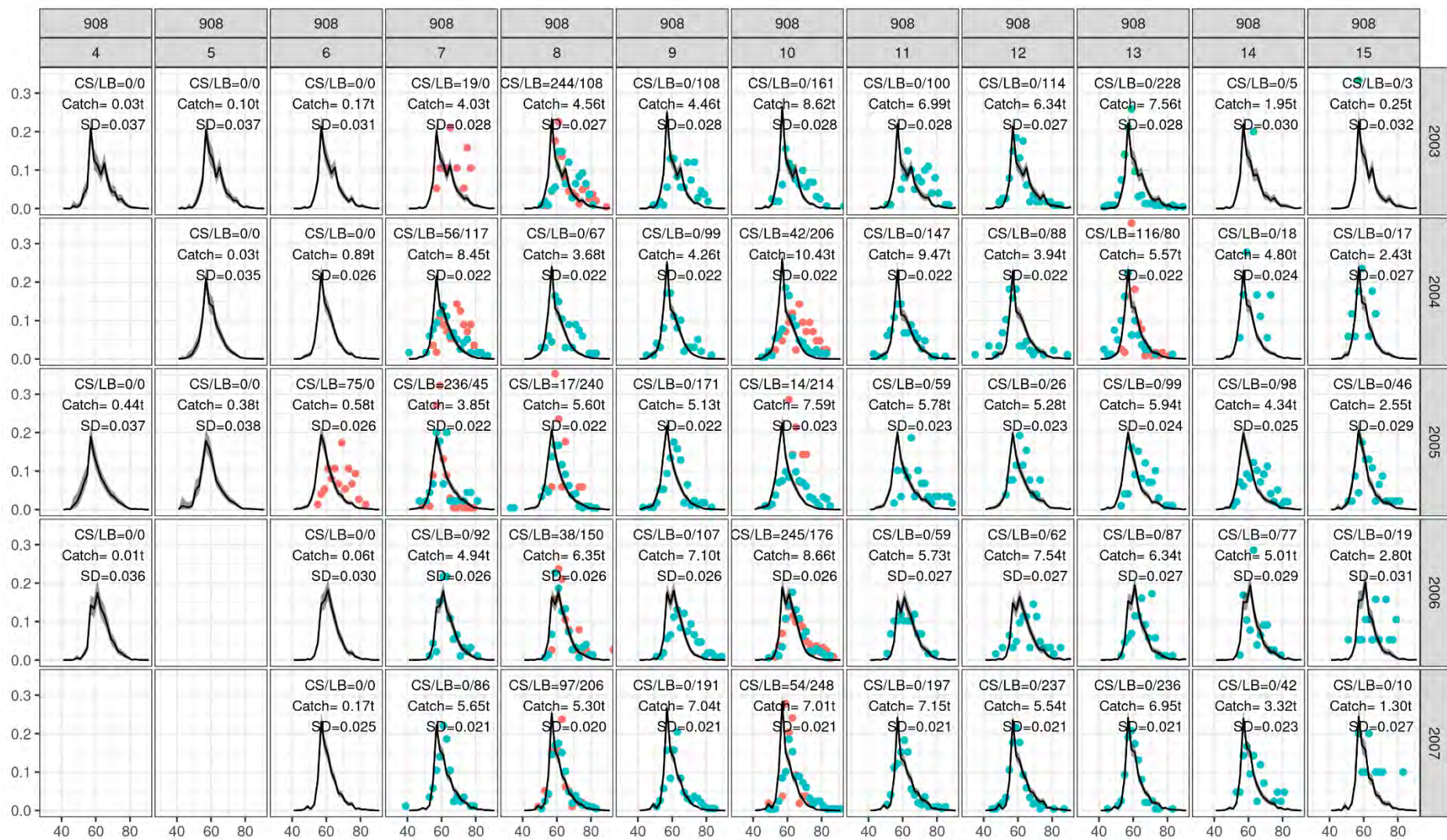


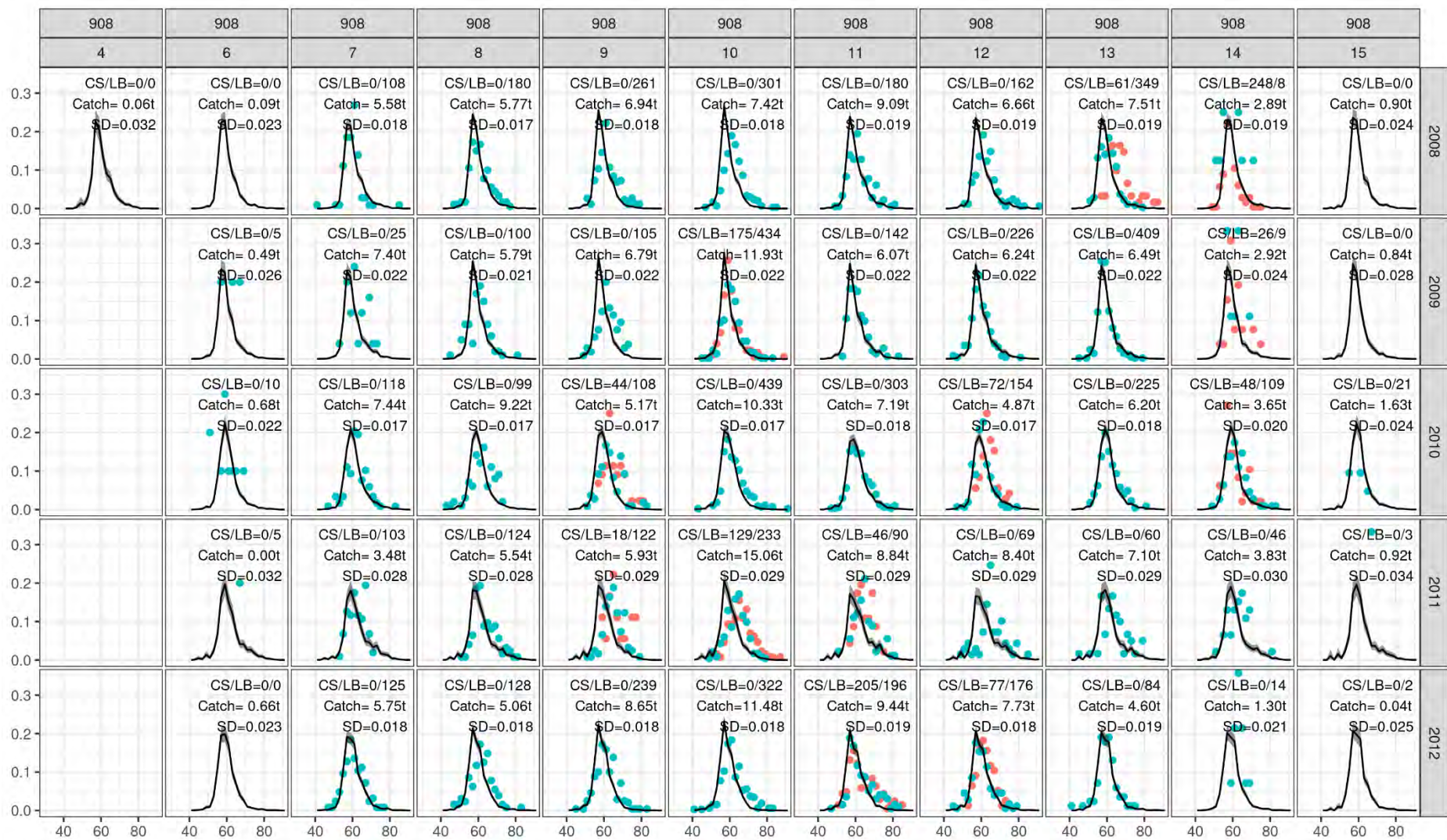


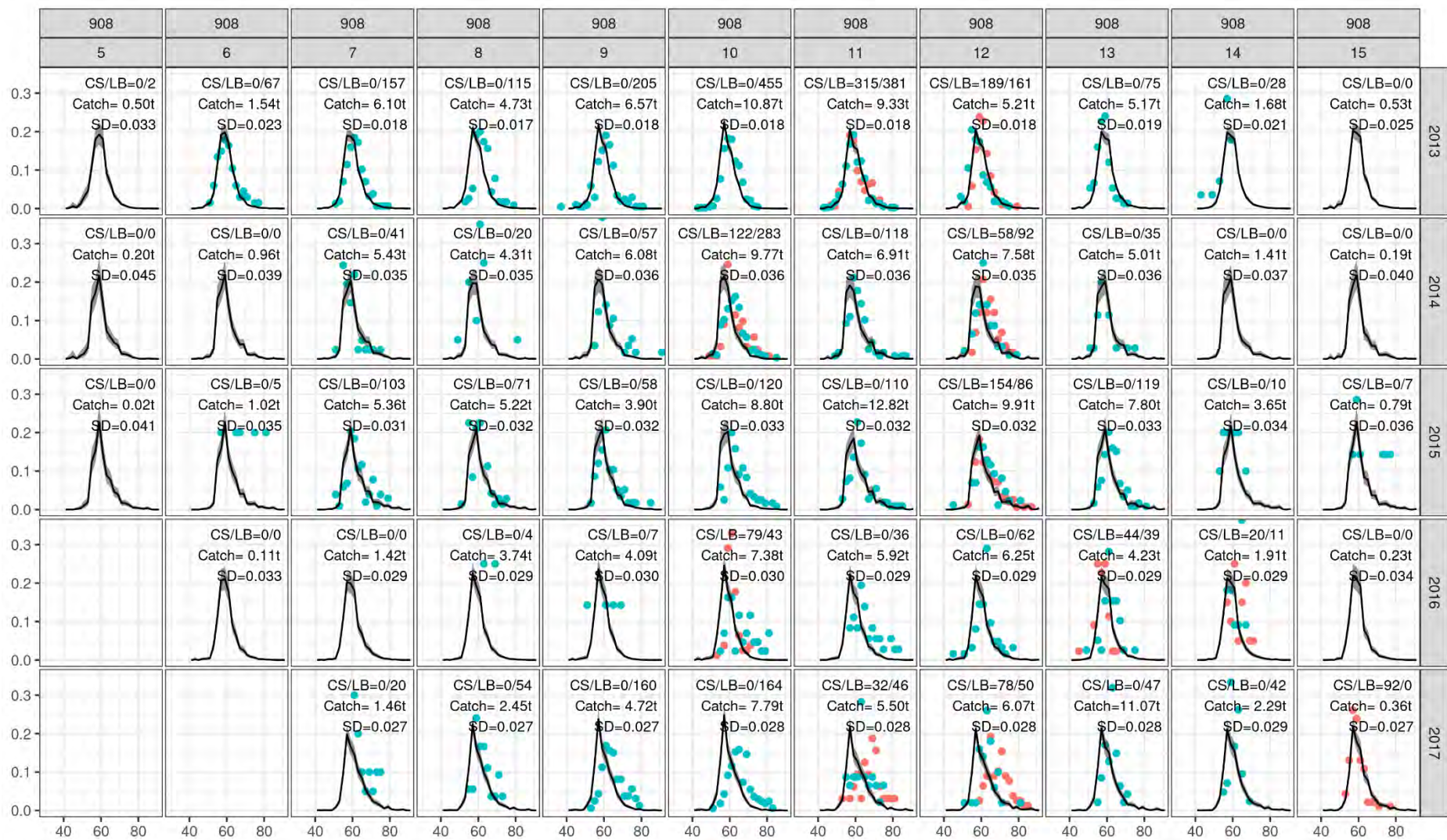


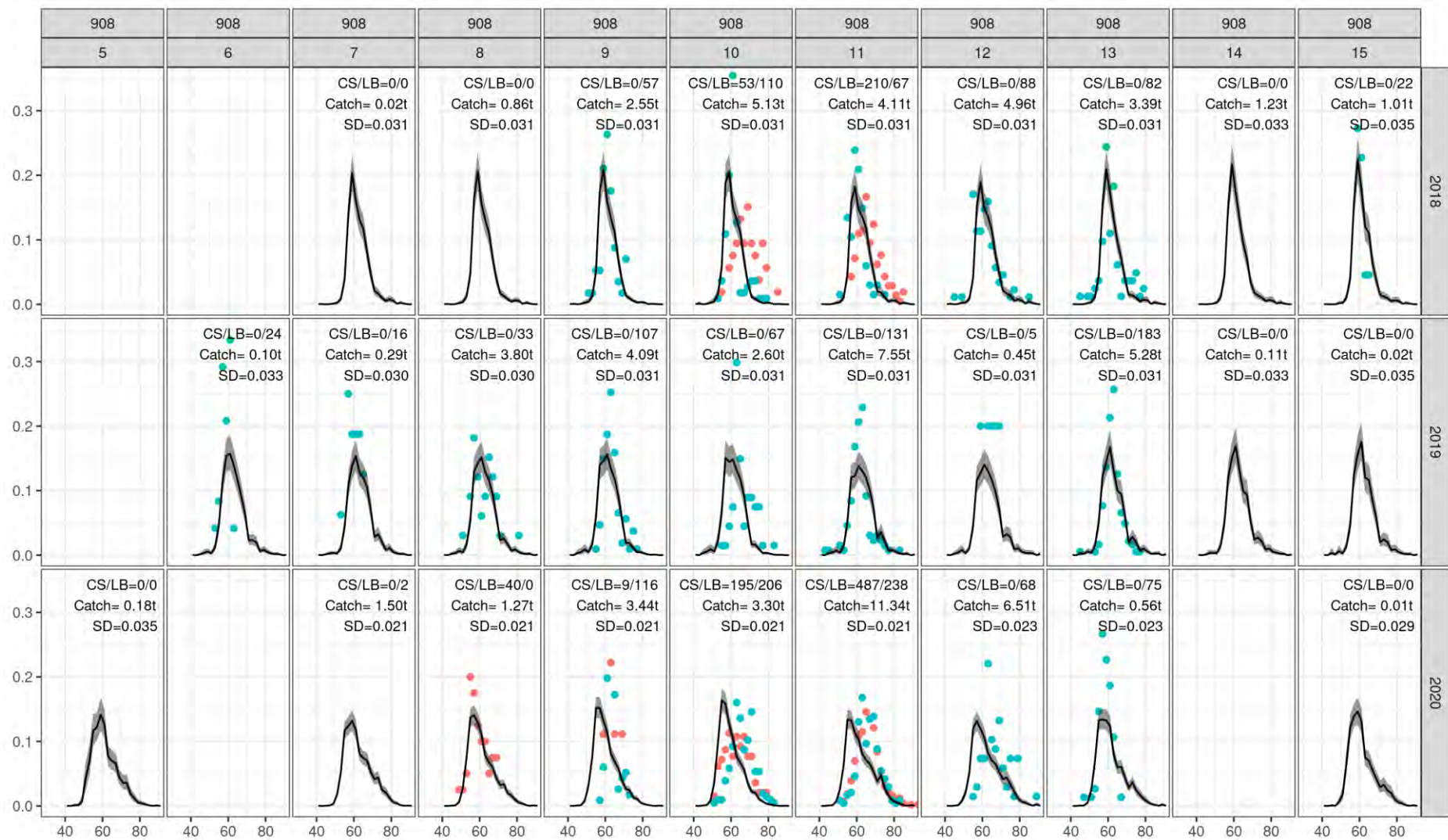




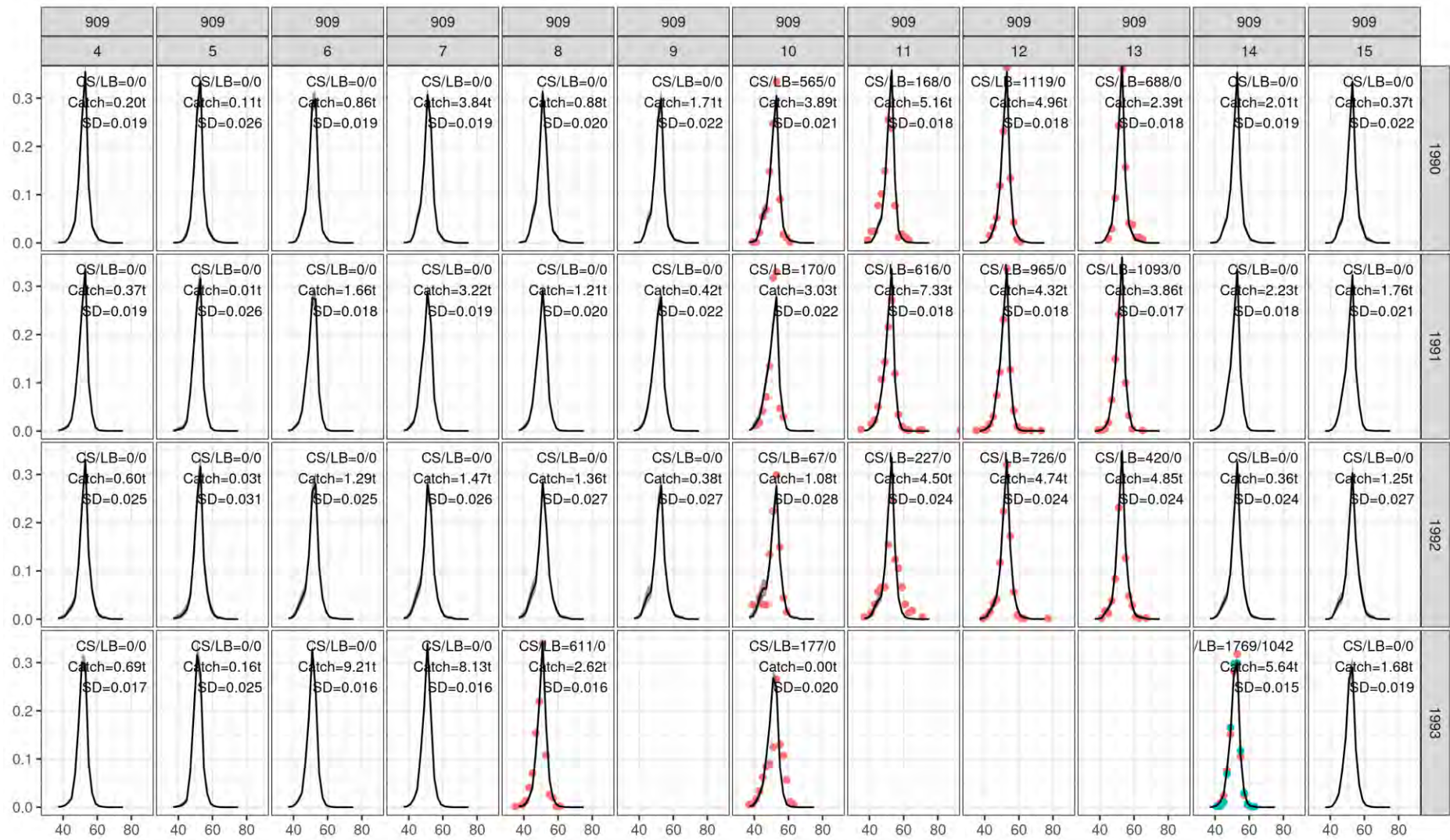


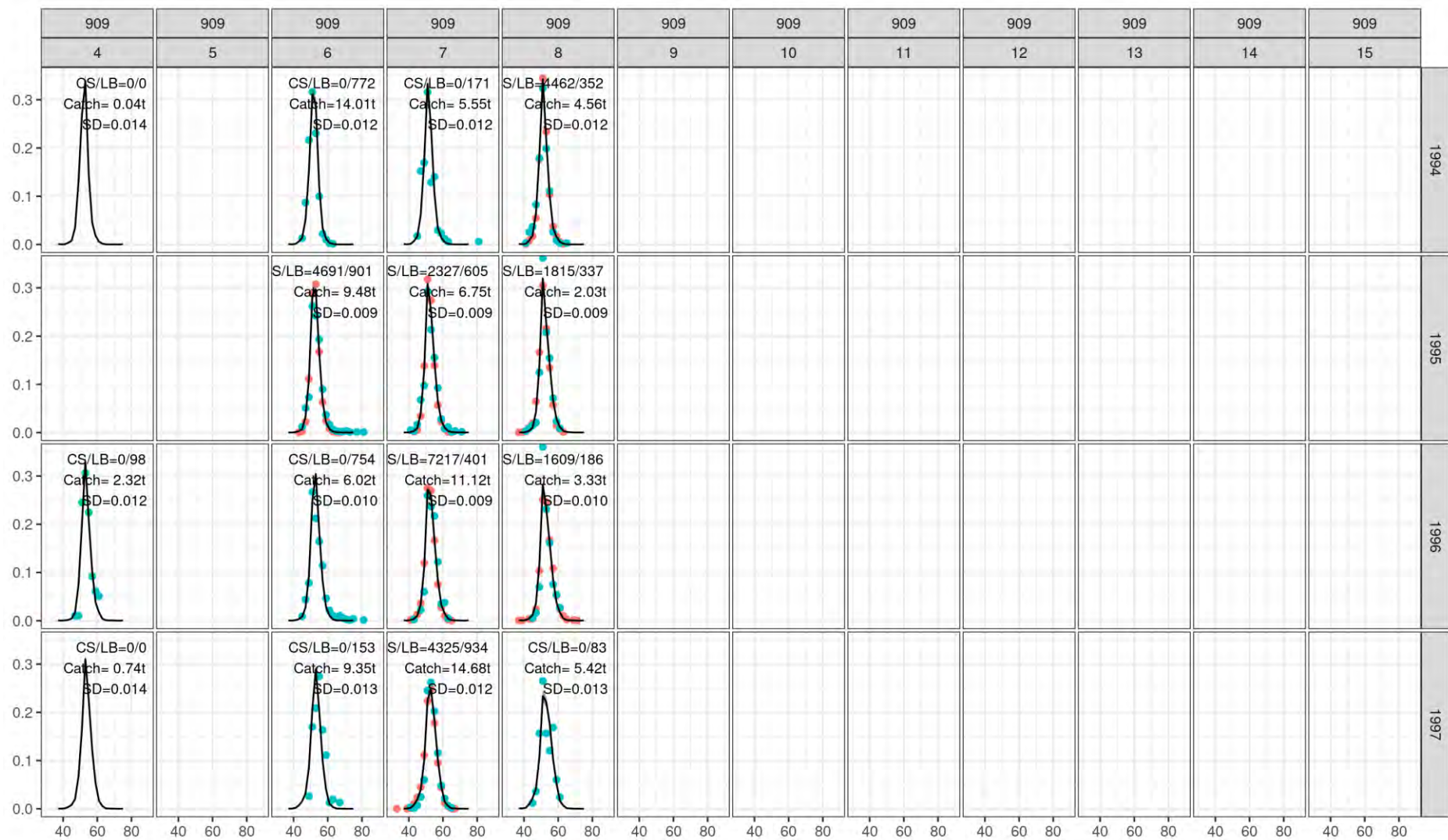


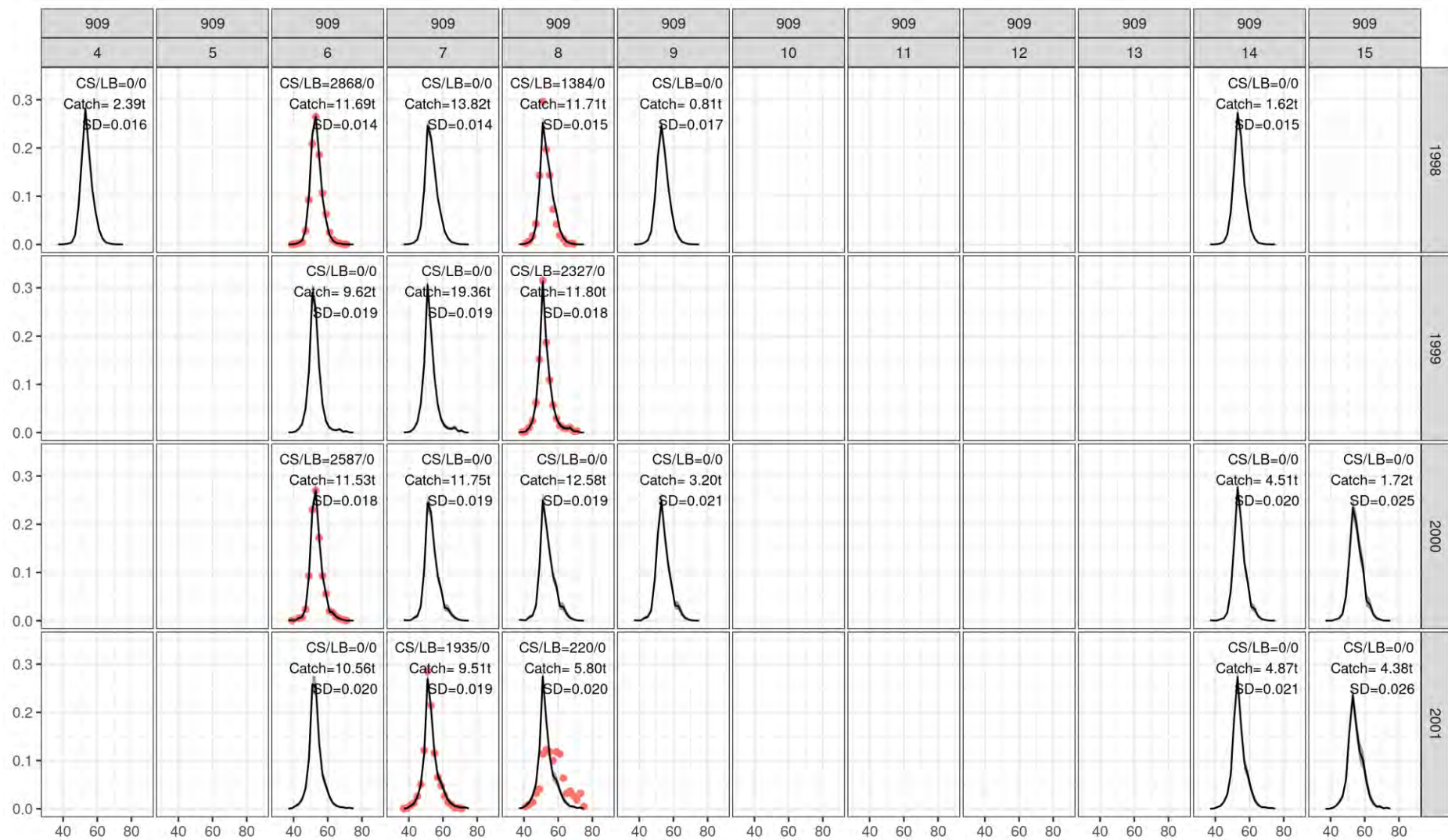


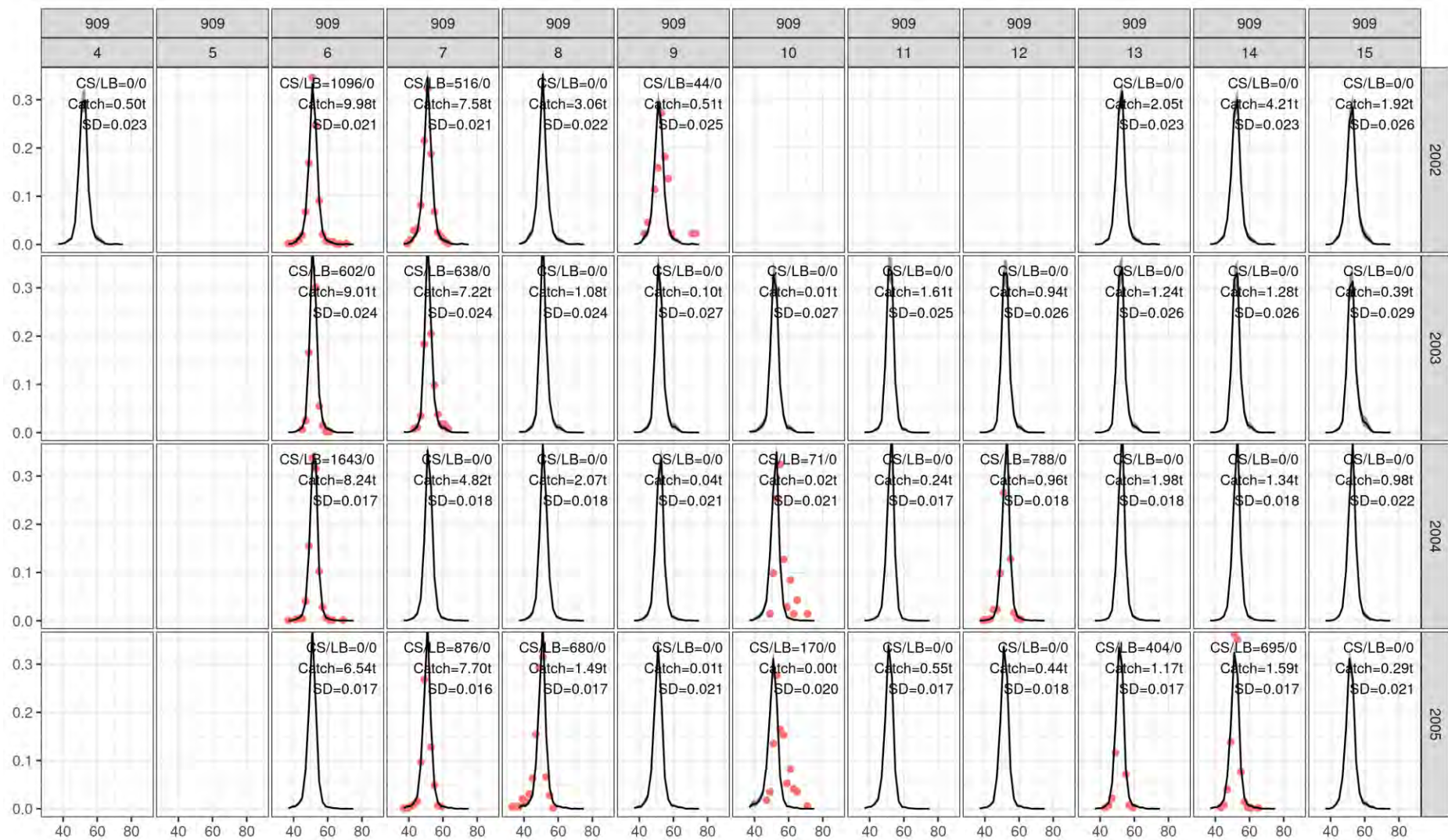


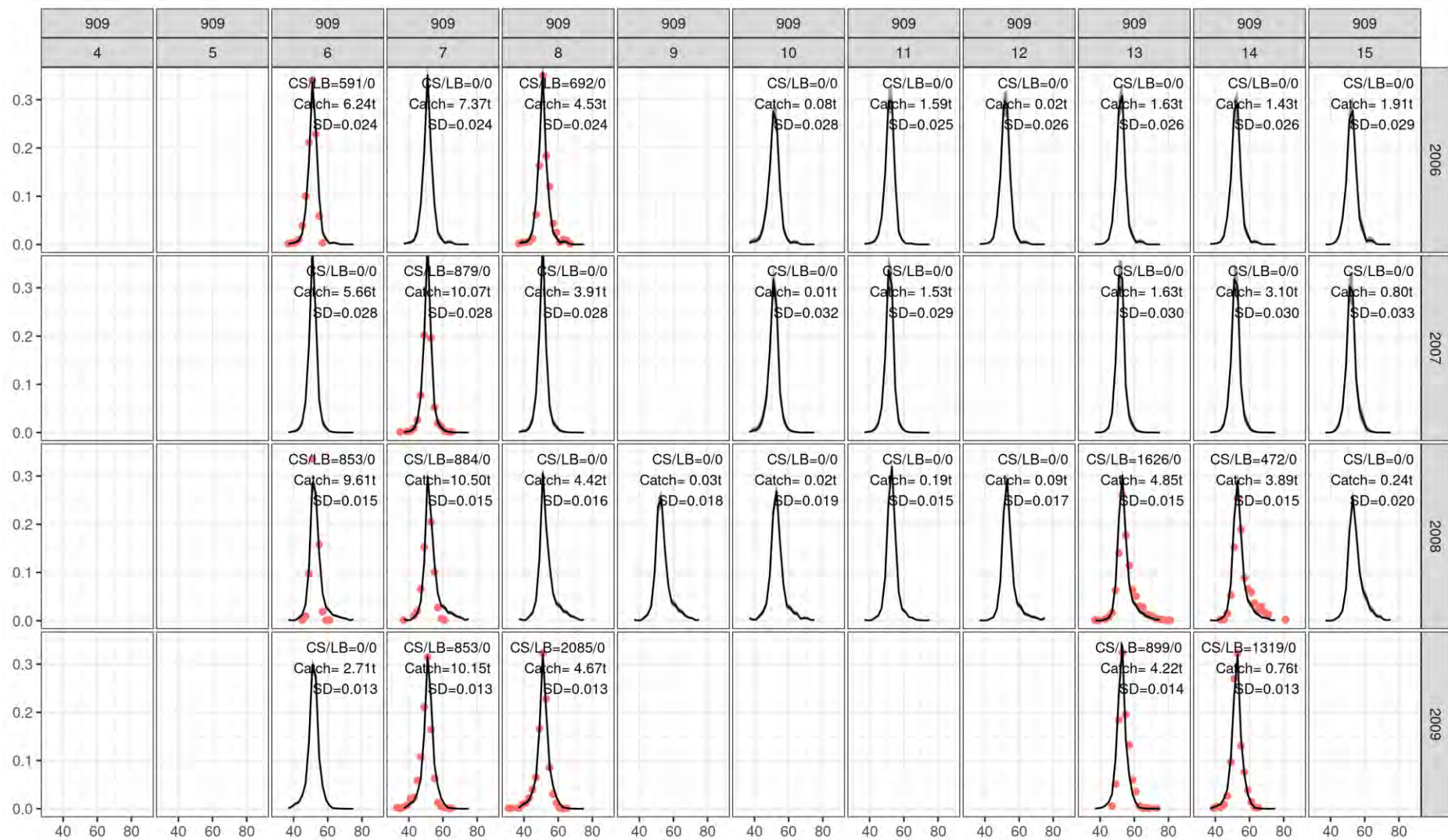
11.4 CRA 3 males

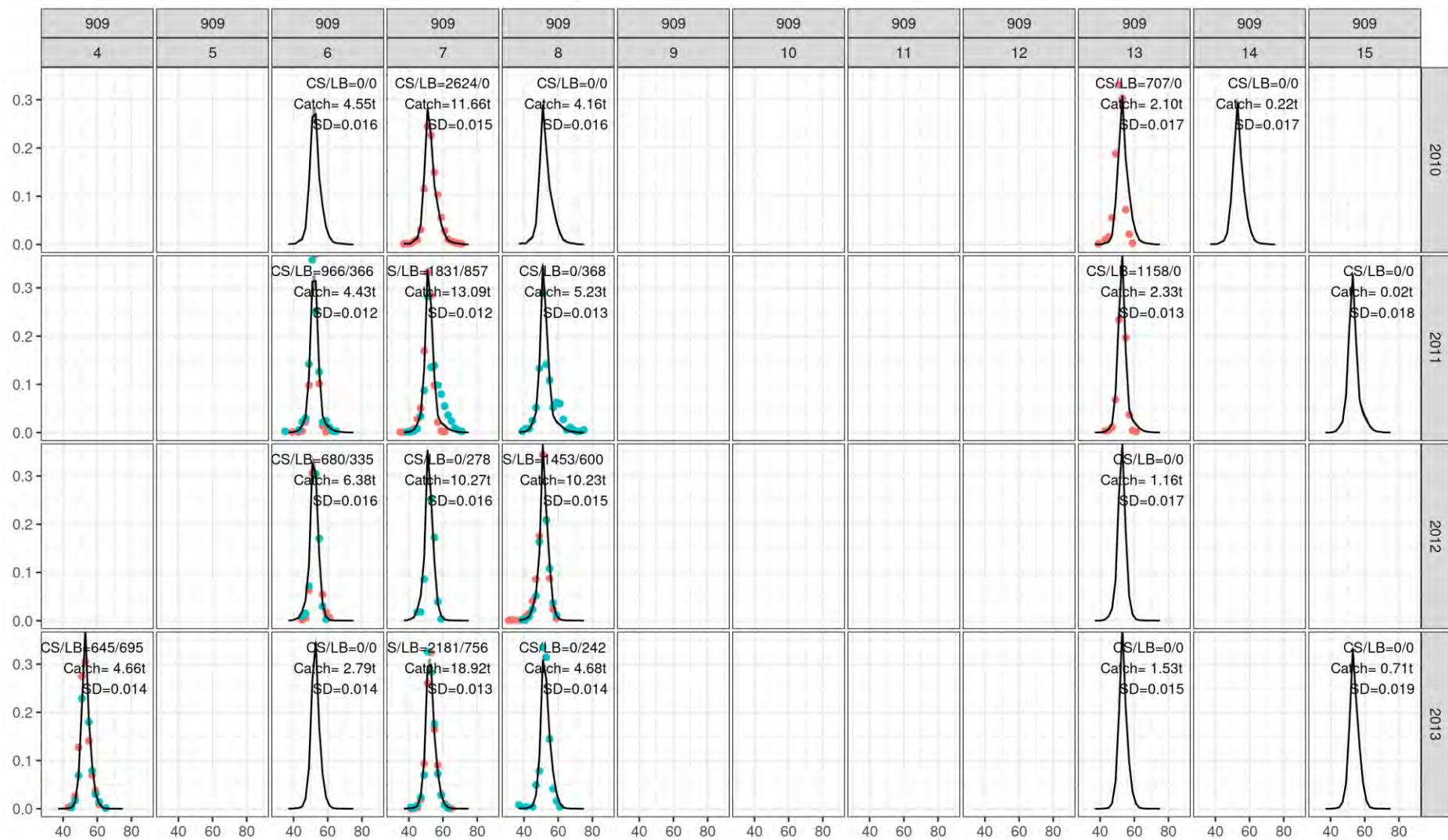


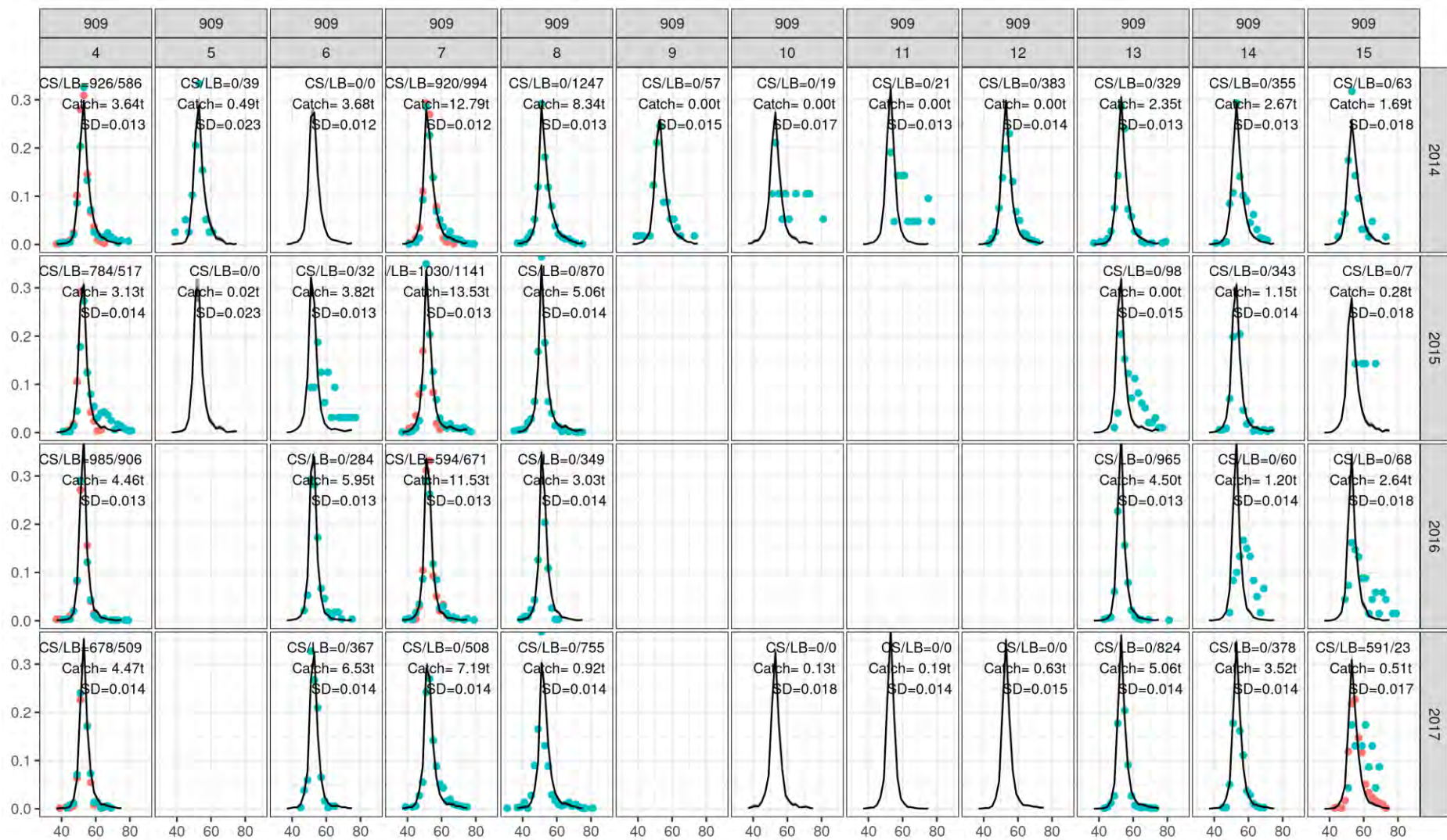


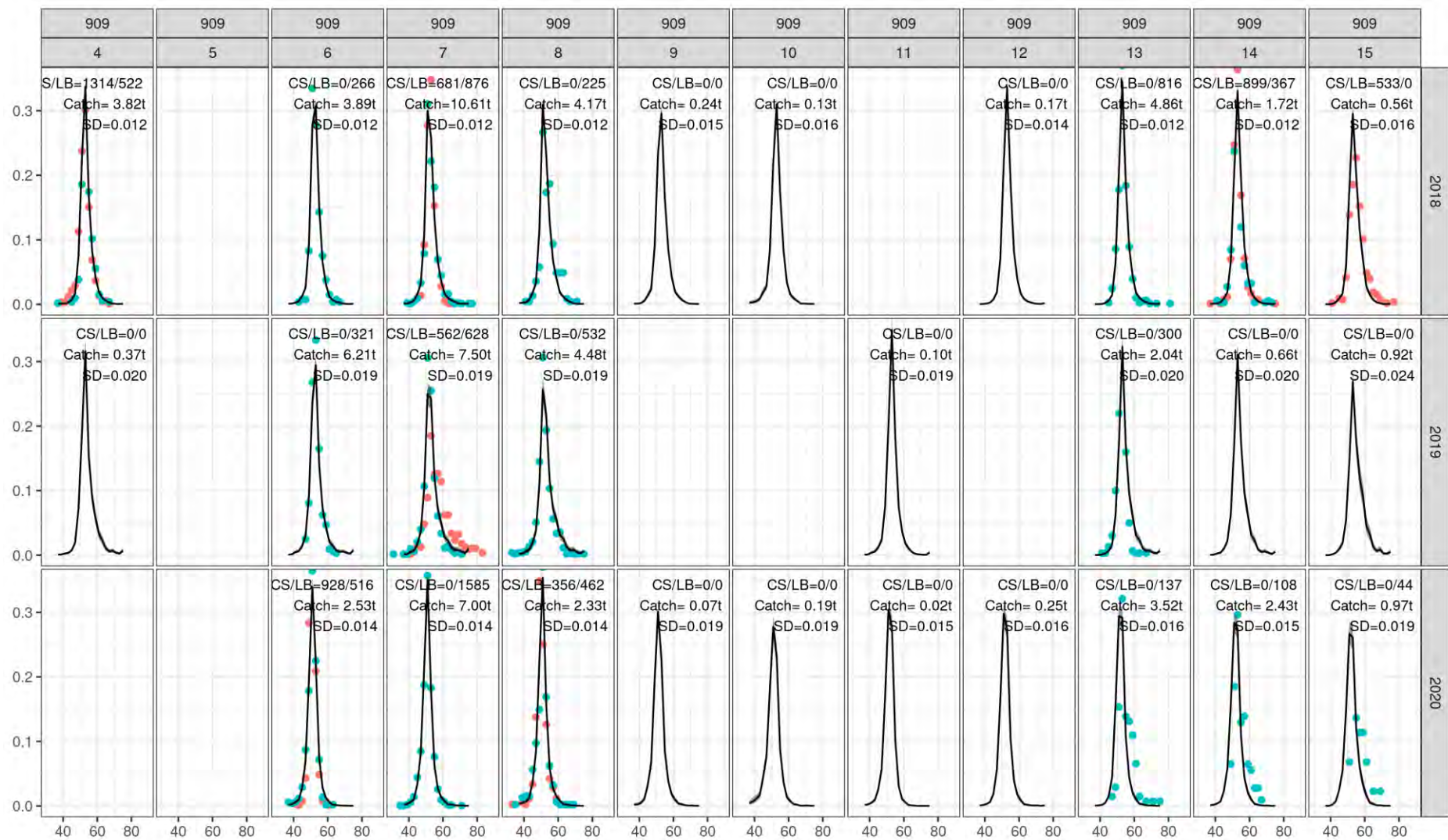


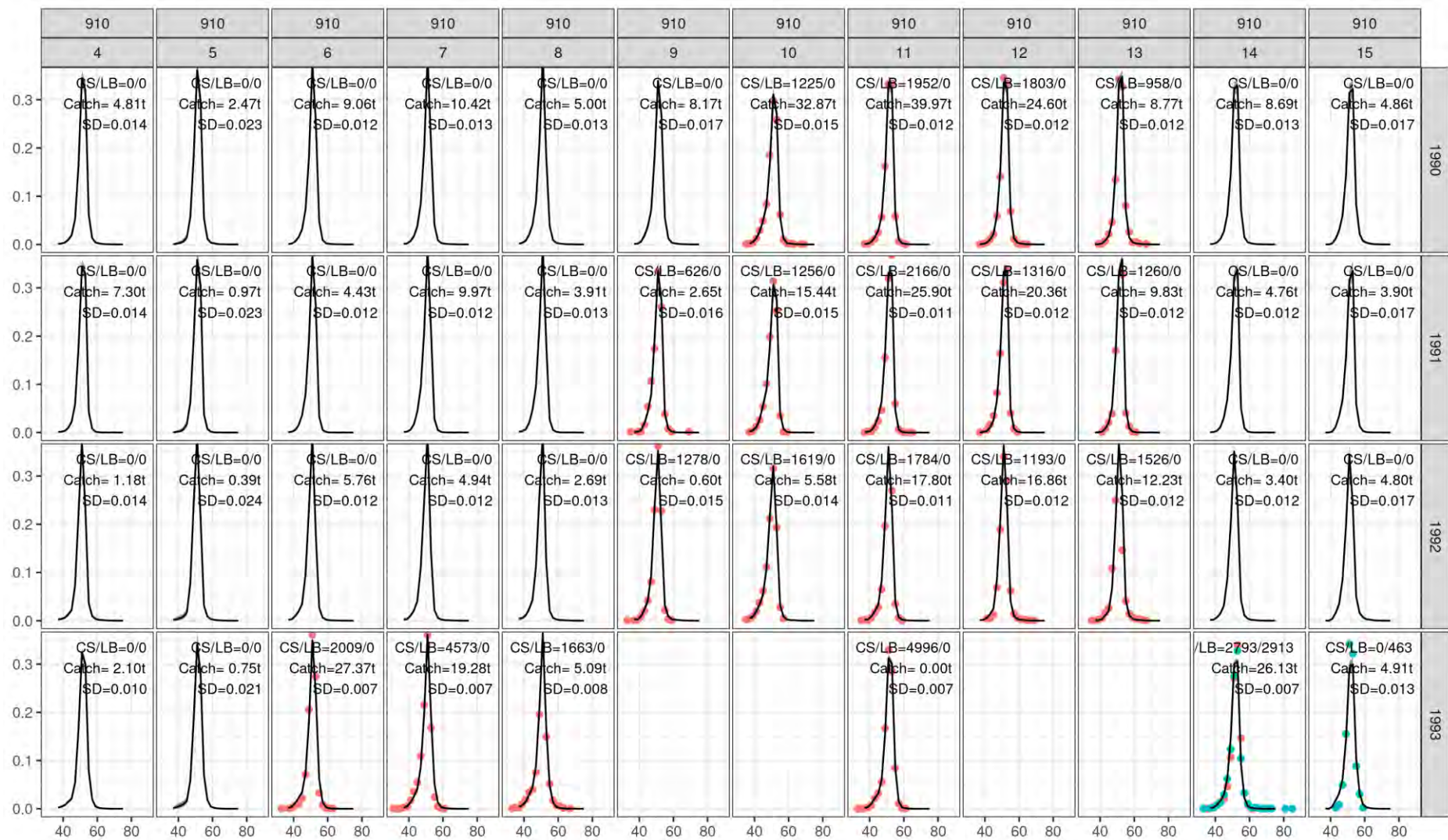


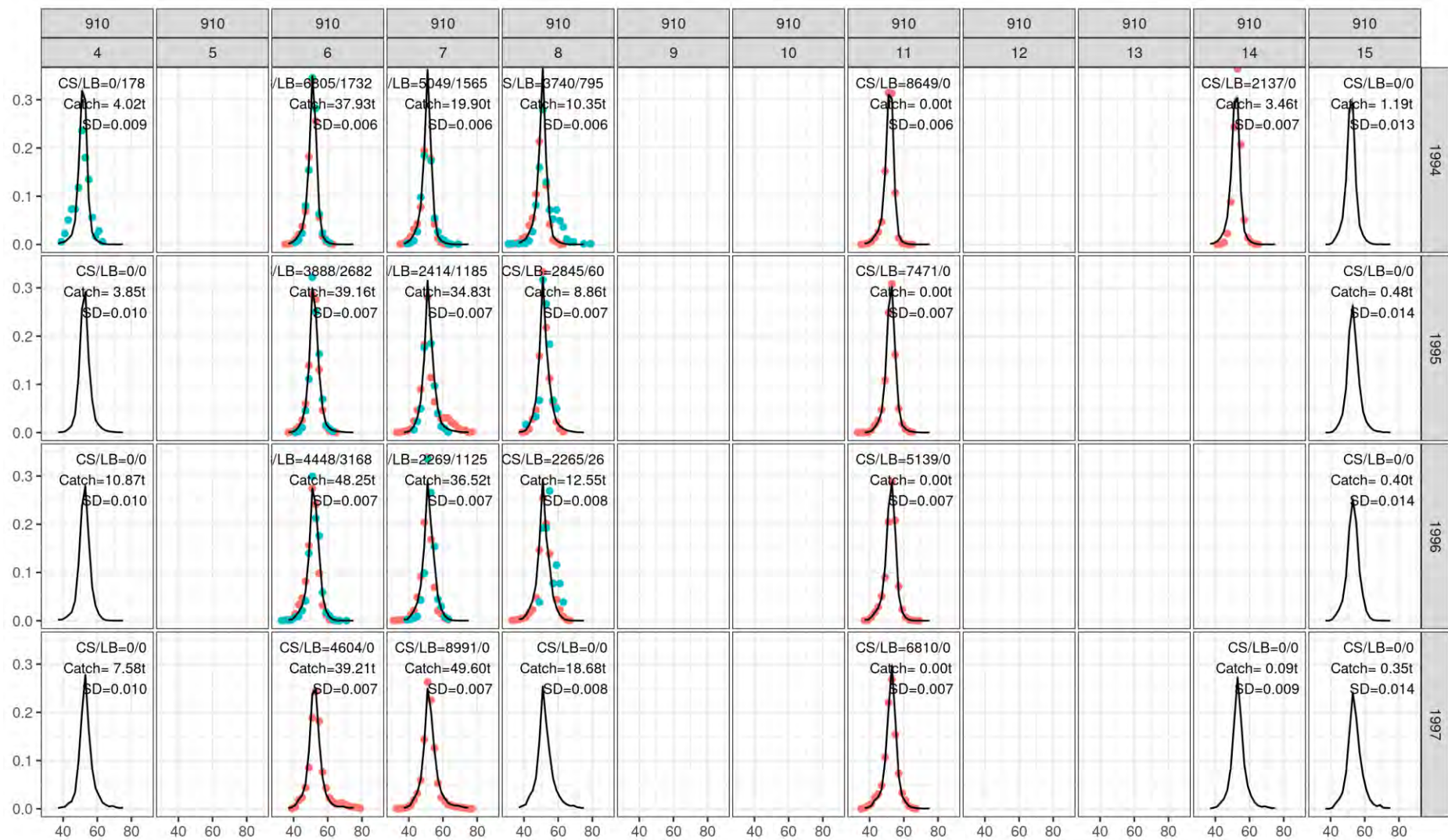


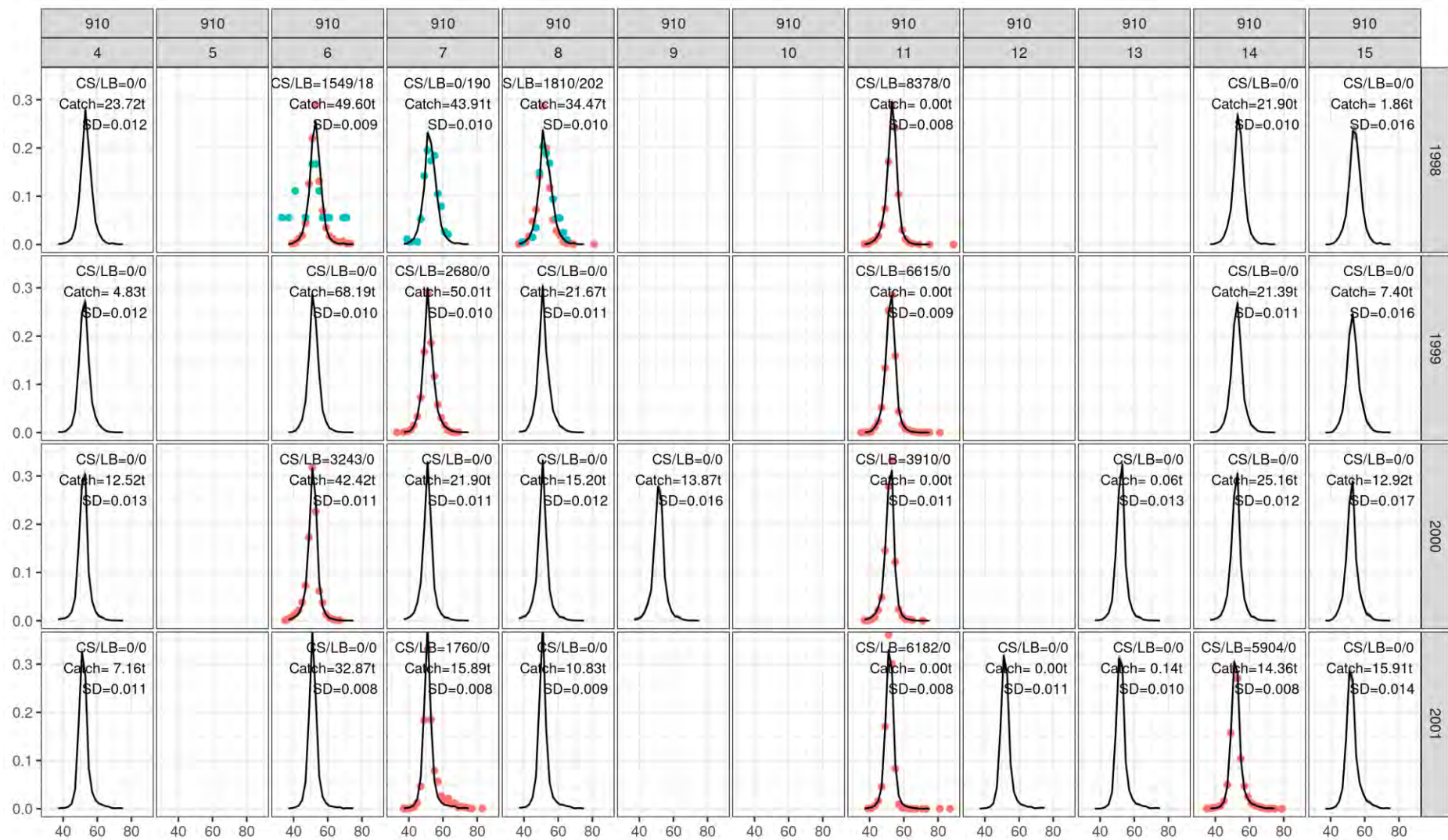


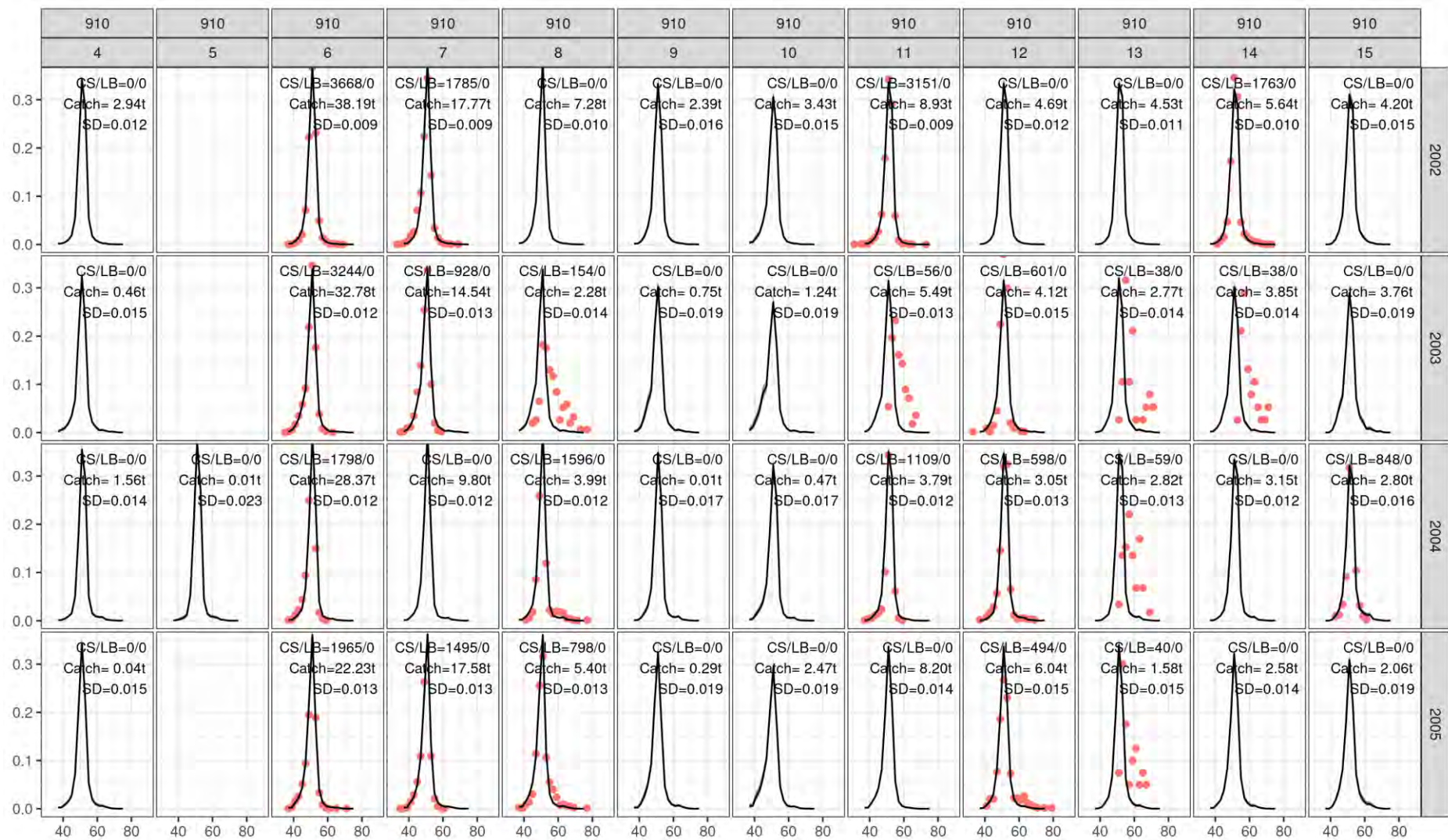


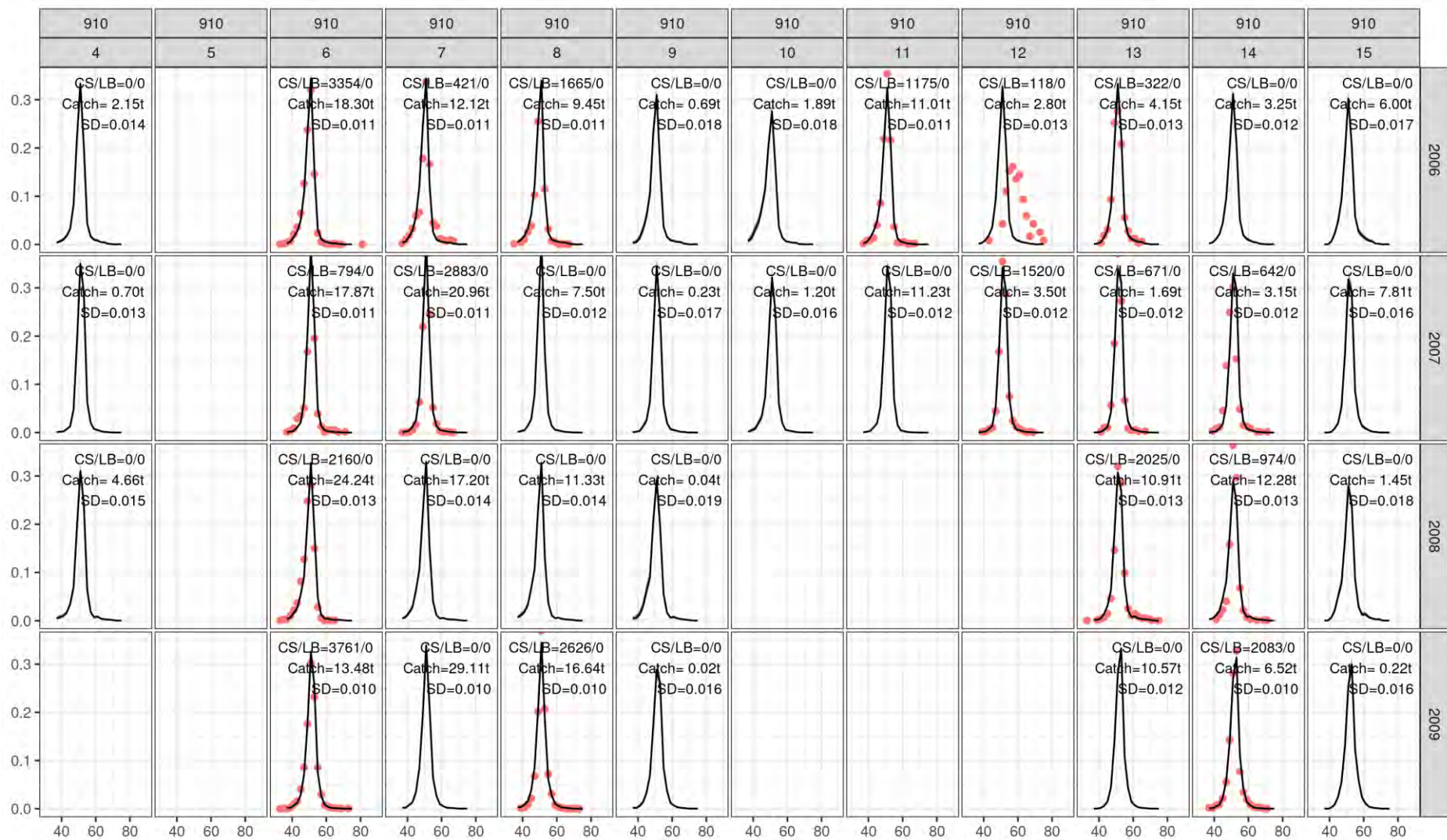


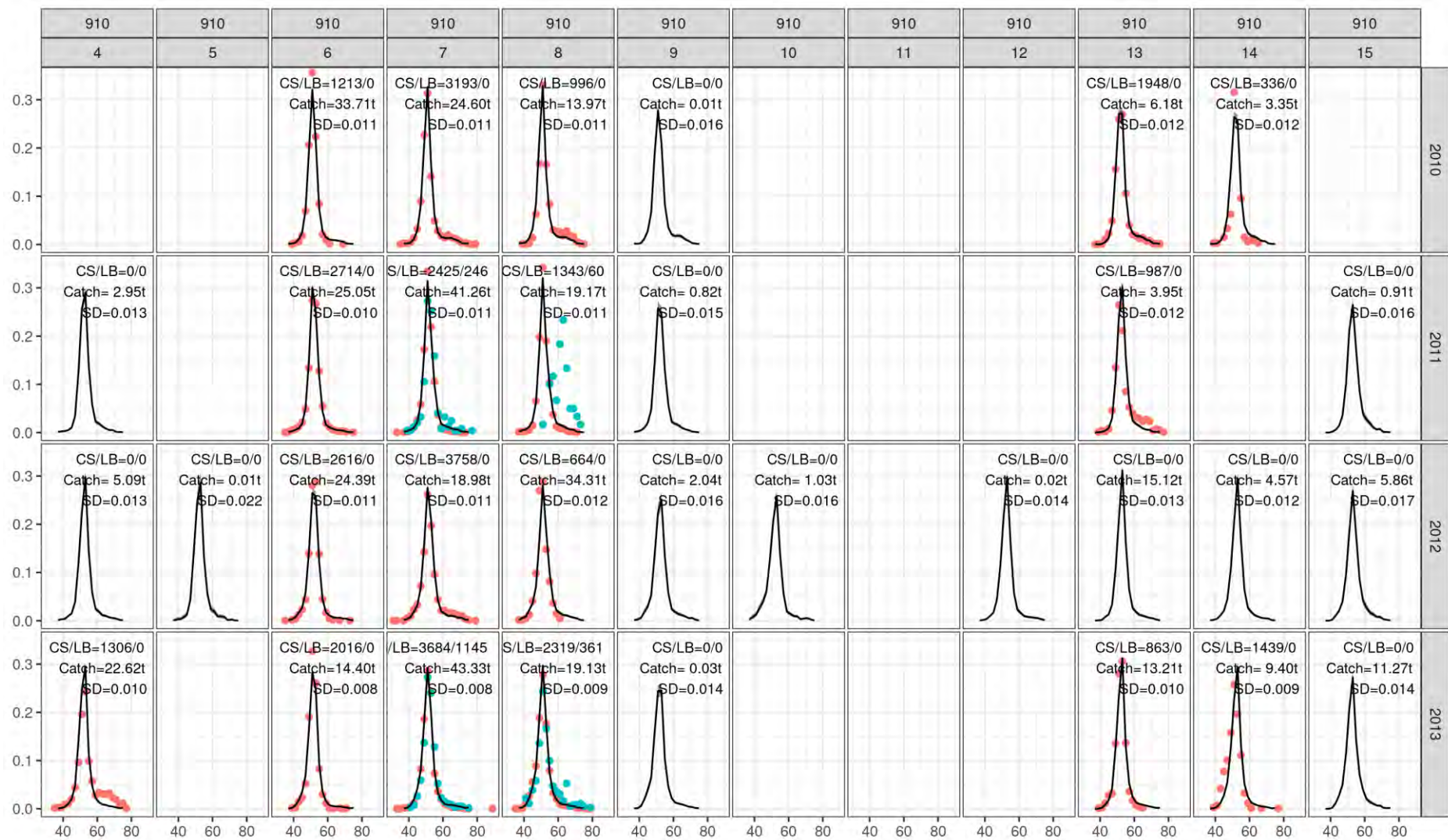


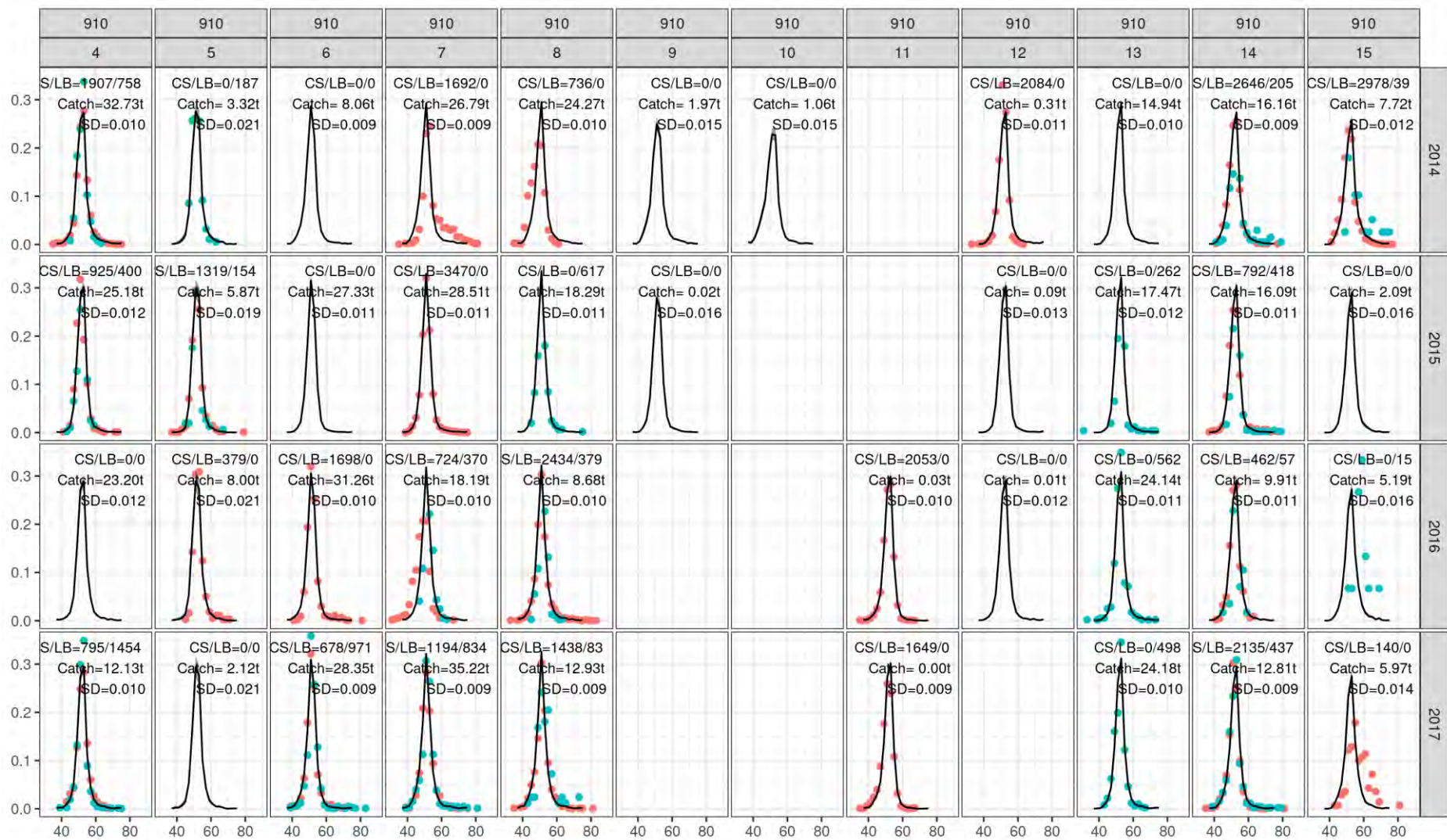


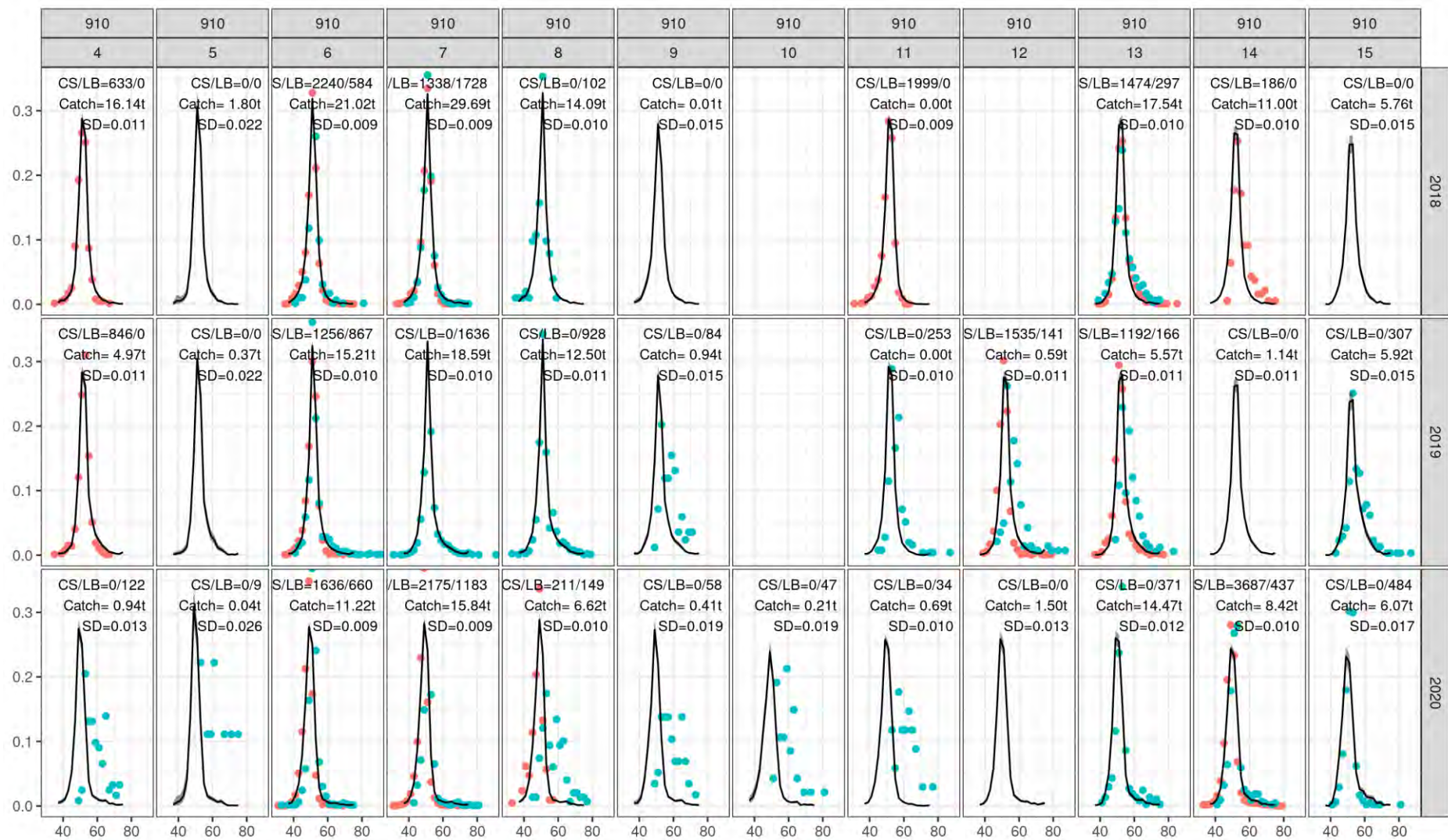


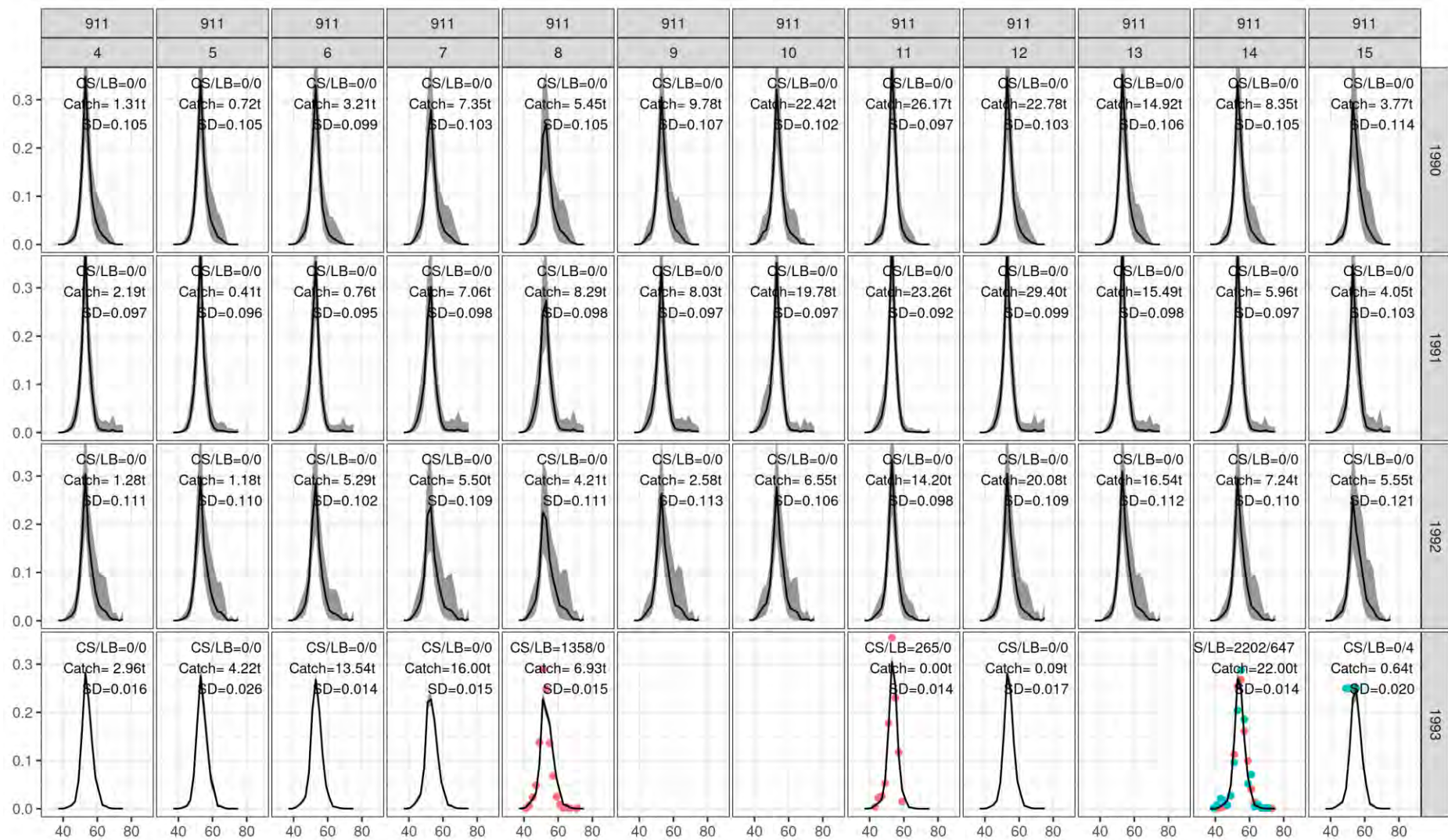


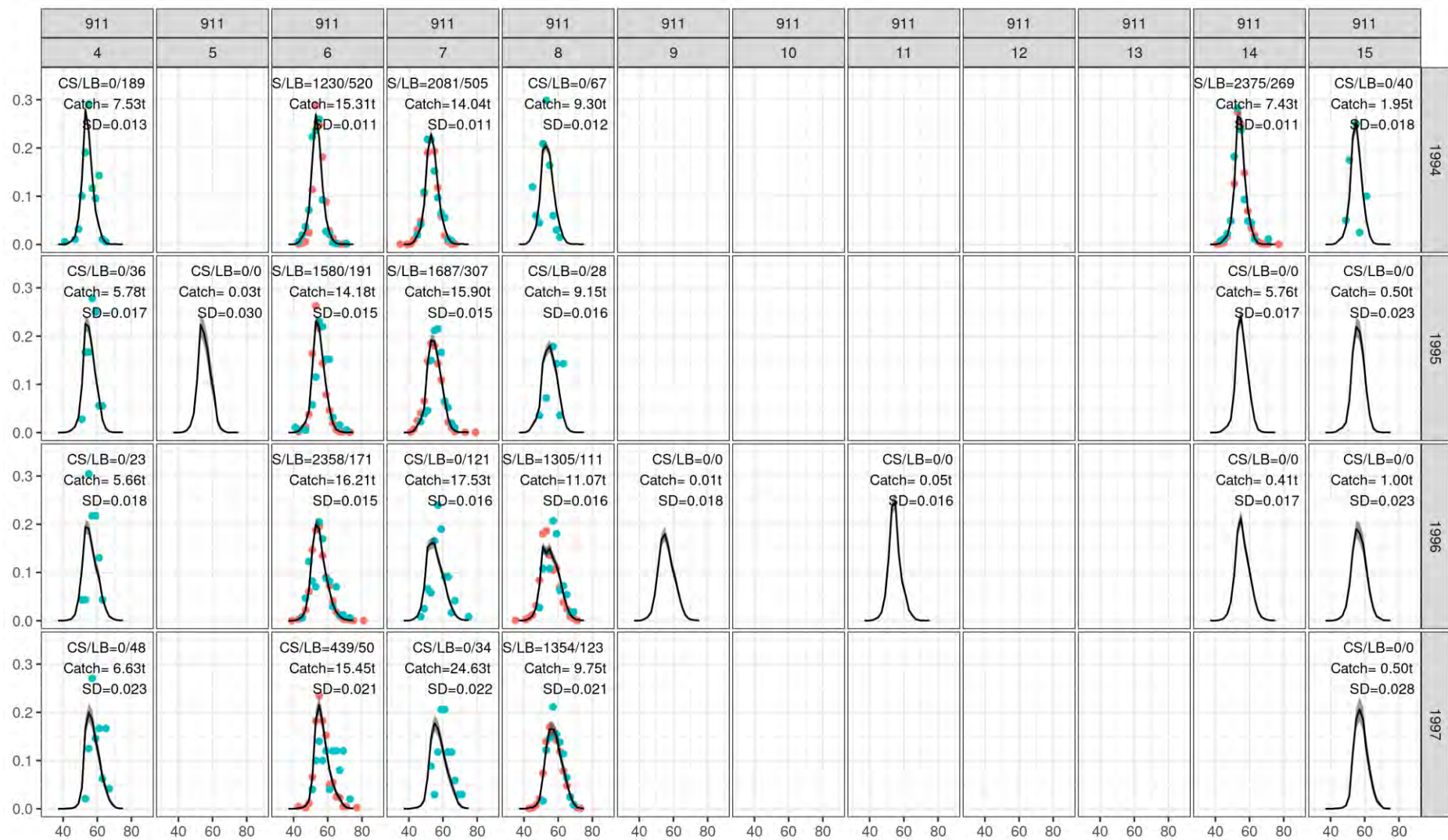


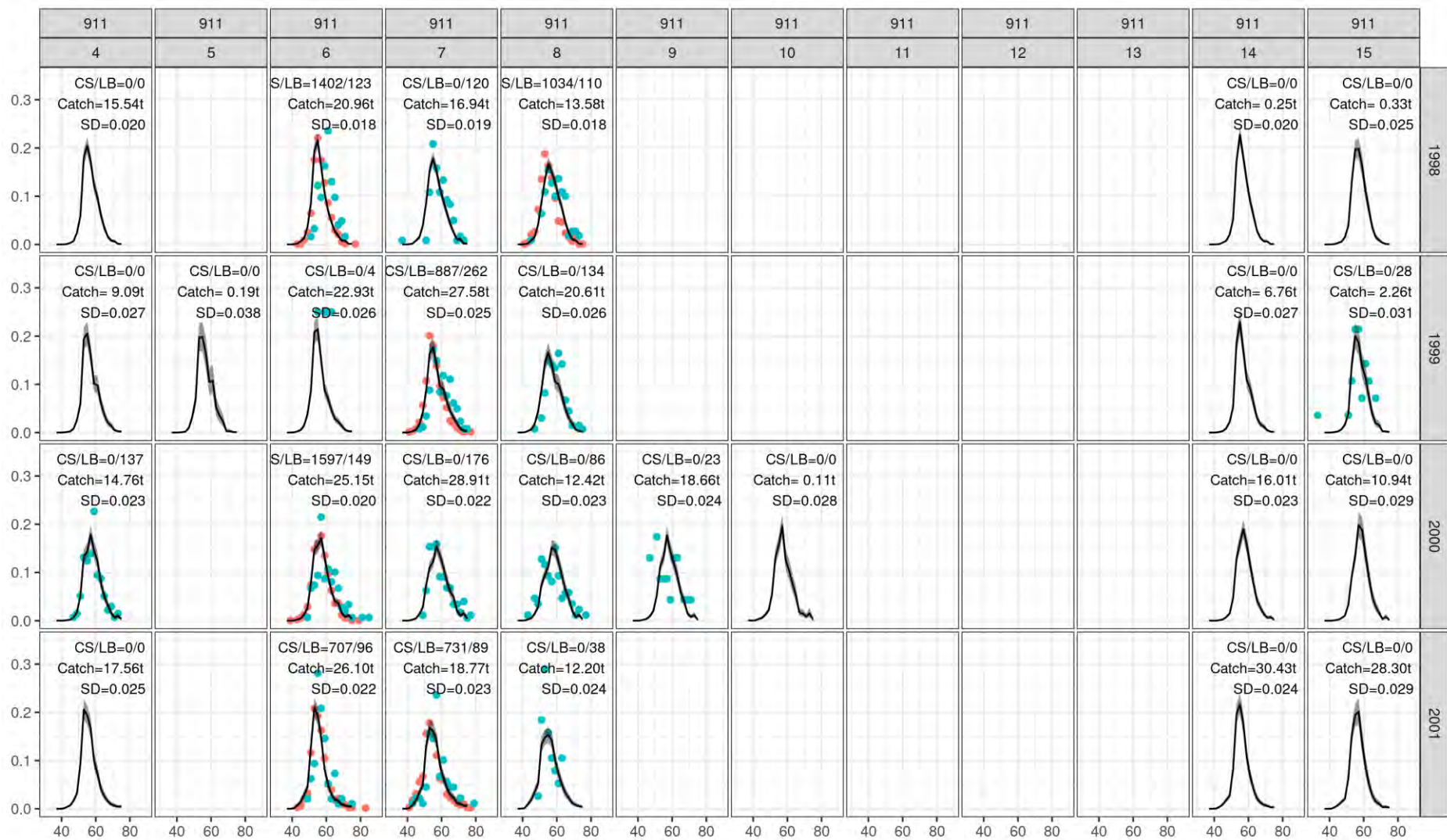


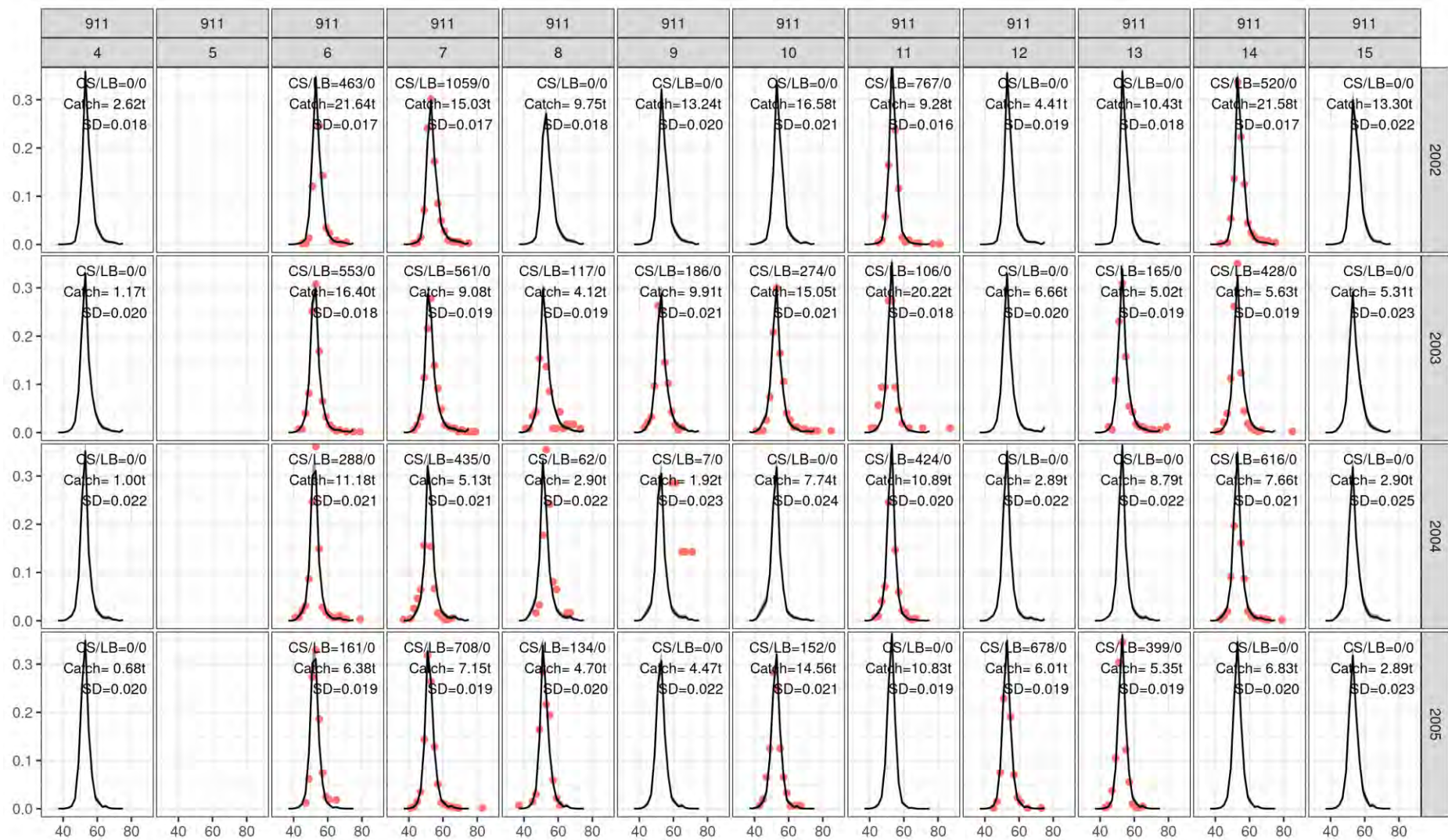


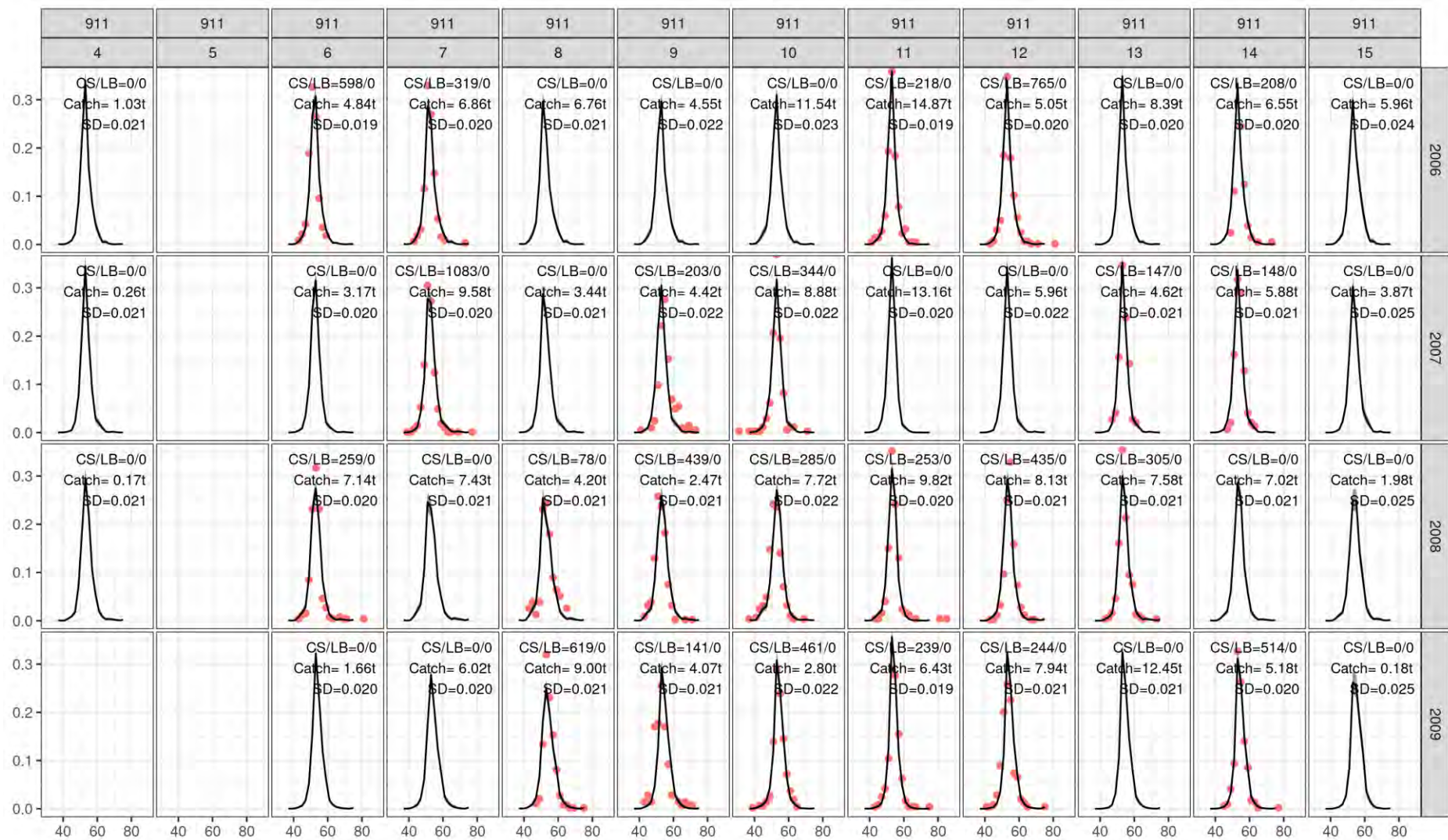


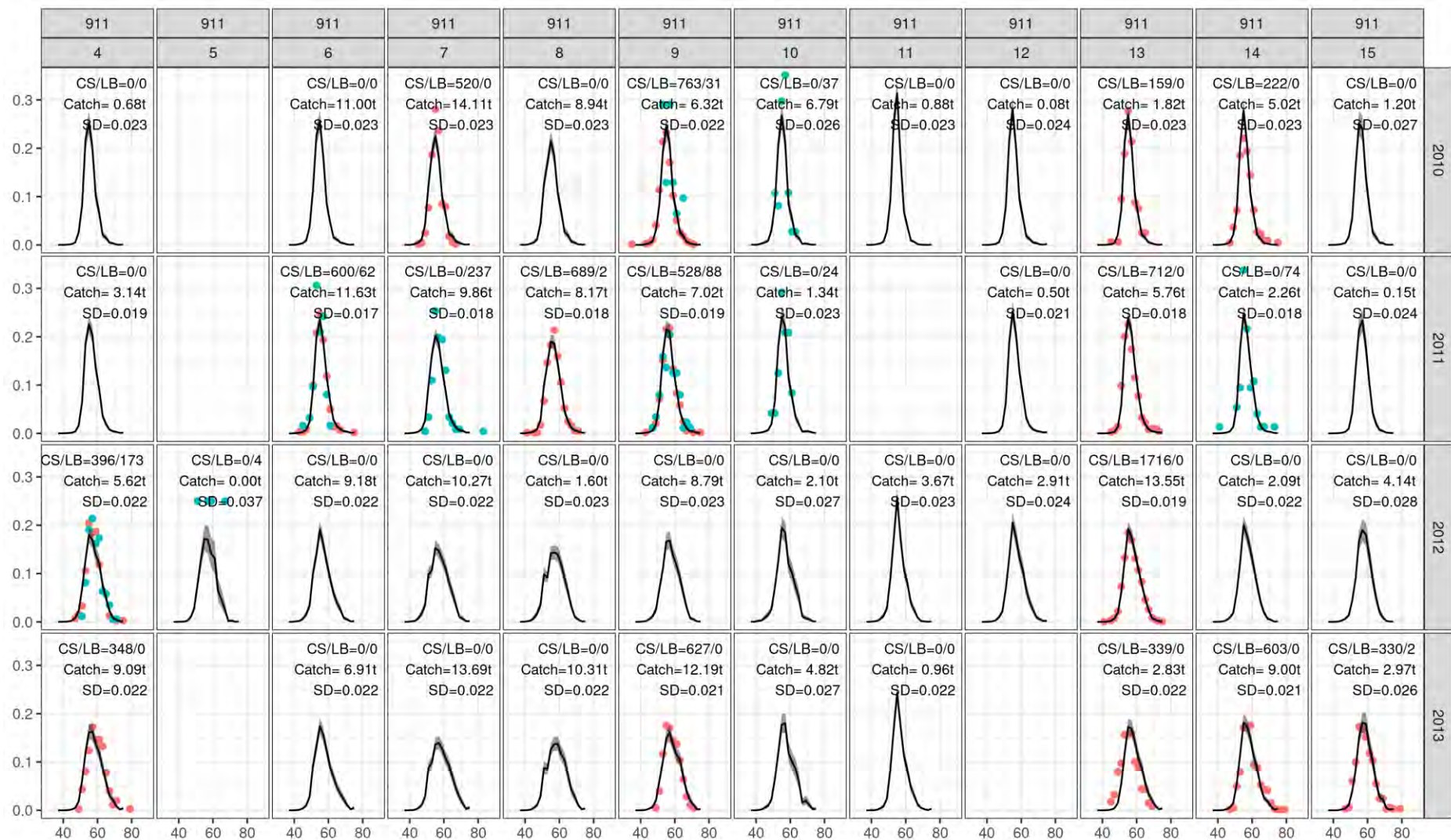


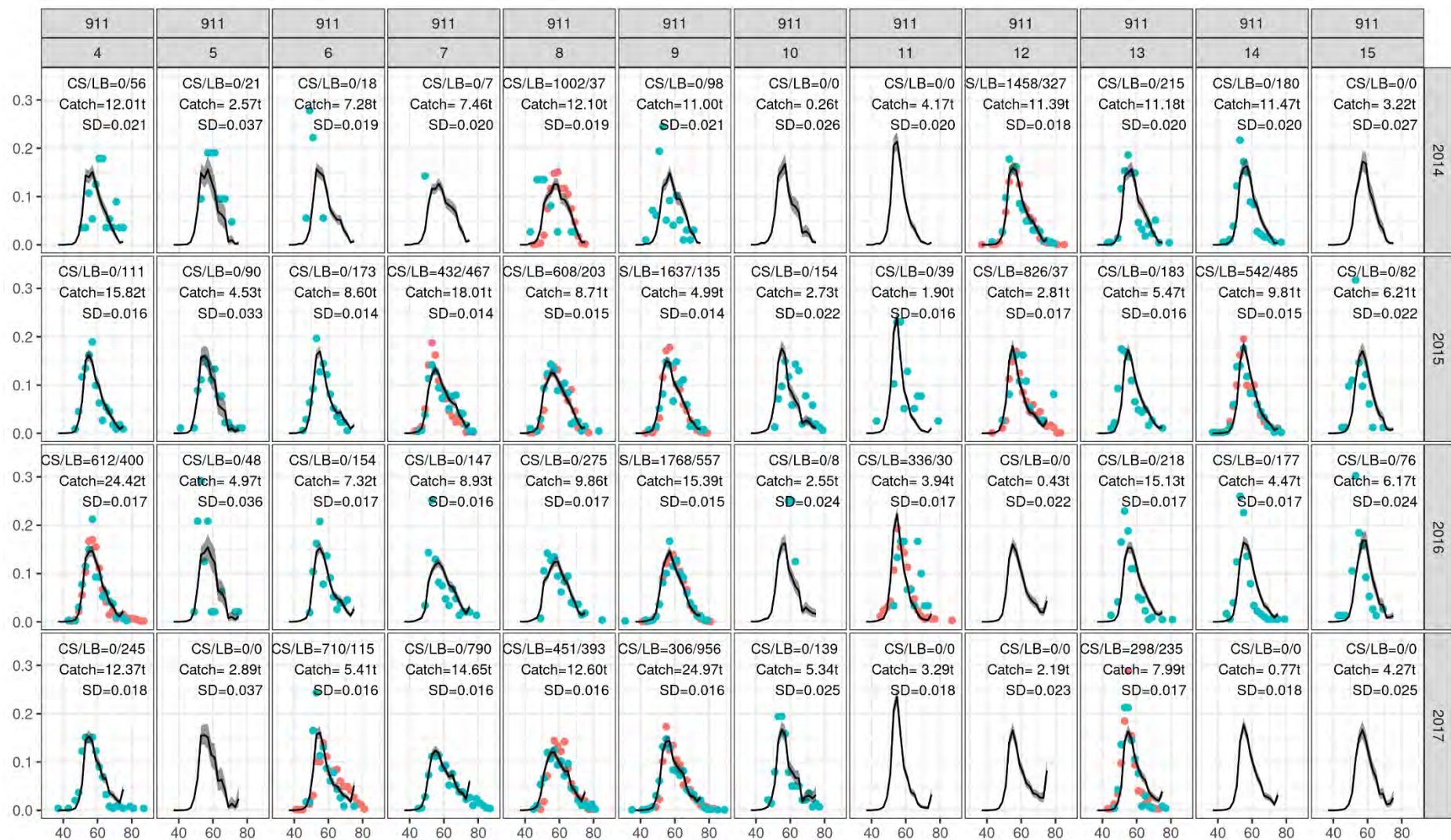


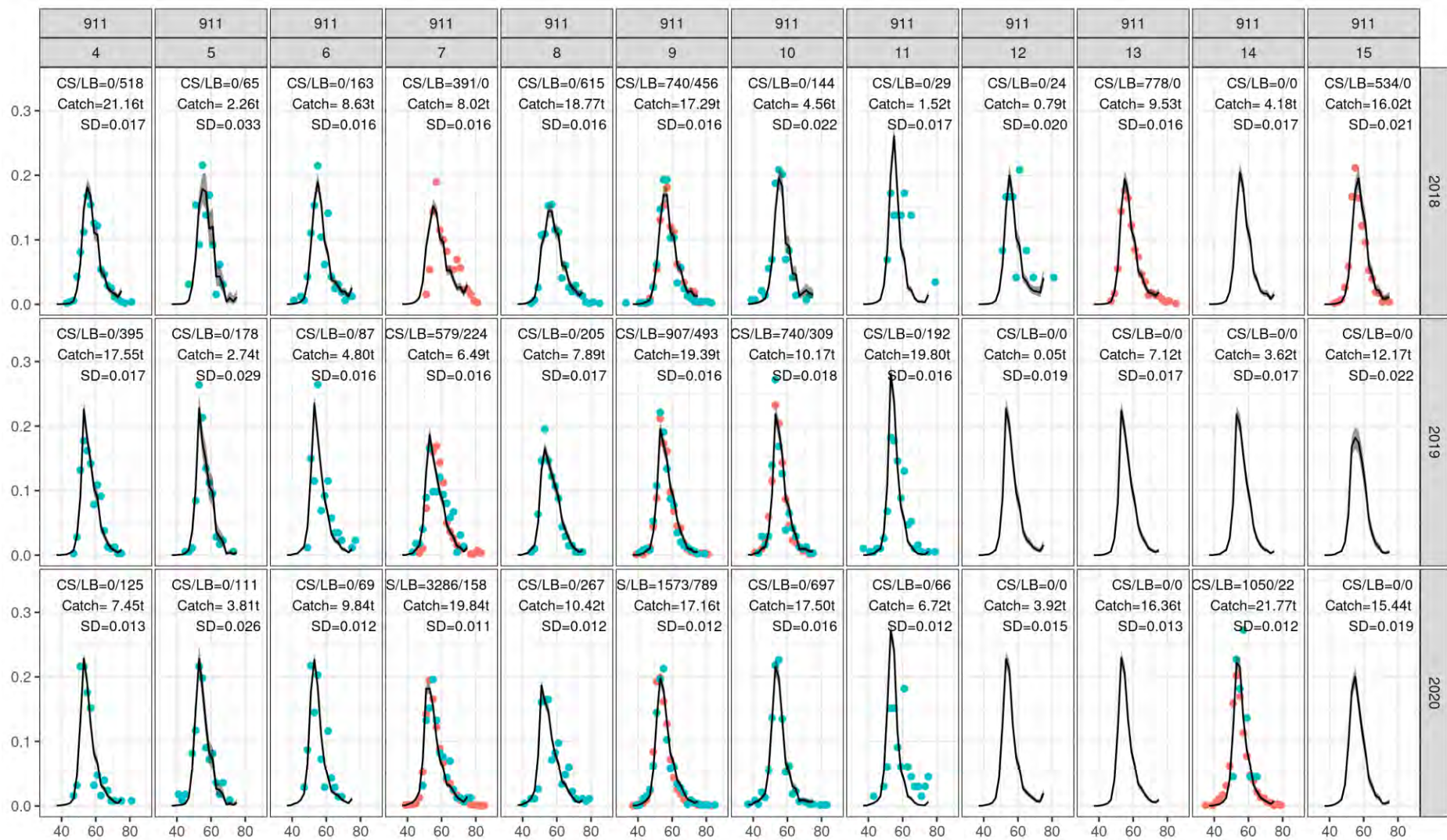




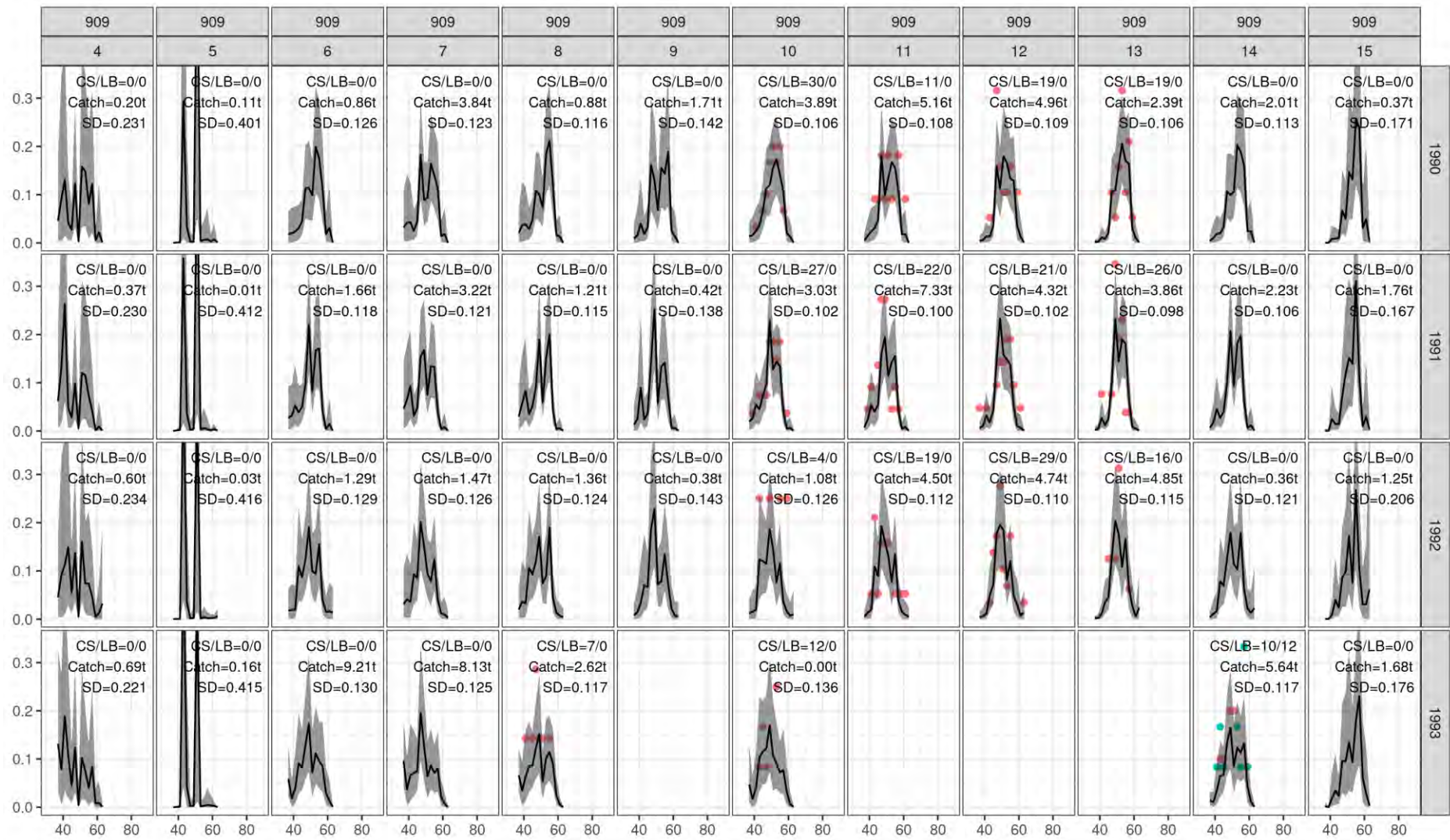


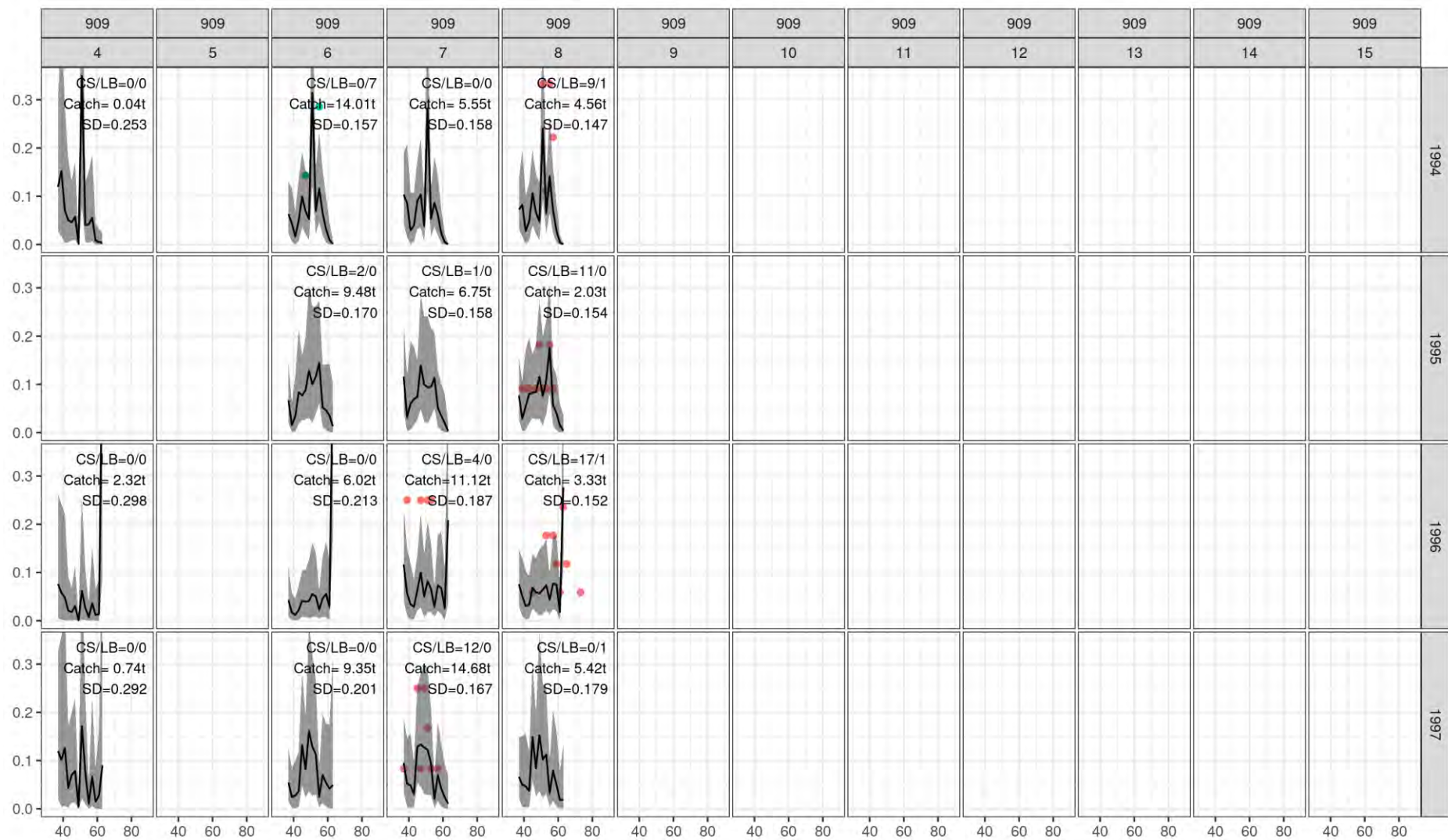


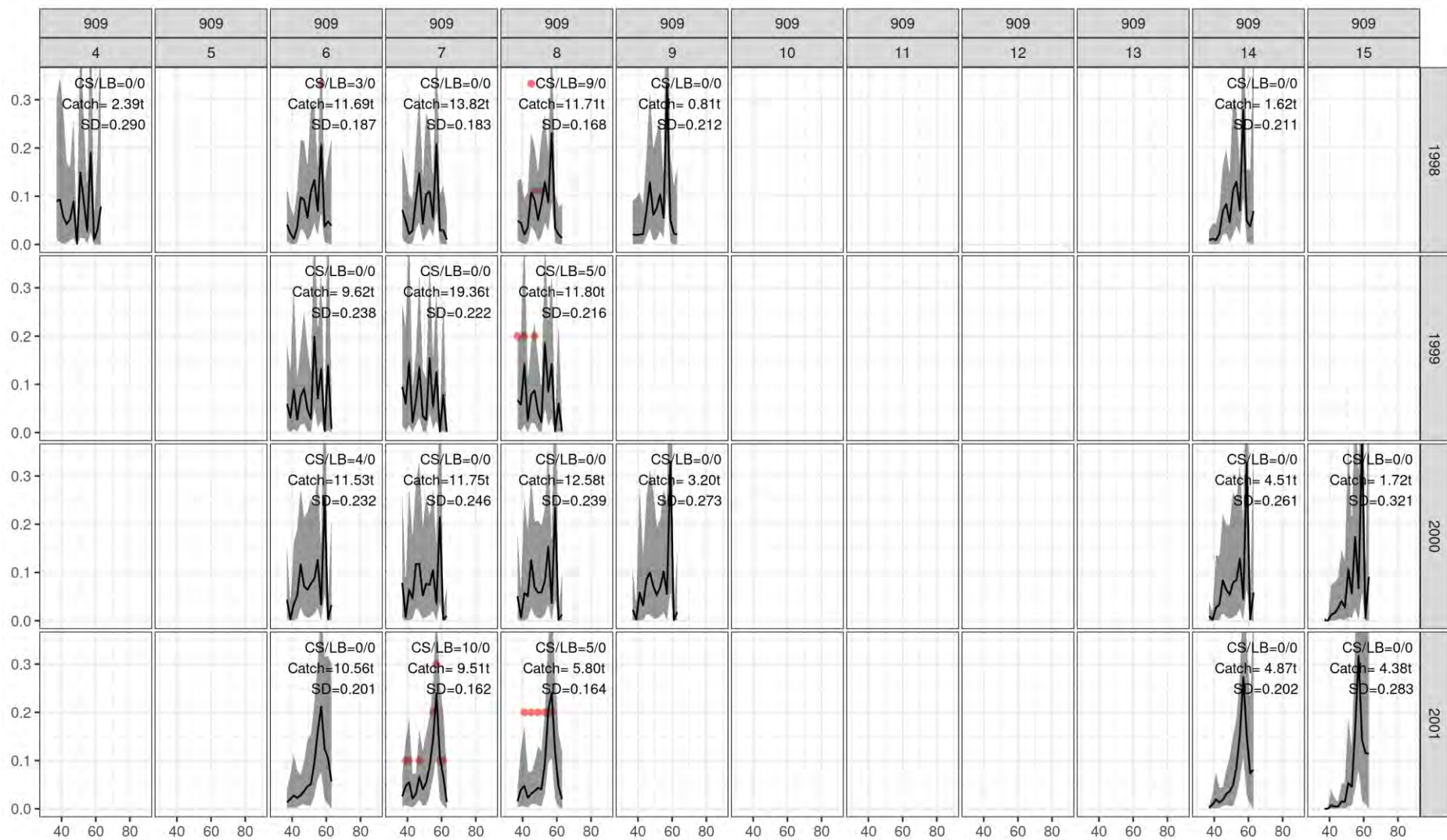


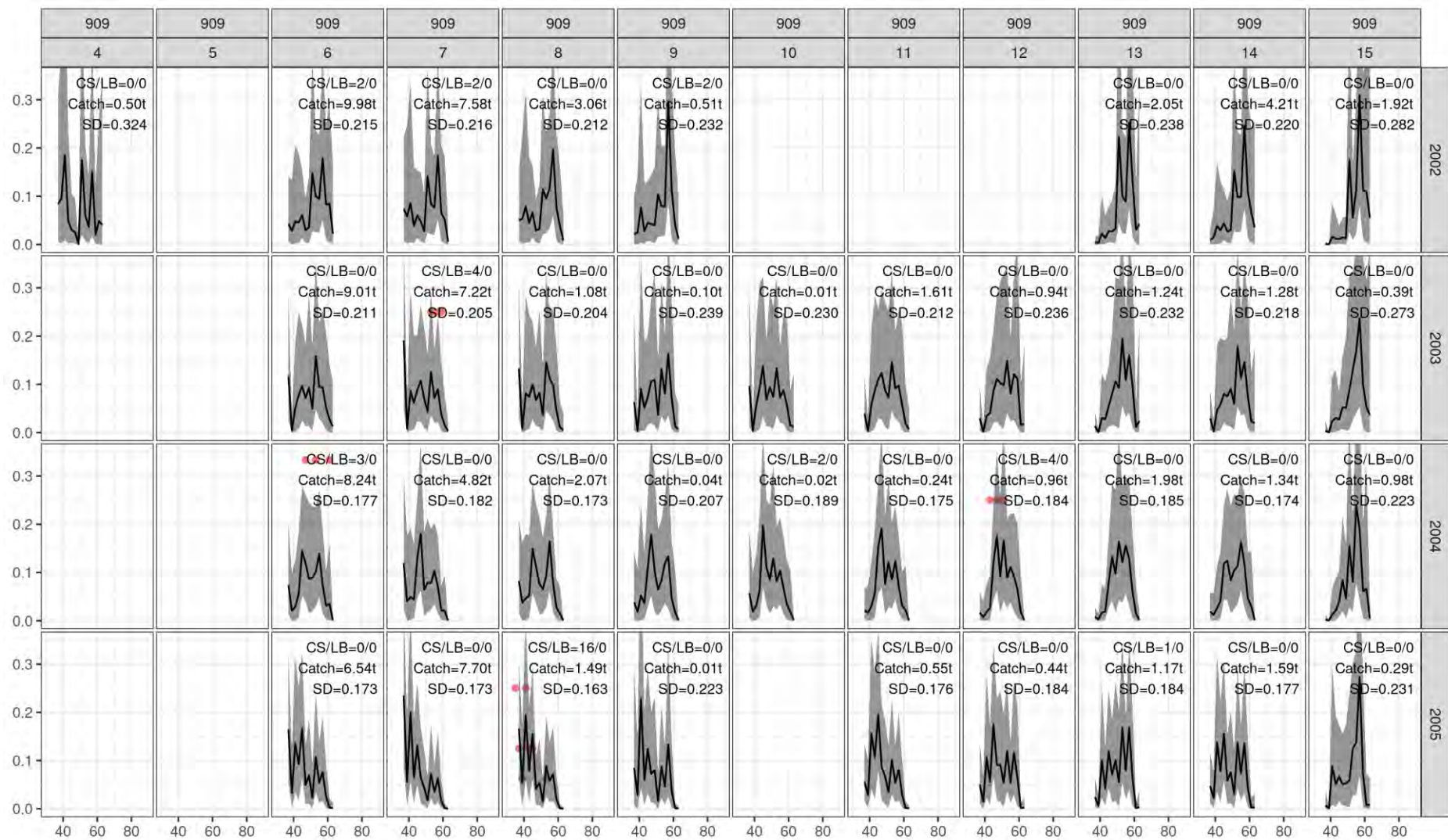


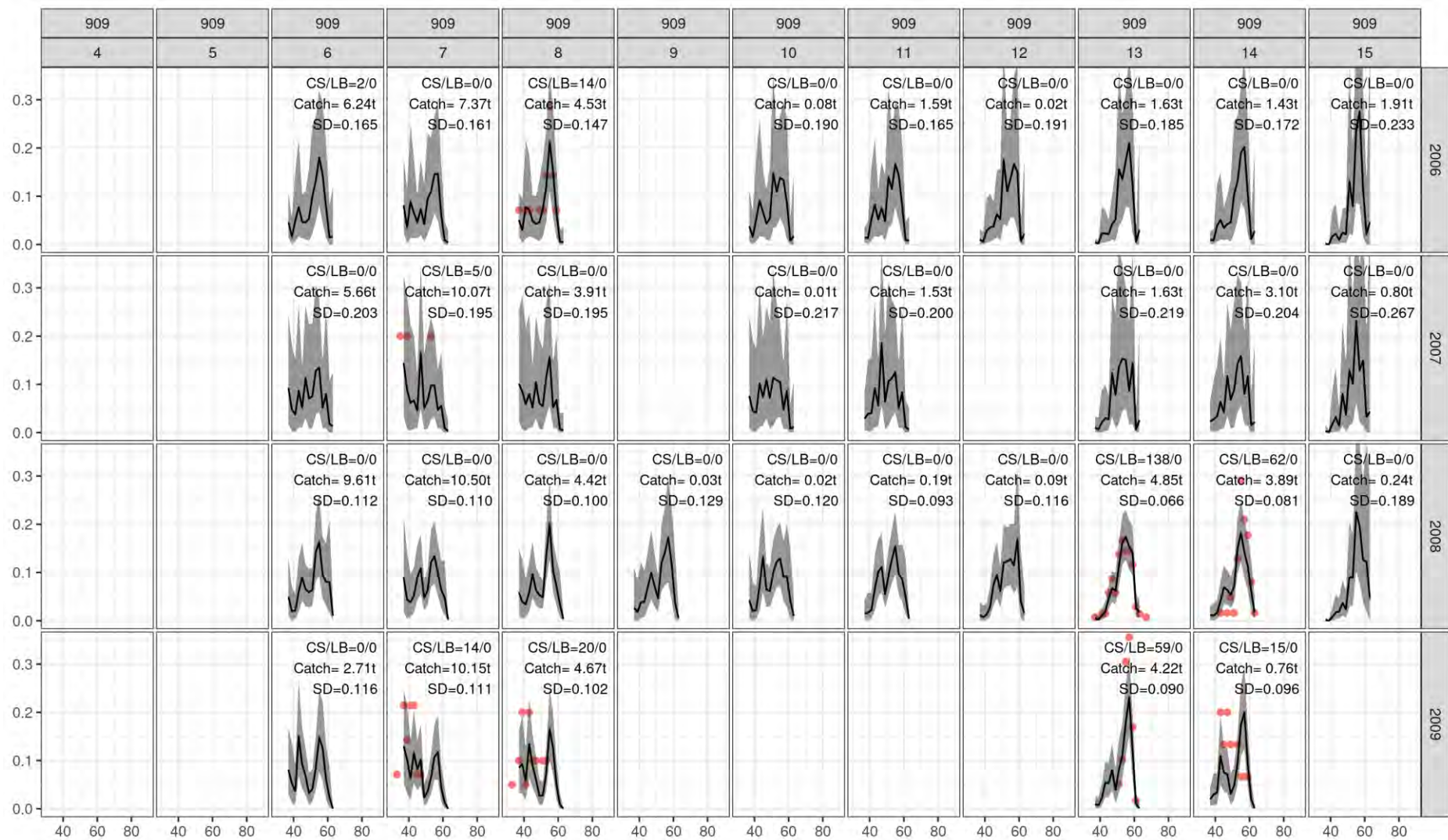
11.5 CRA 3 immature females

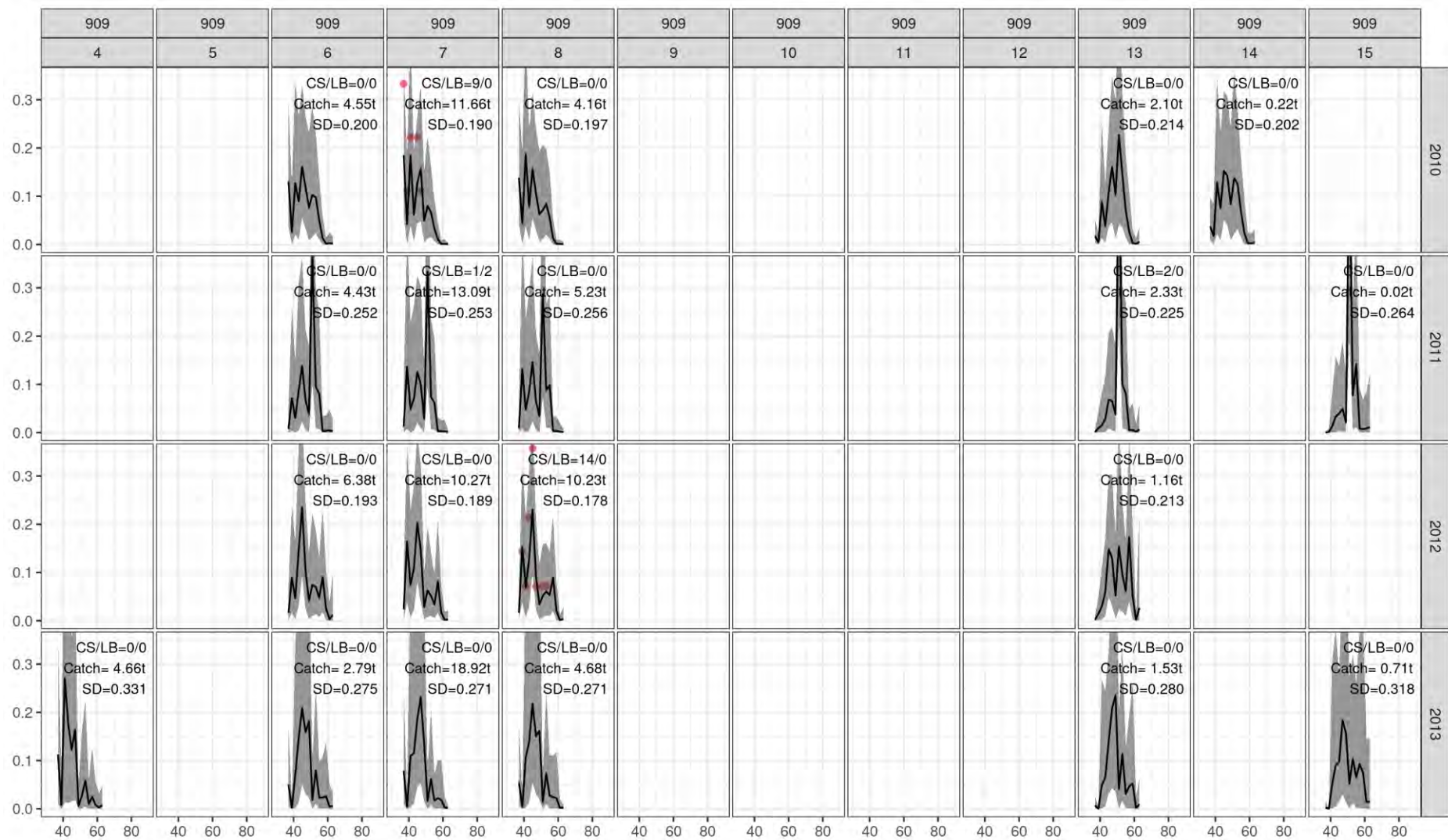


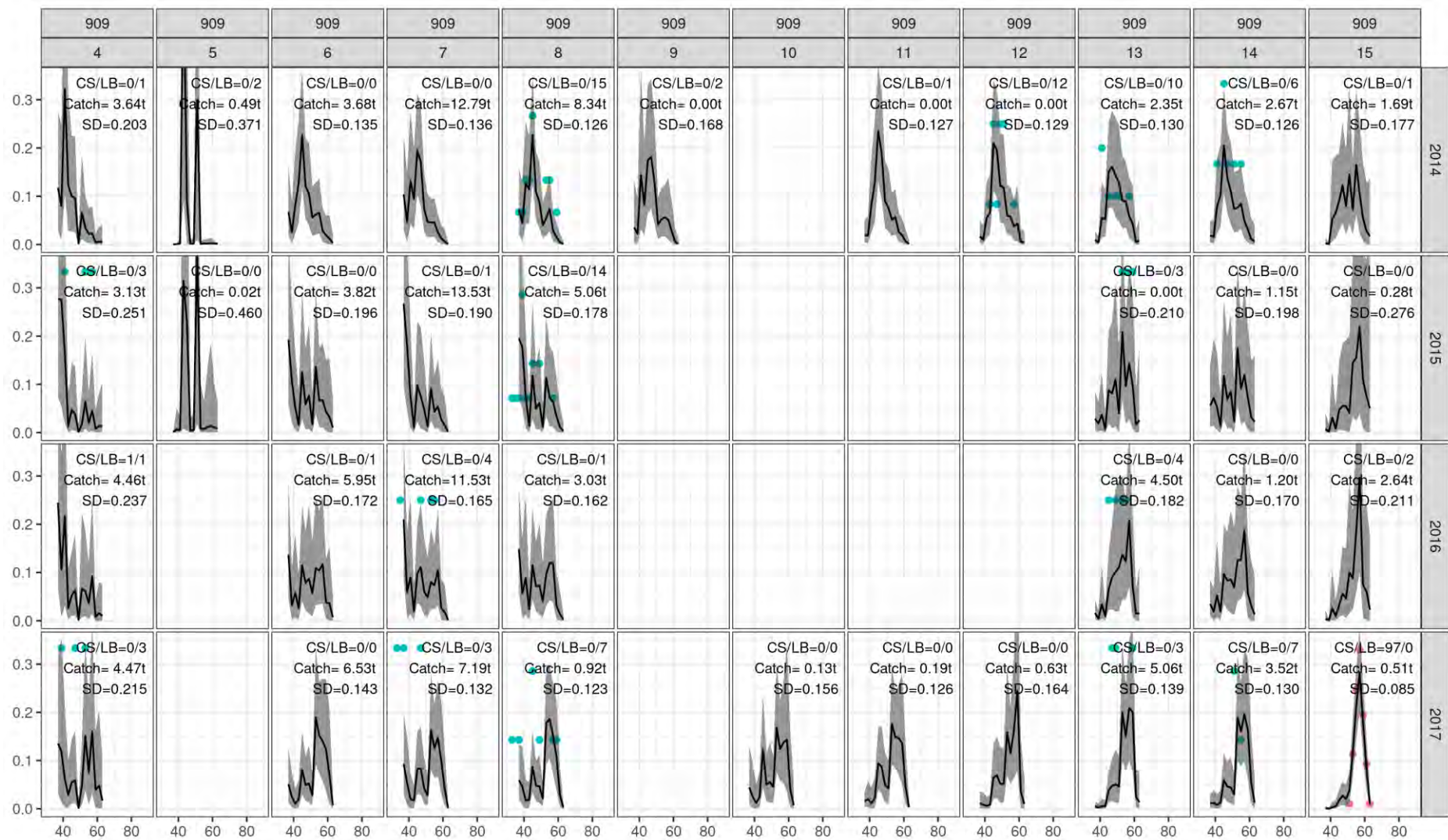


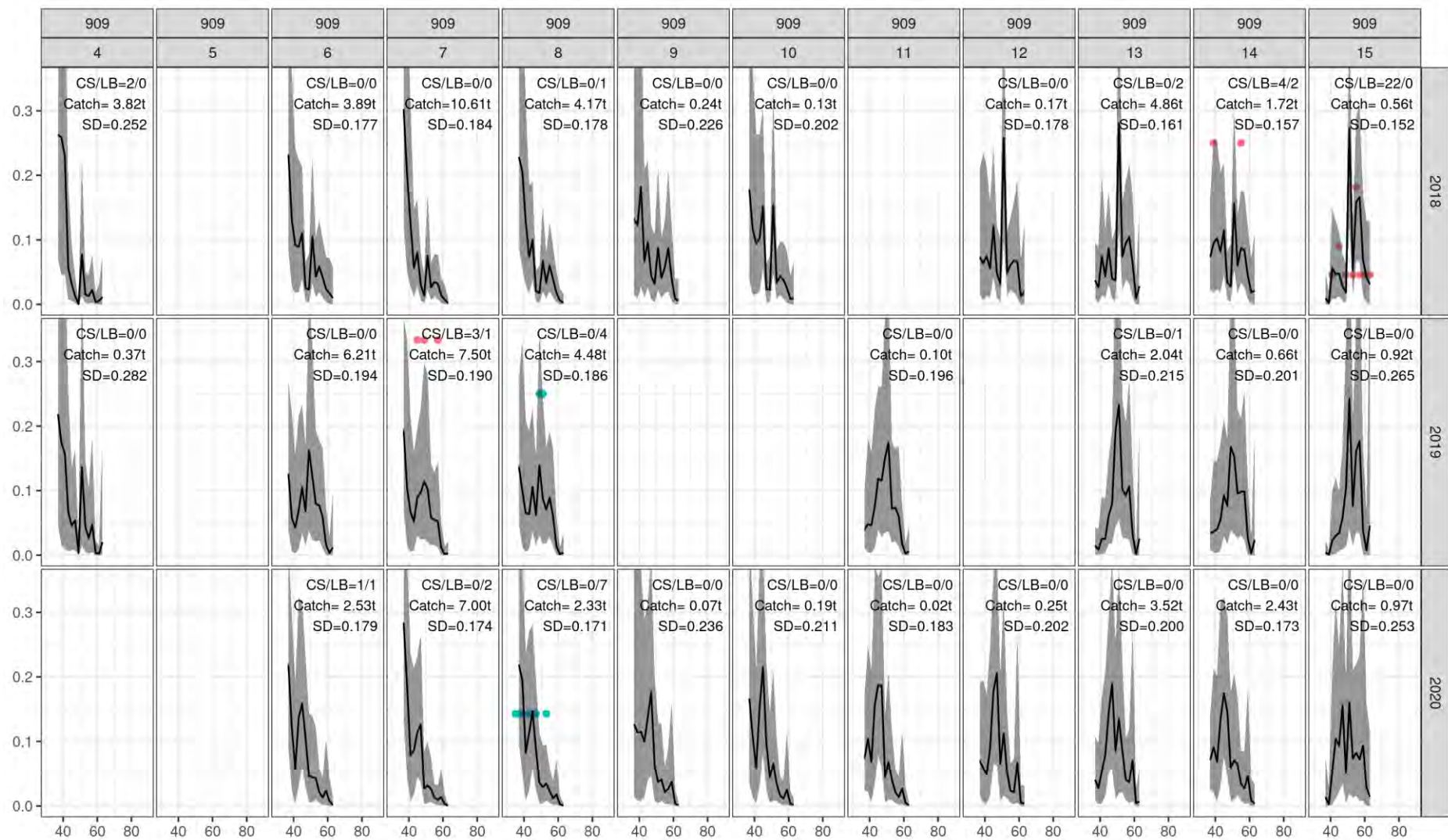


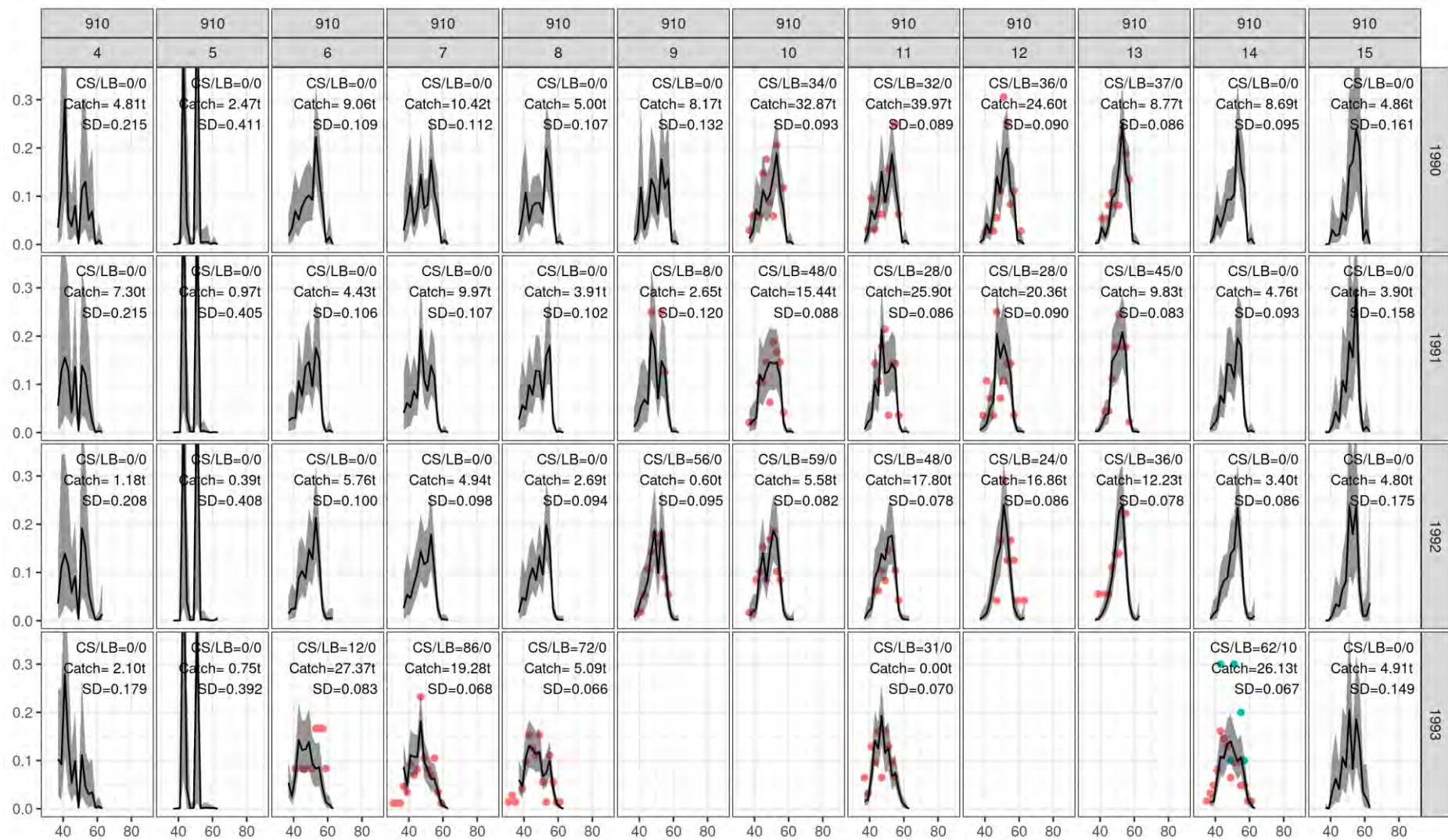


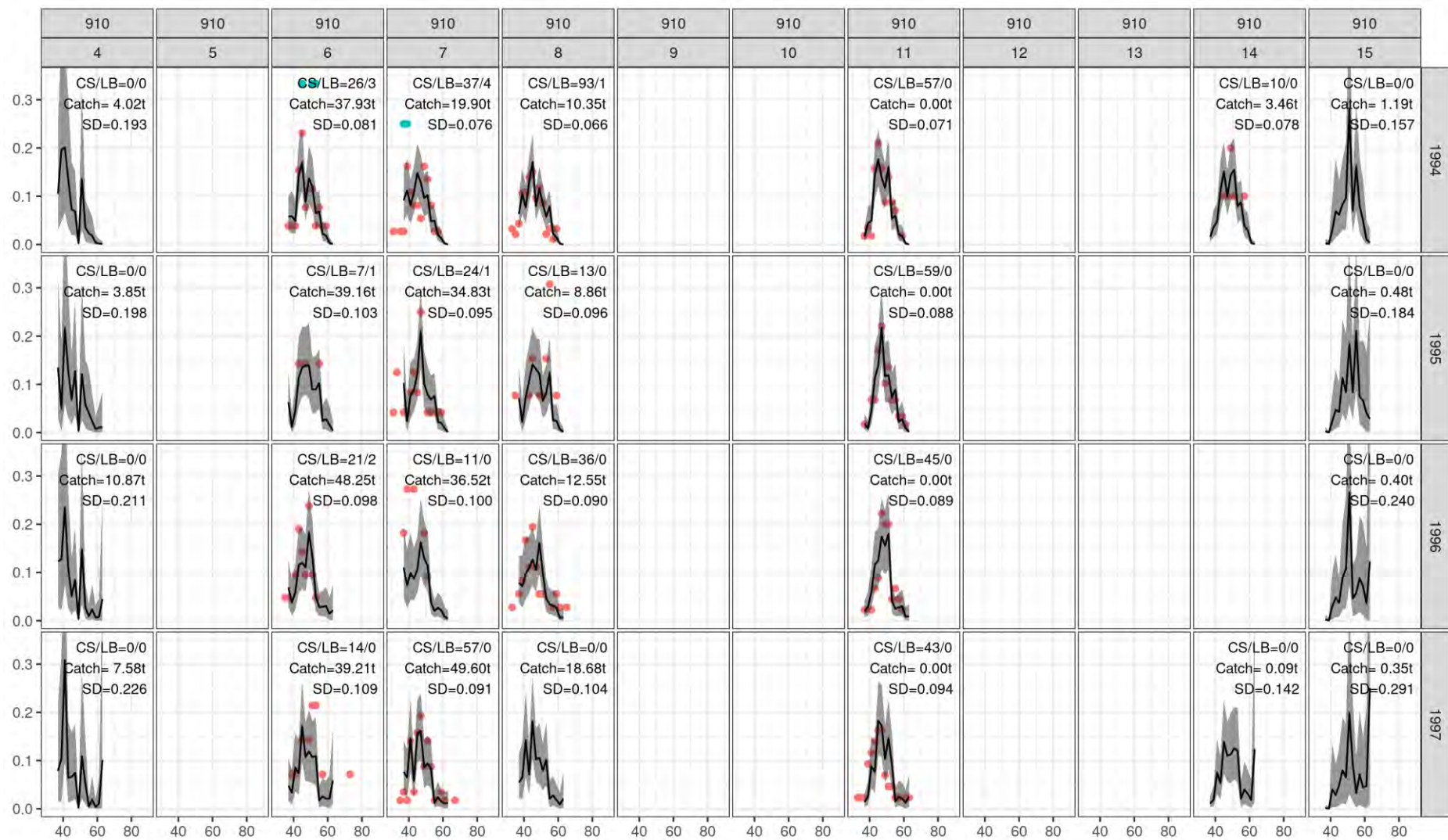


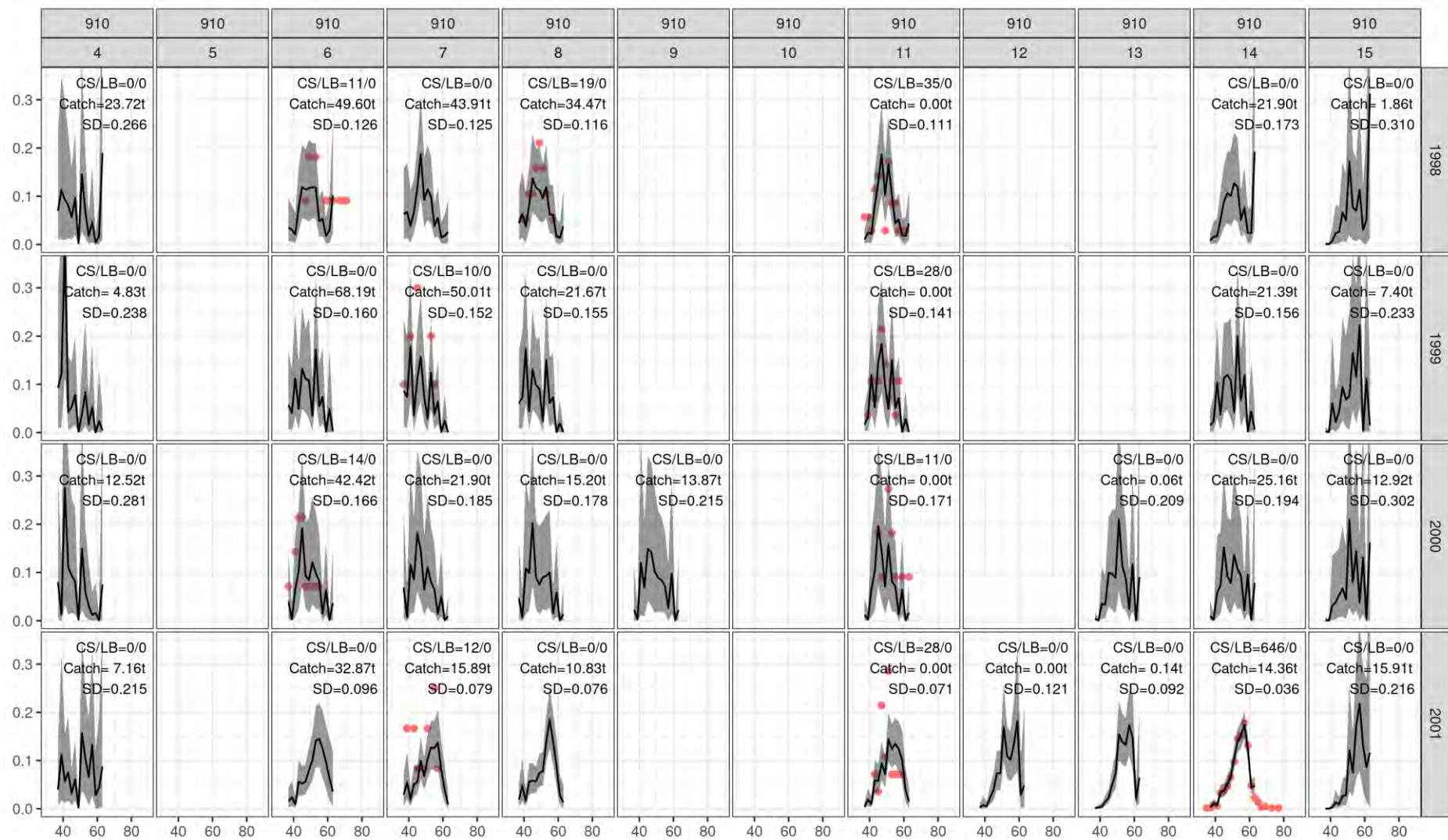


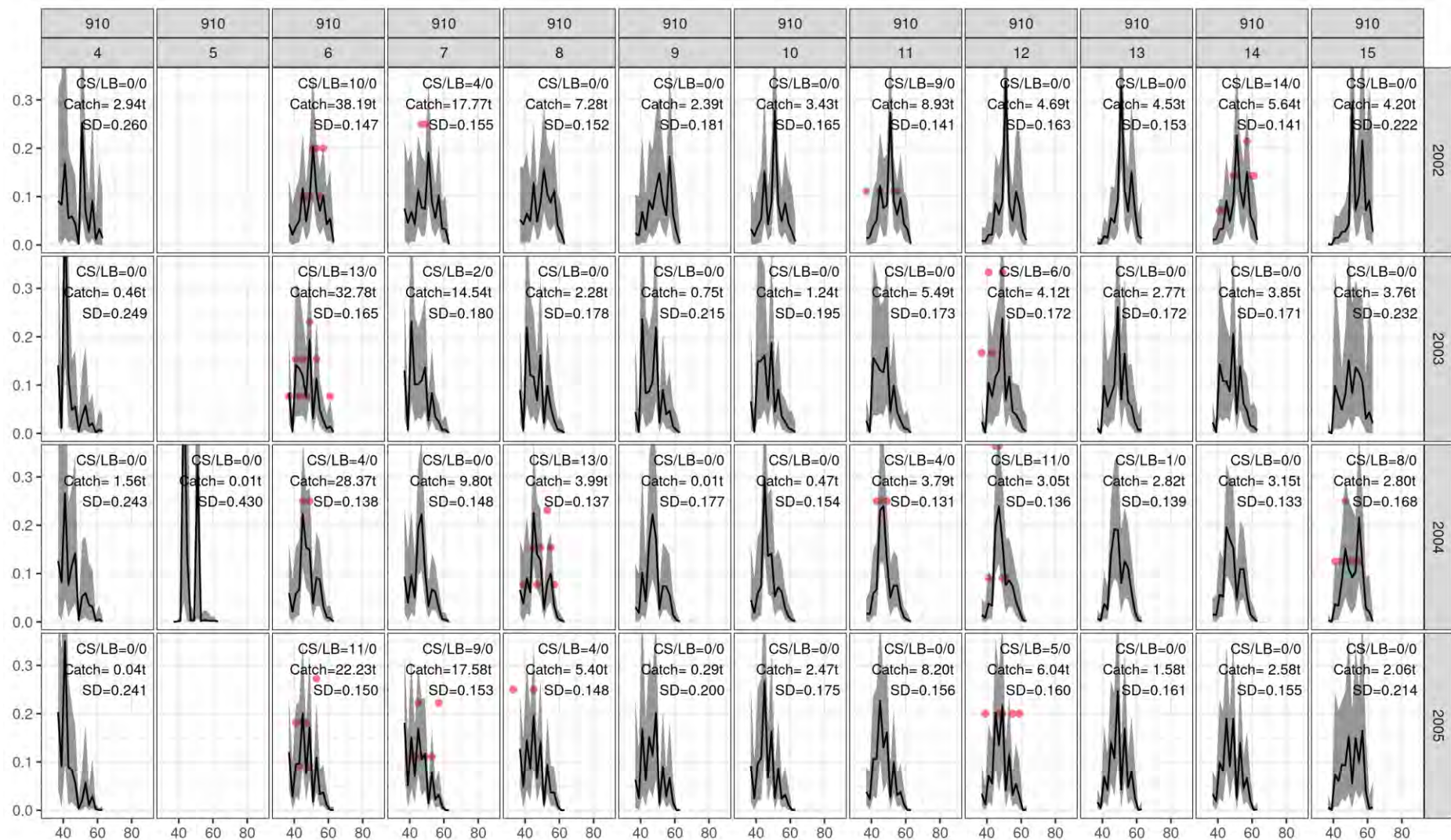


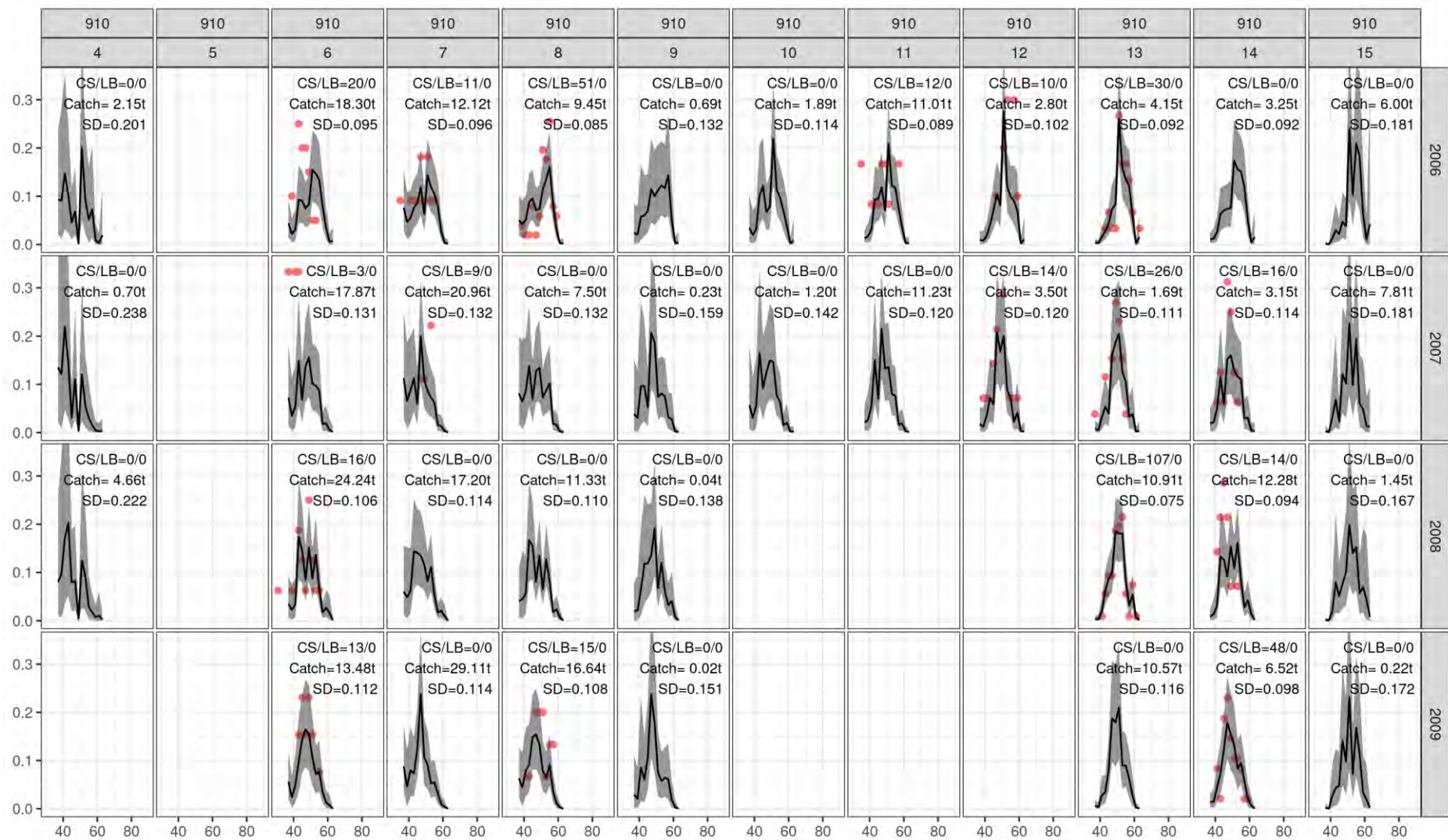


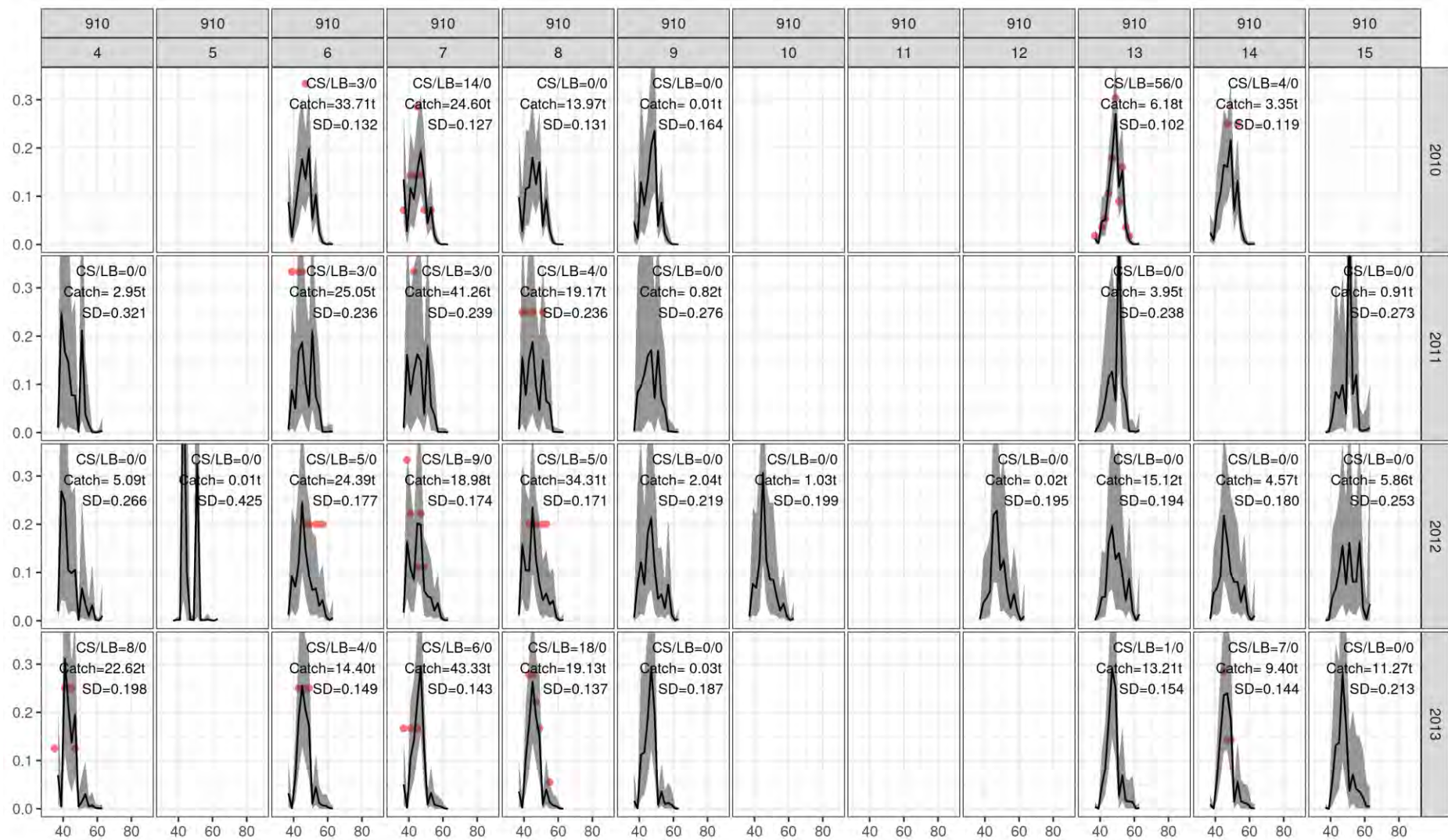


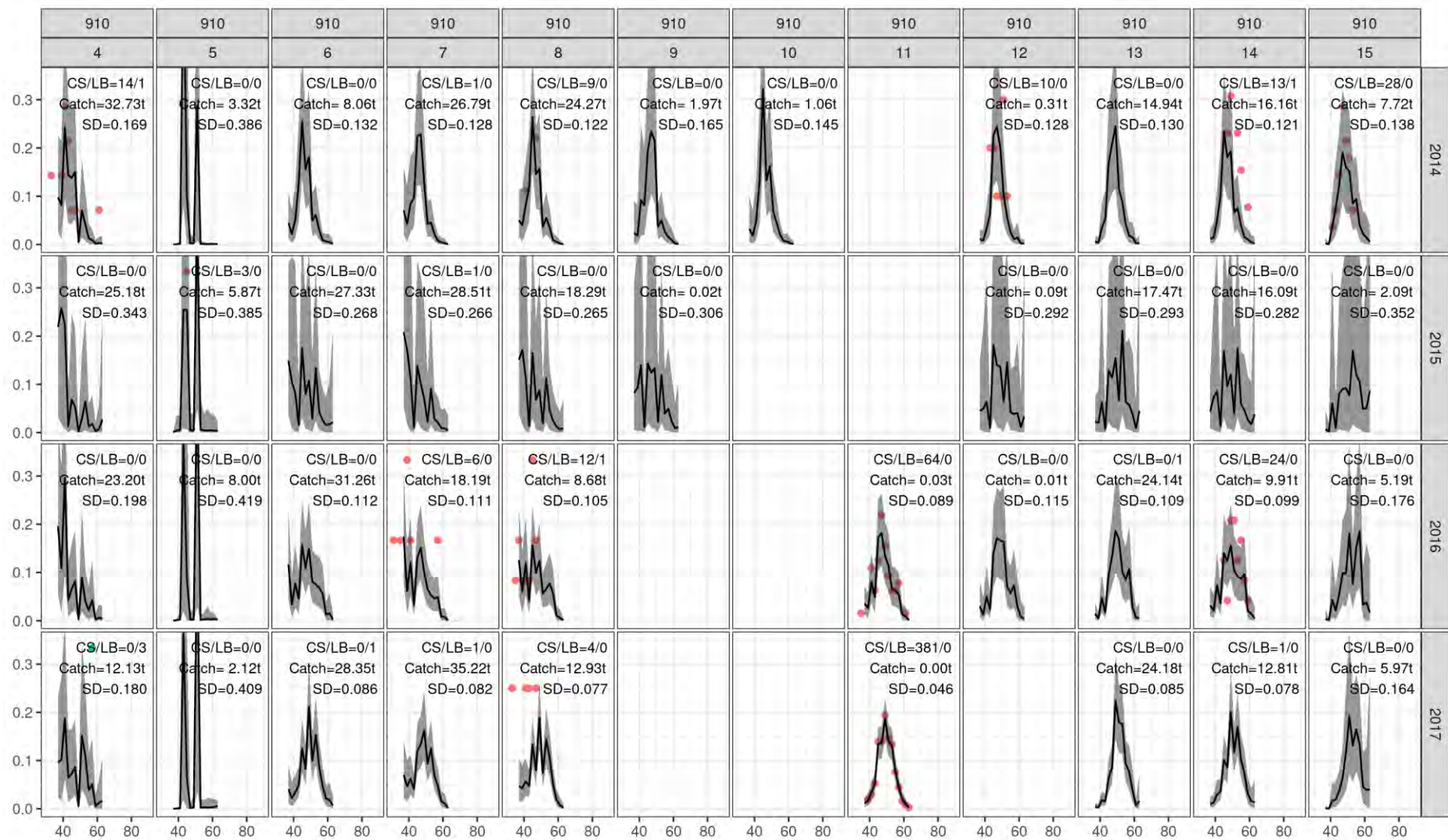


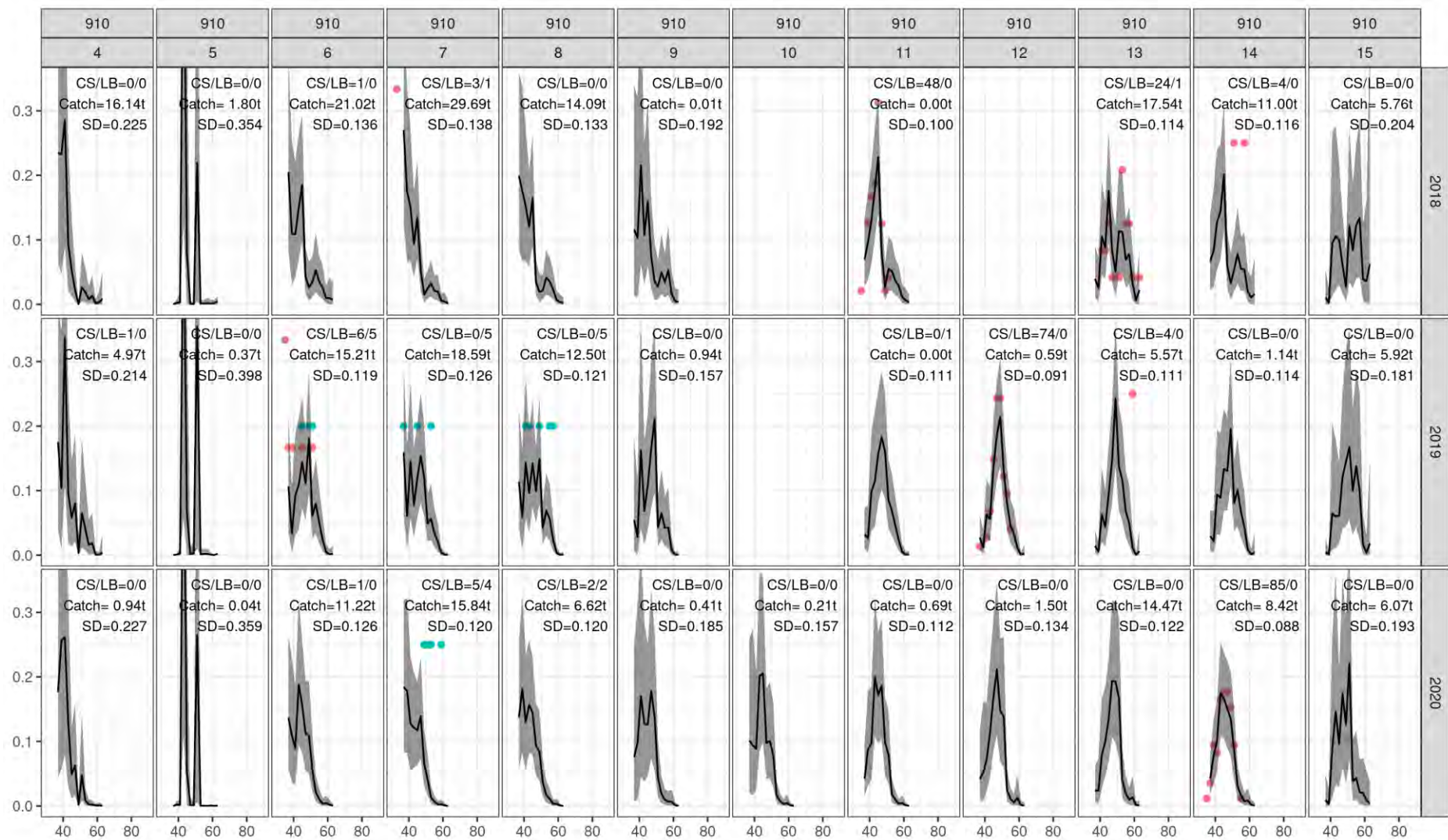


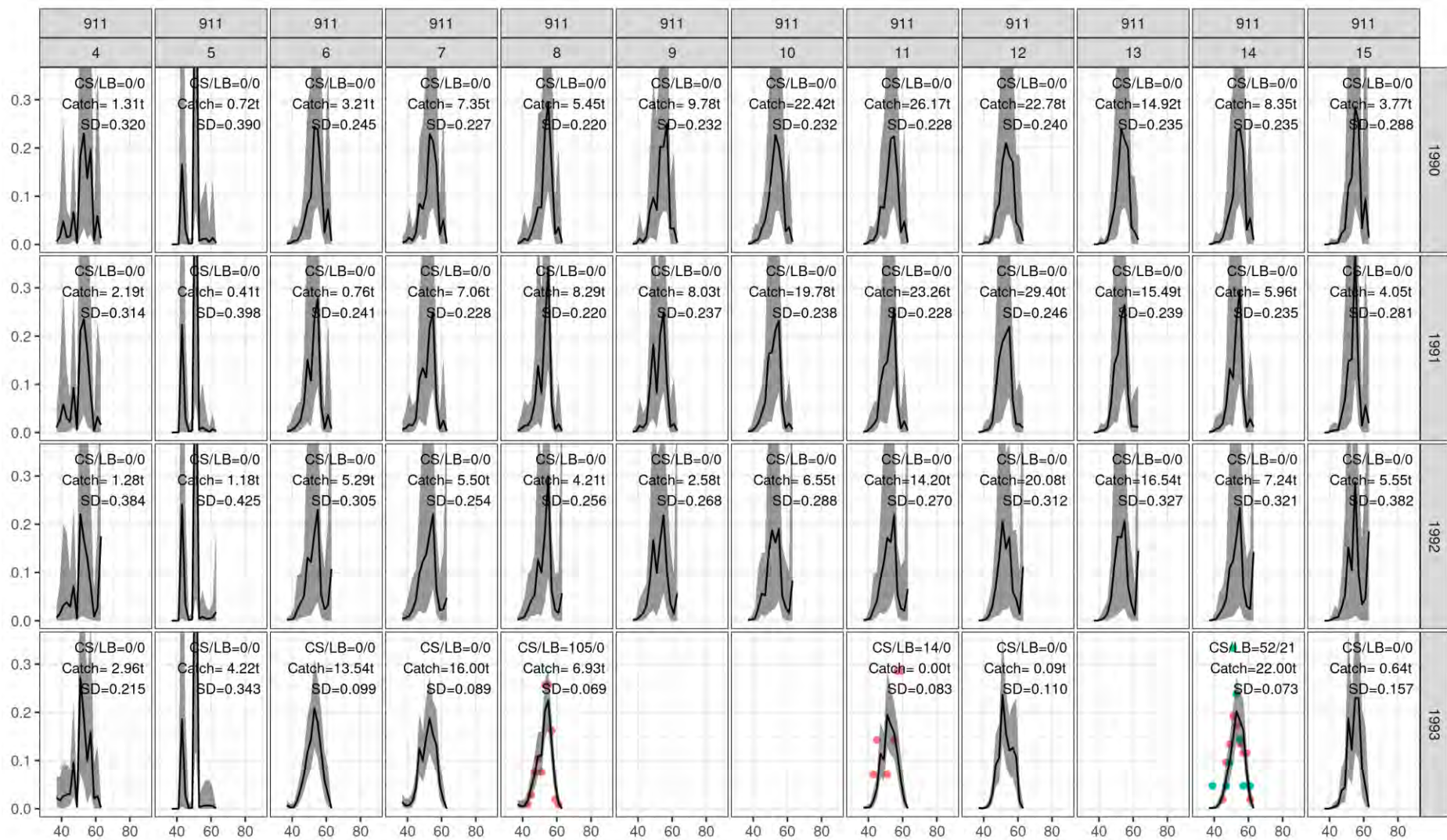


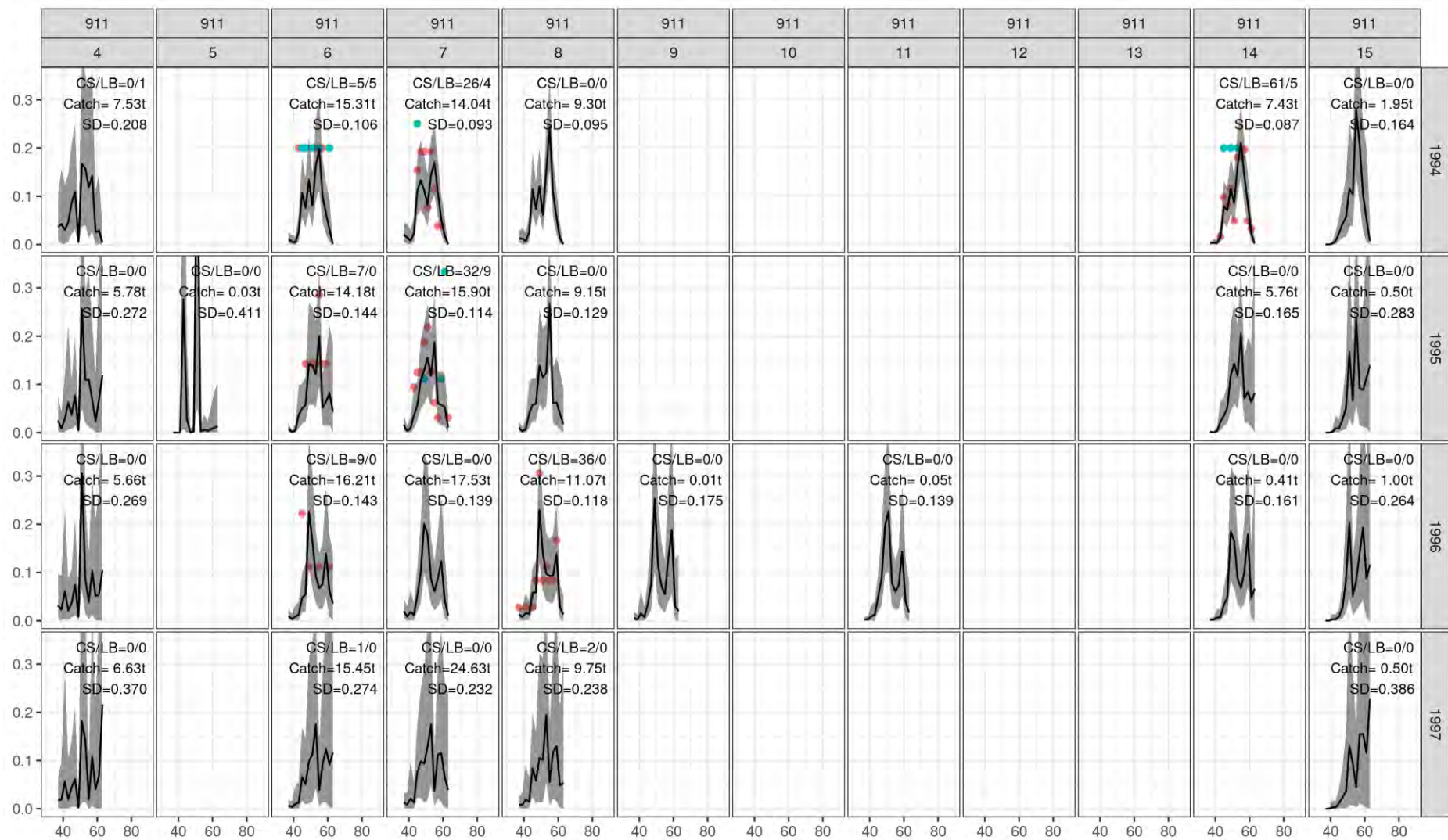


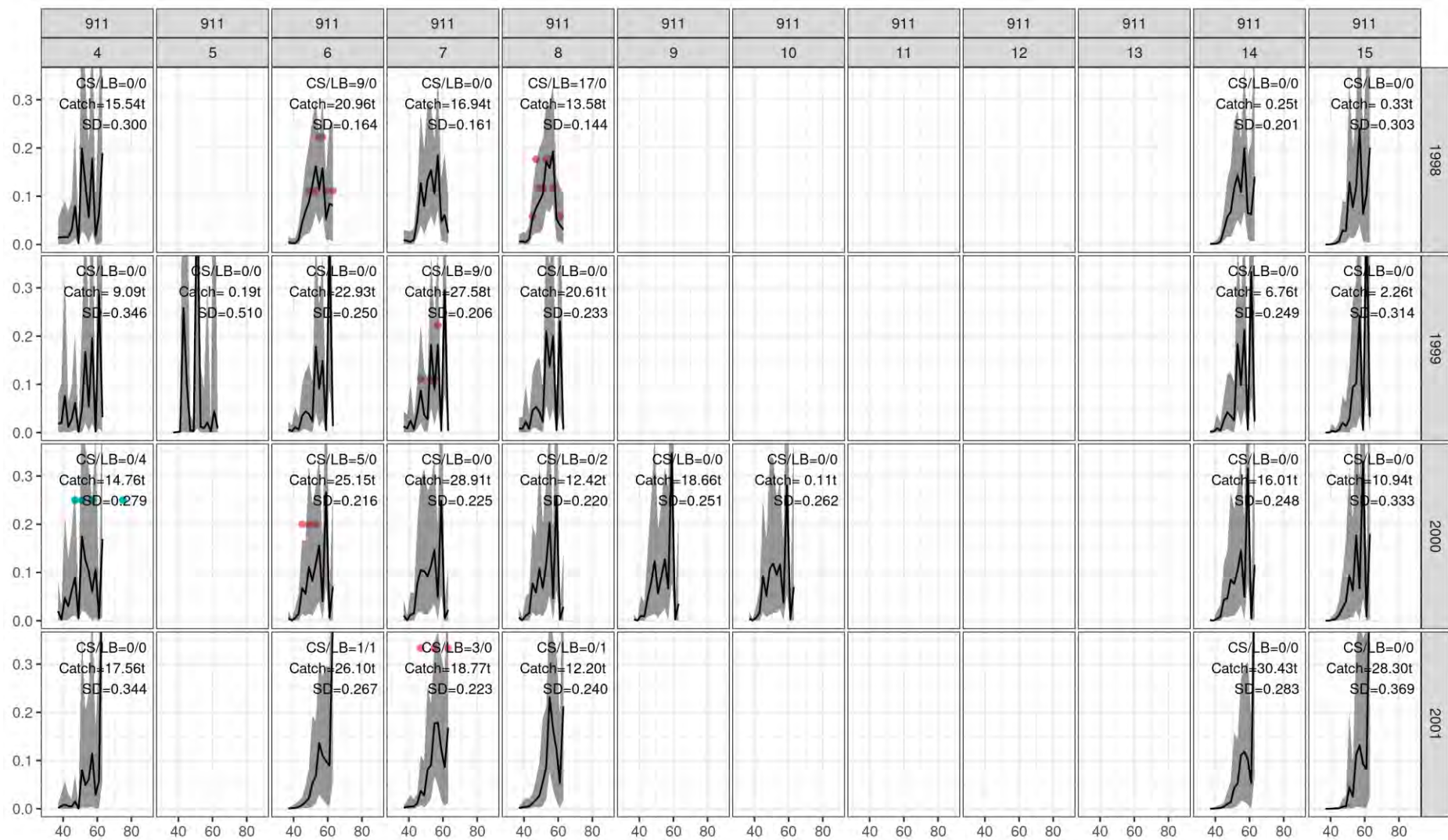


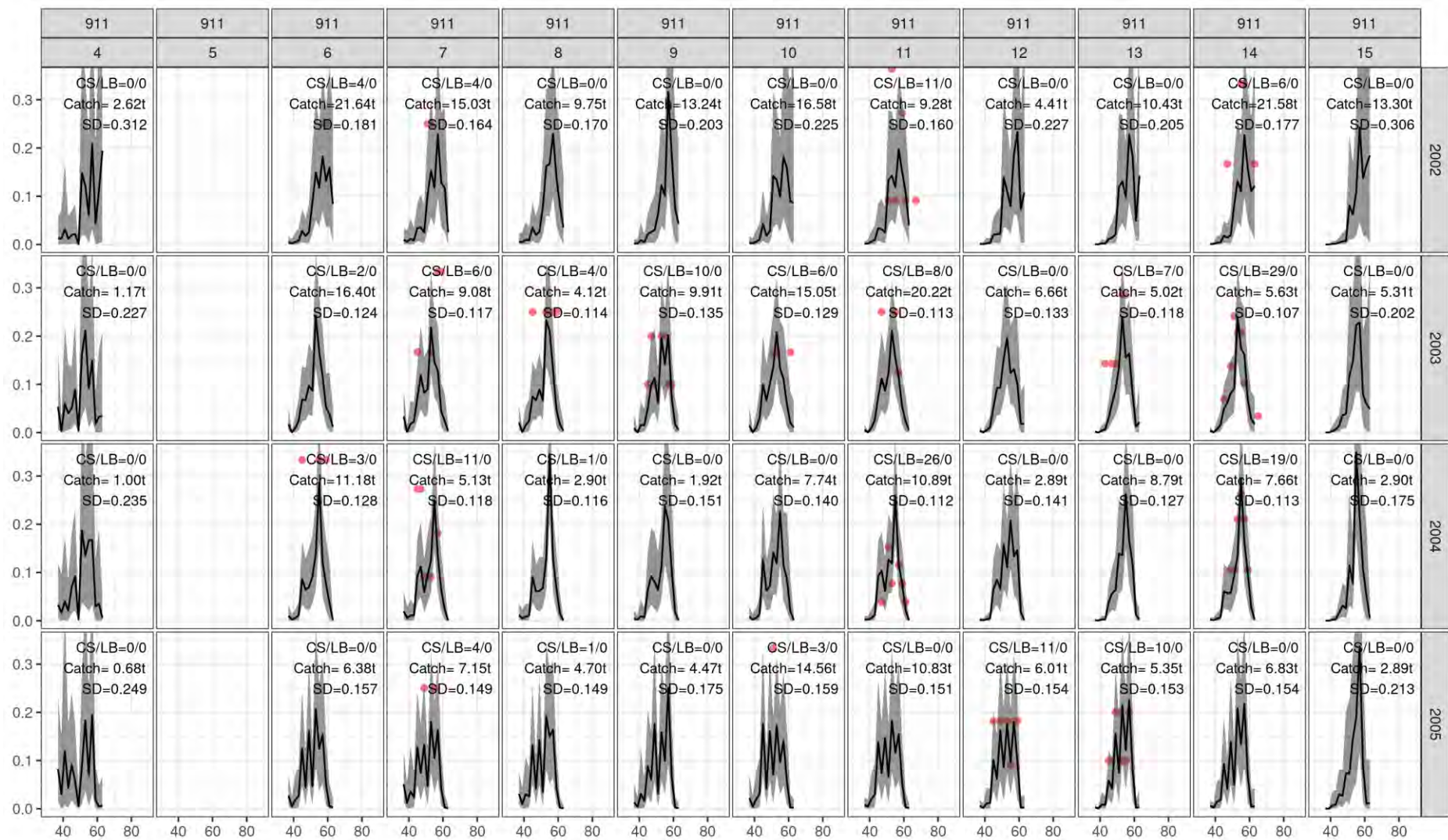


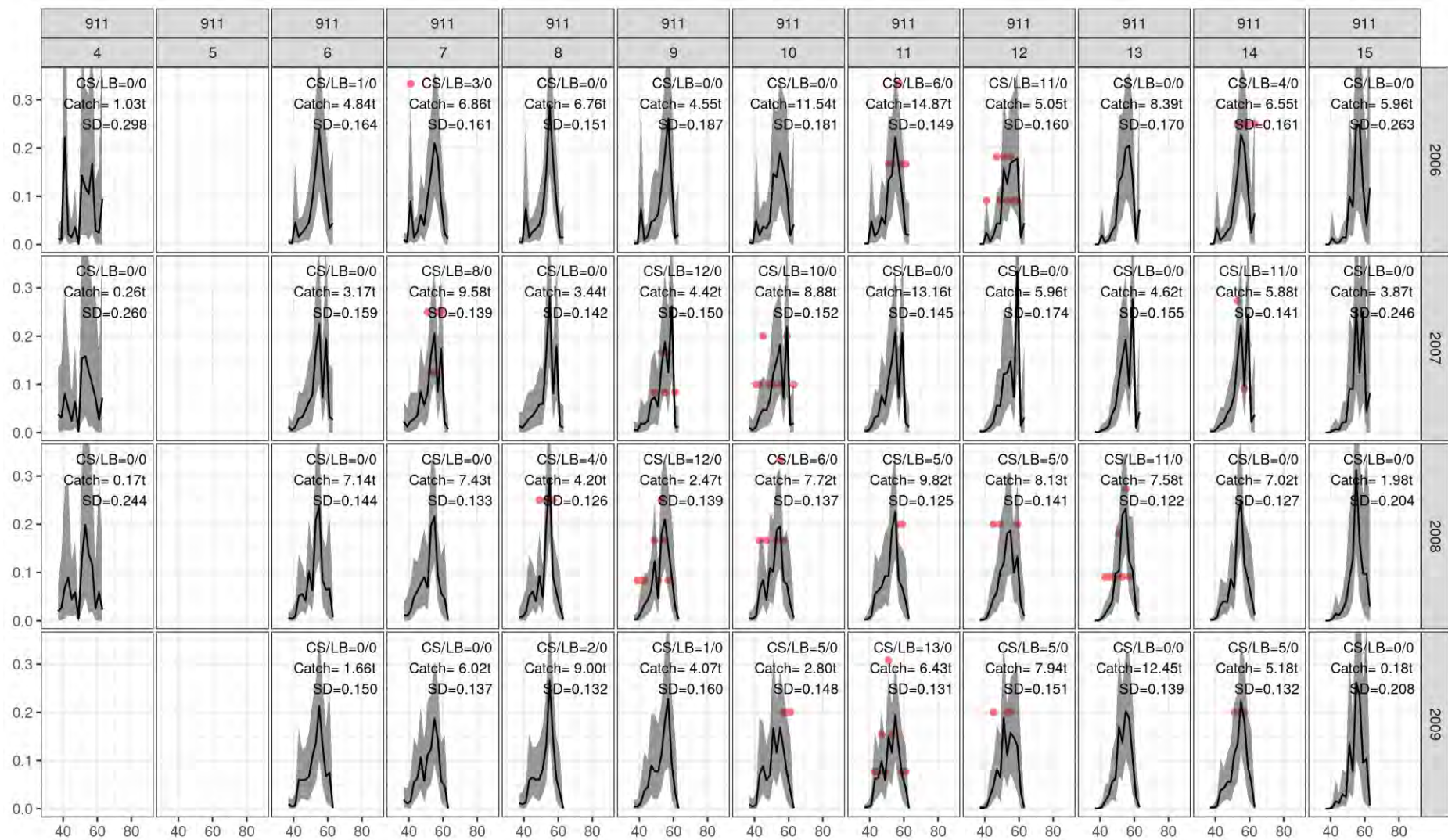


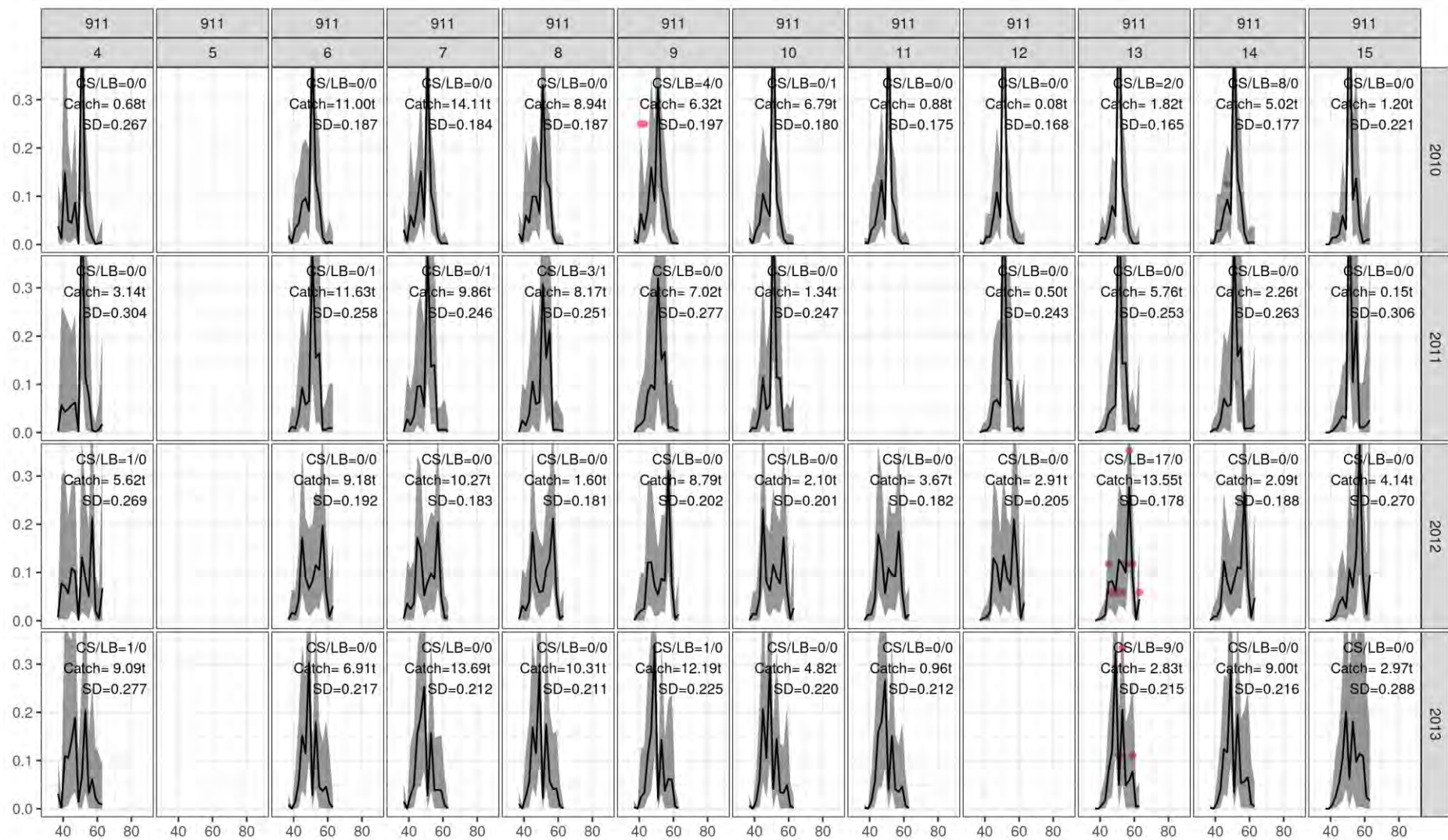


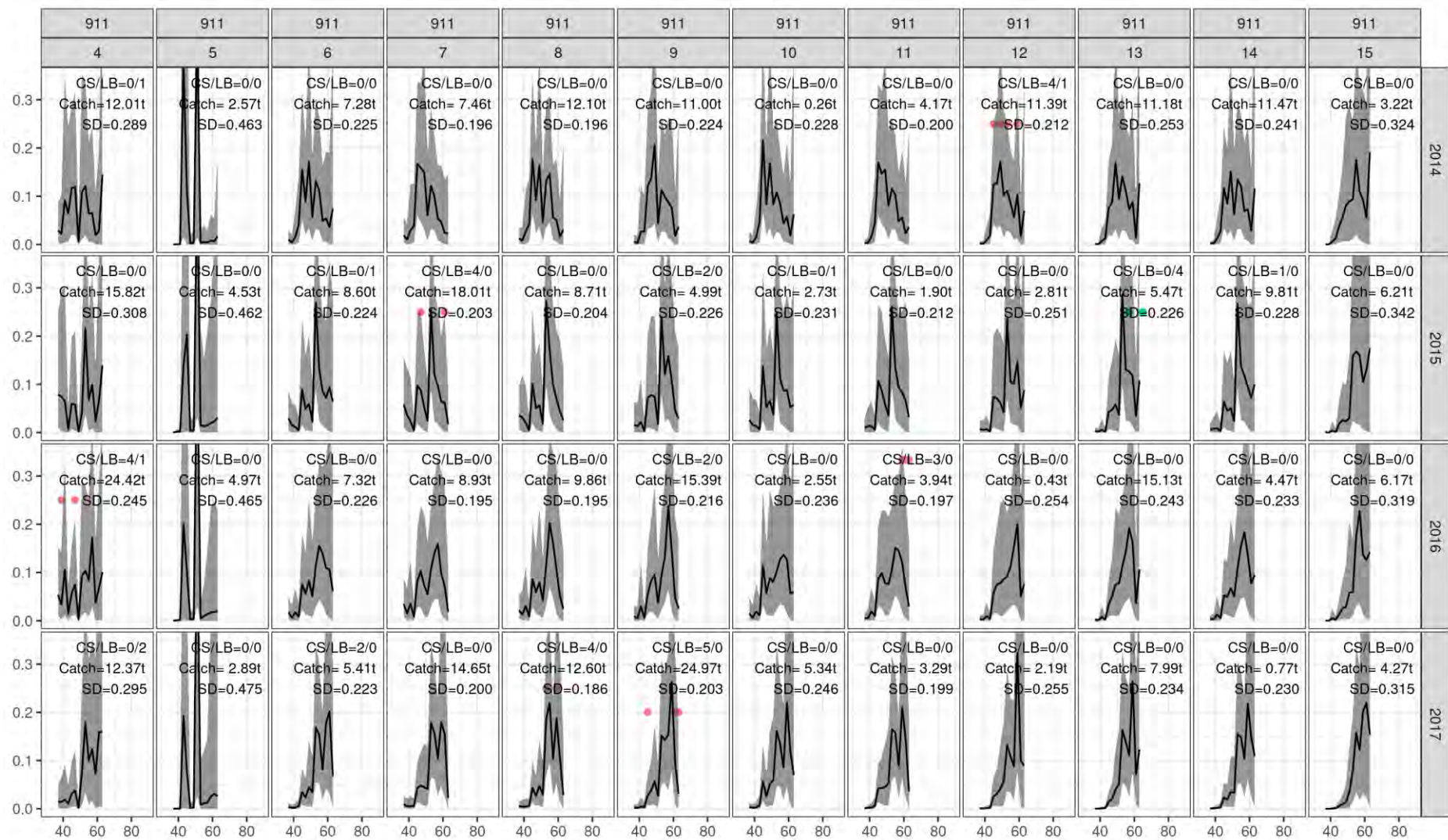


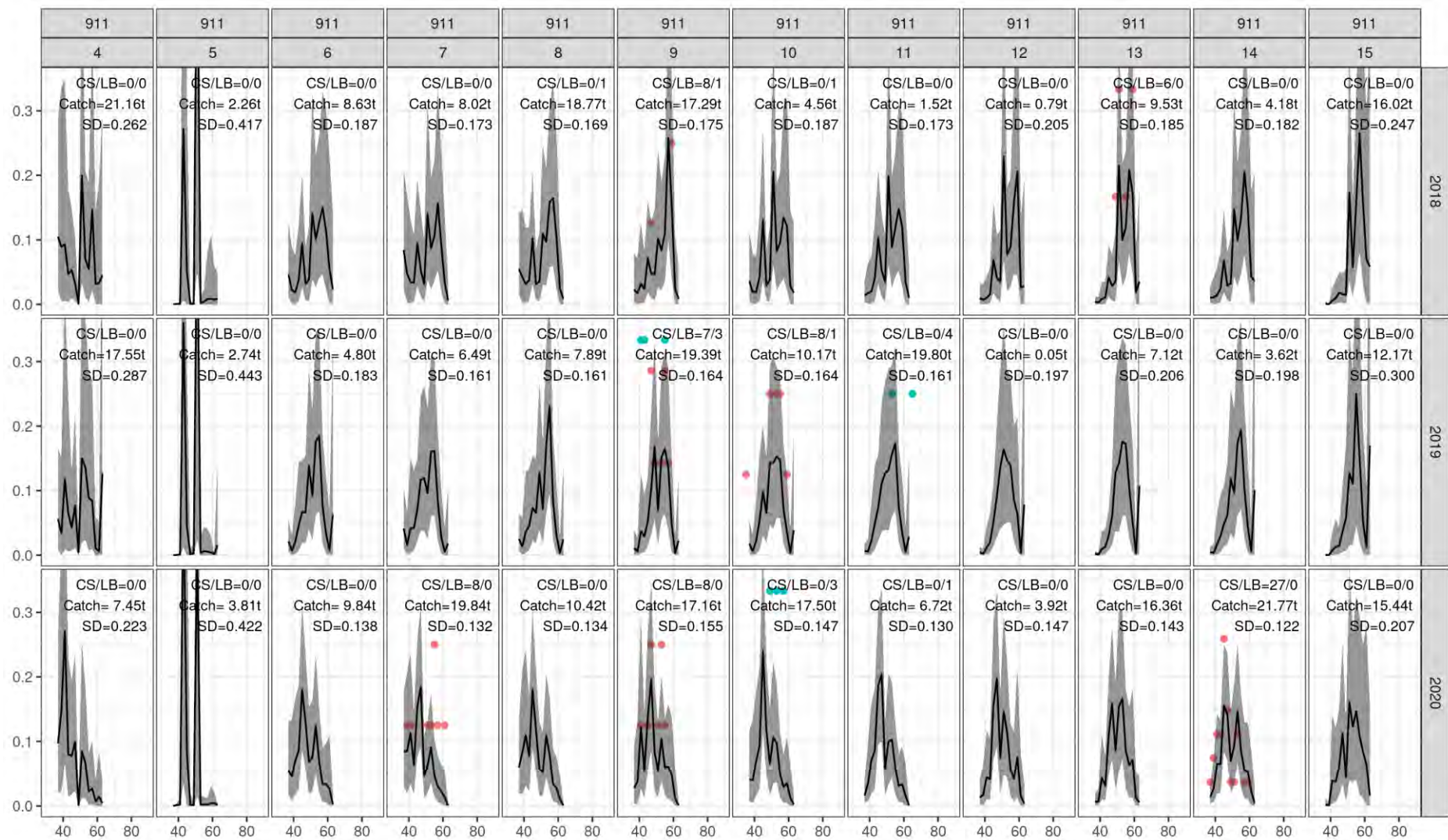




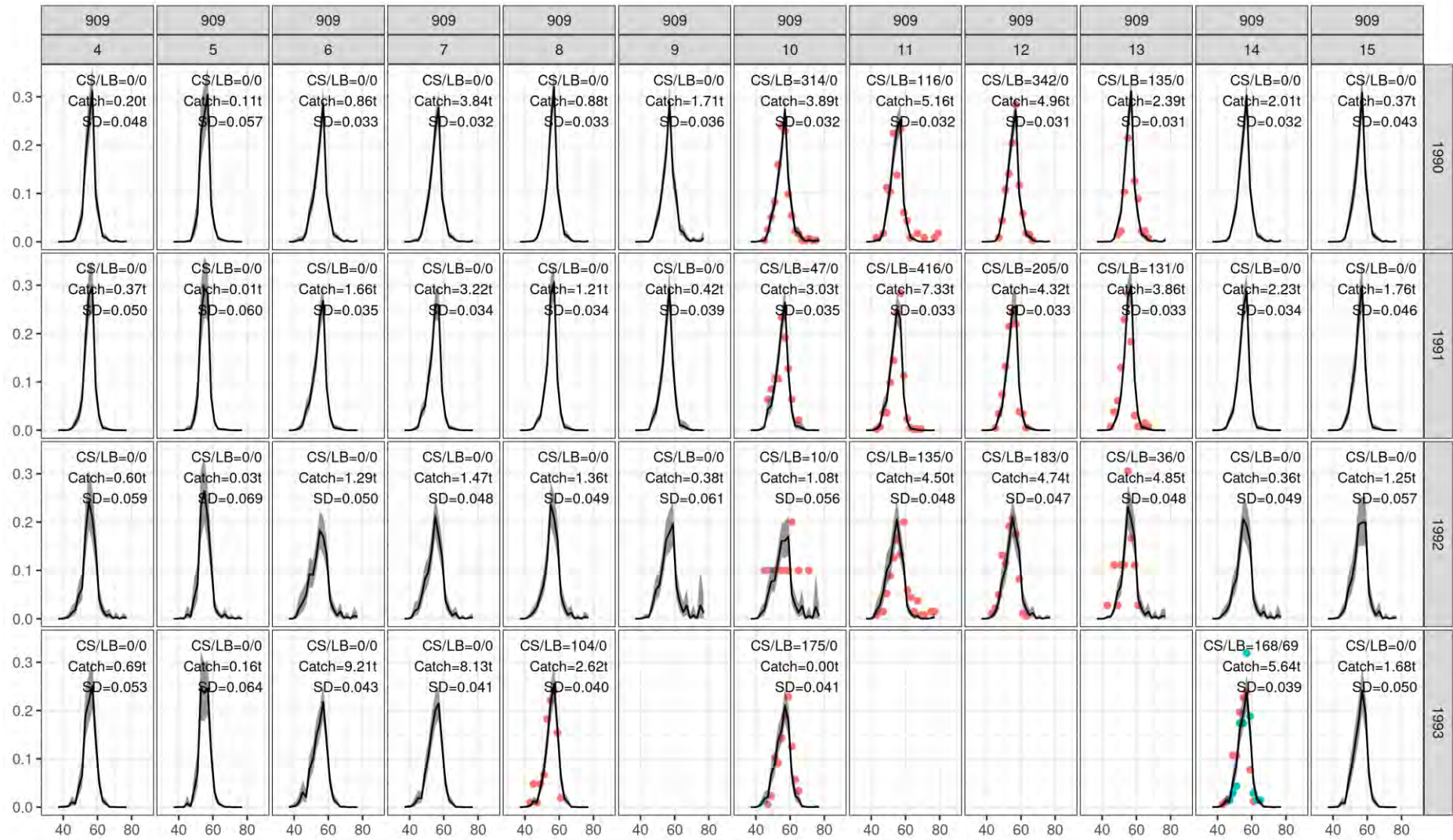


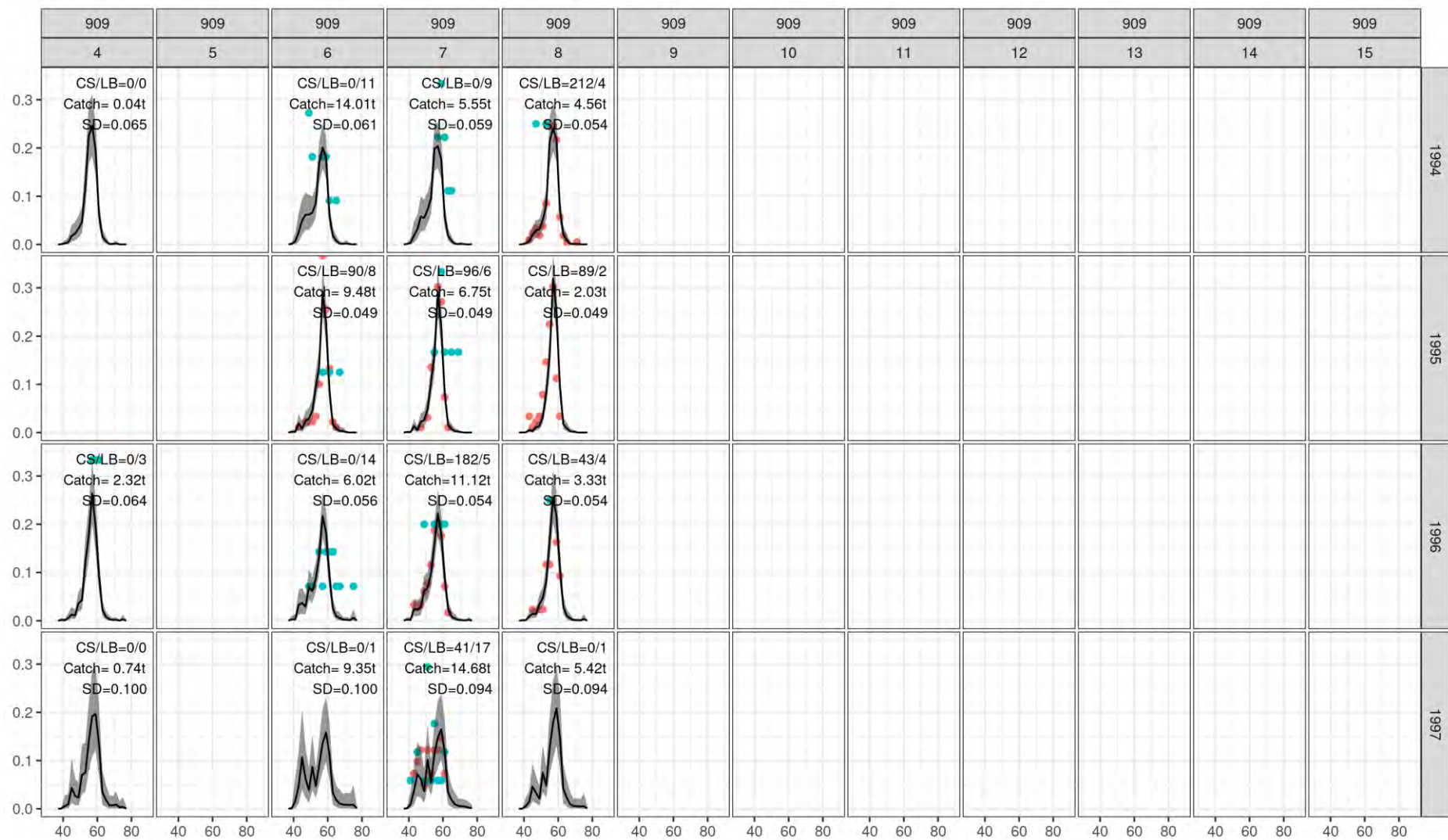


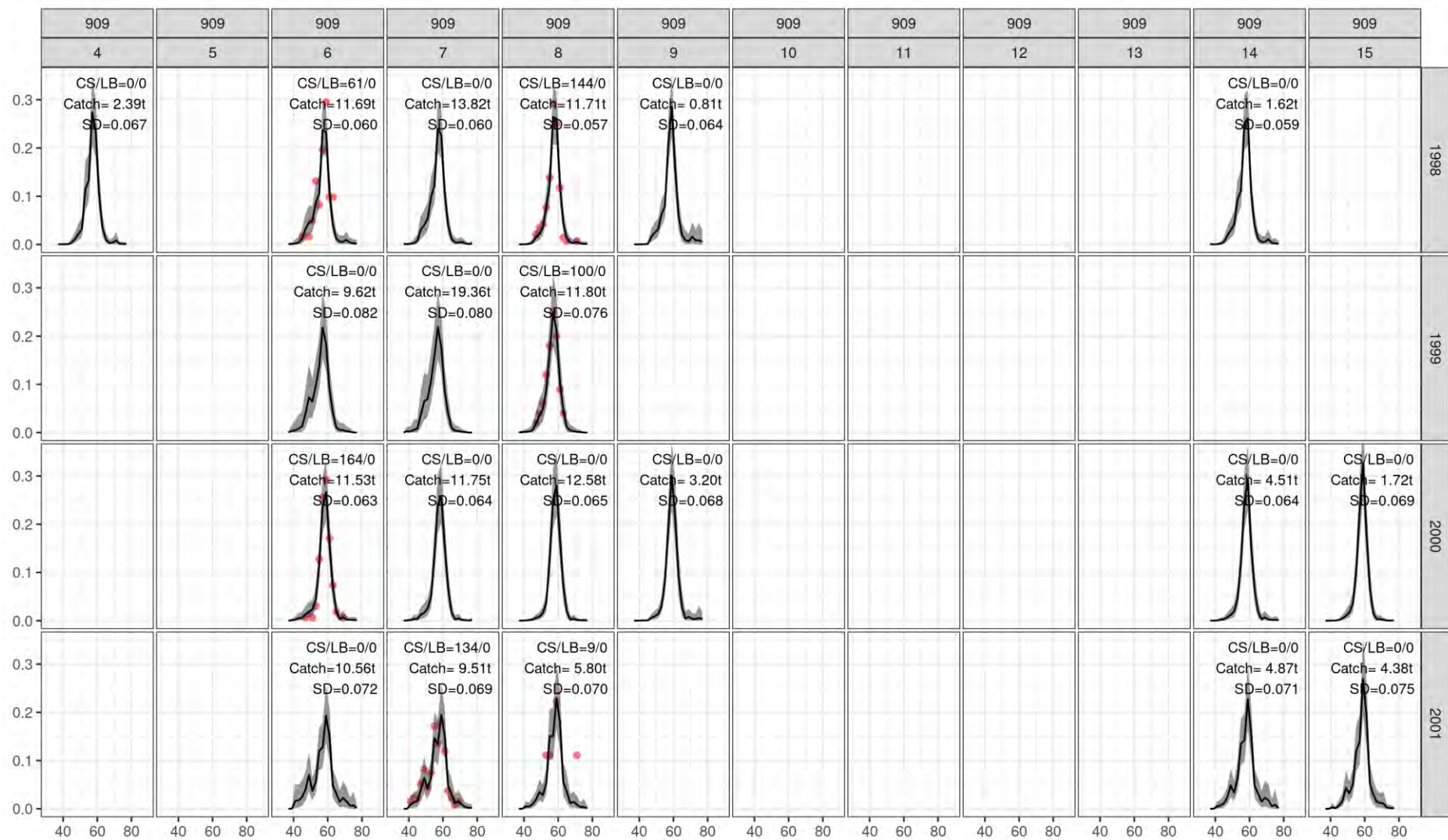


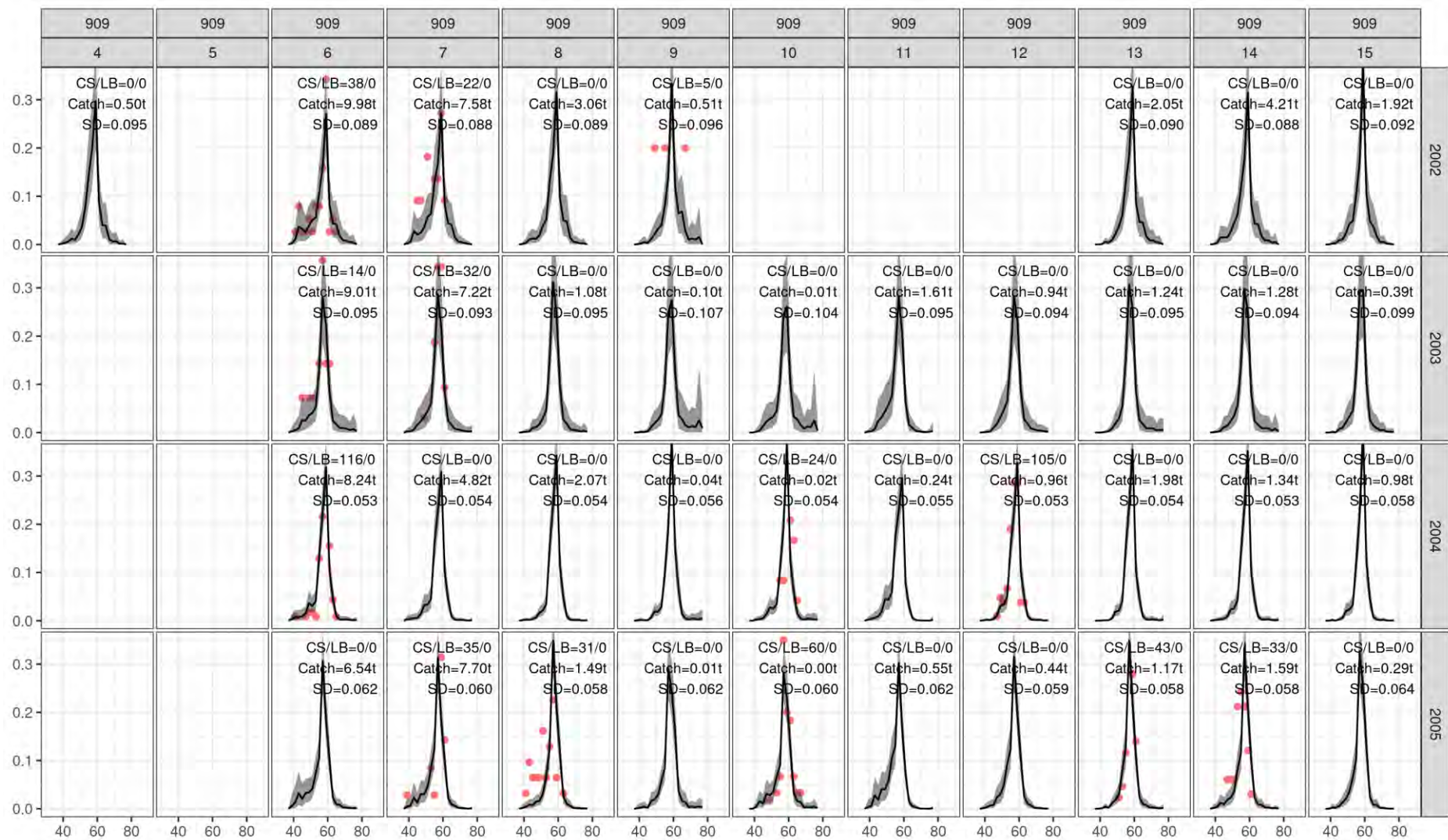


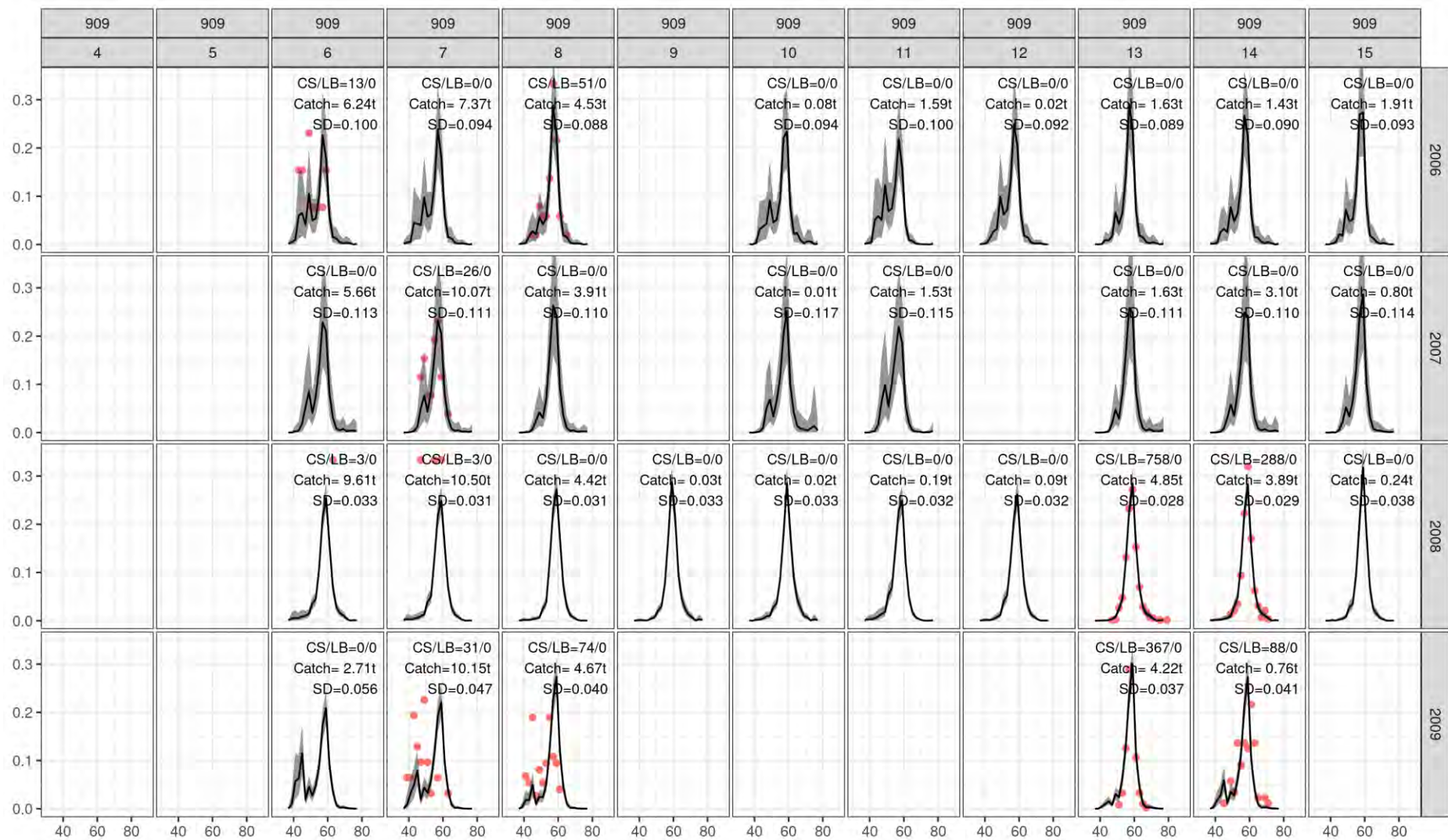
11.6 CRA 3 mature females

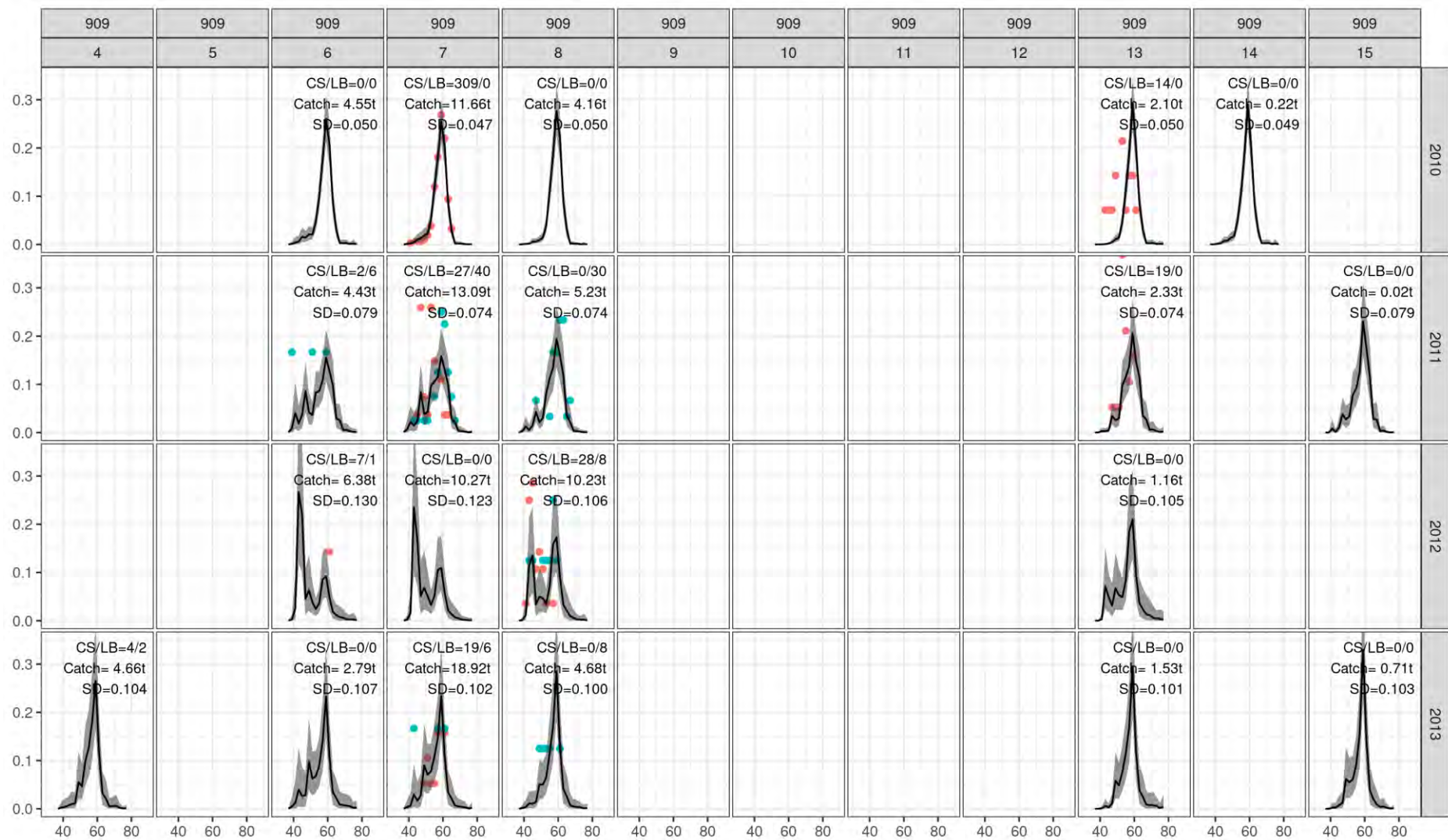


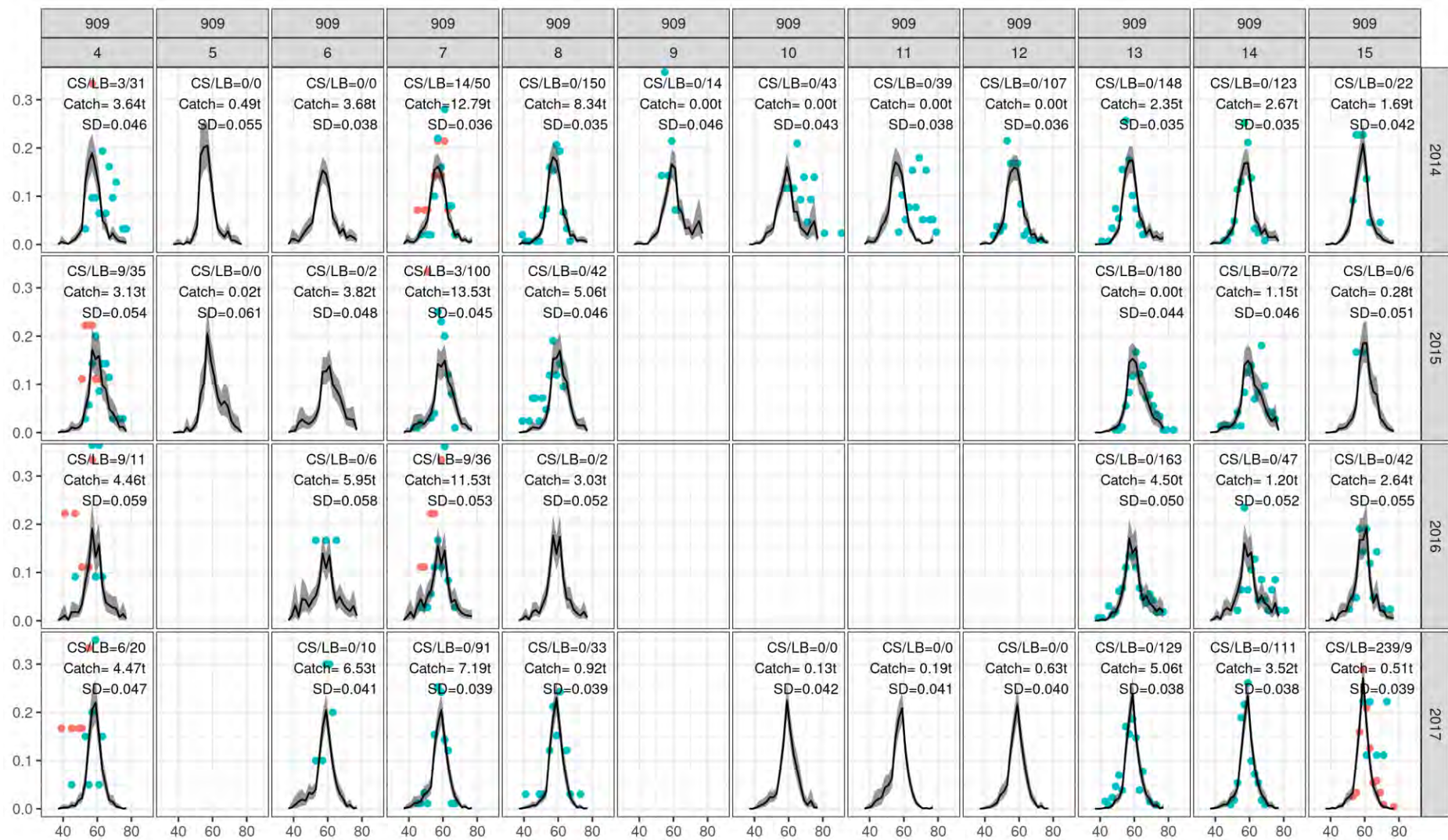


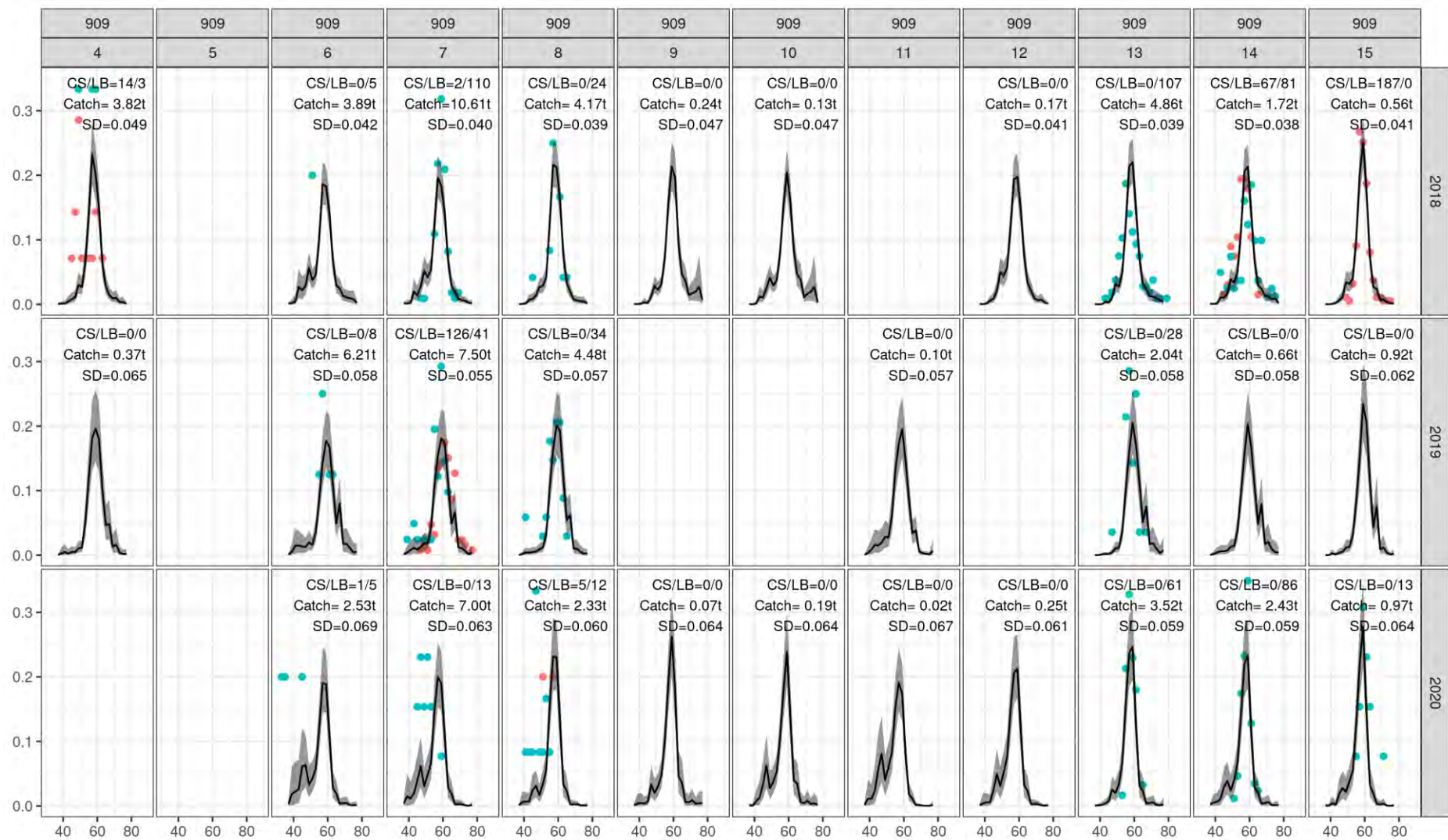


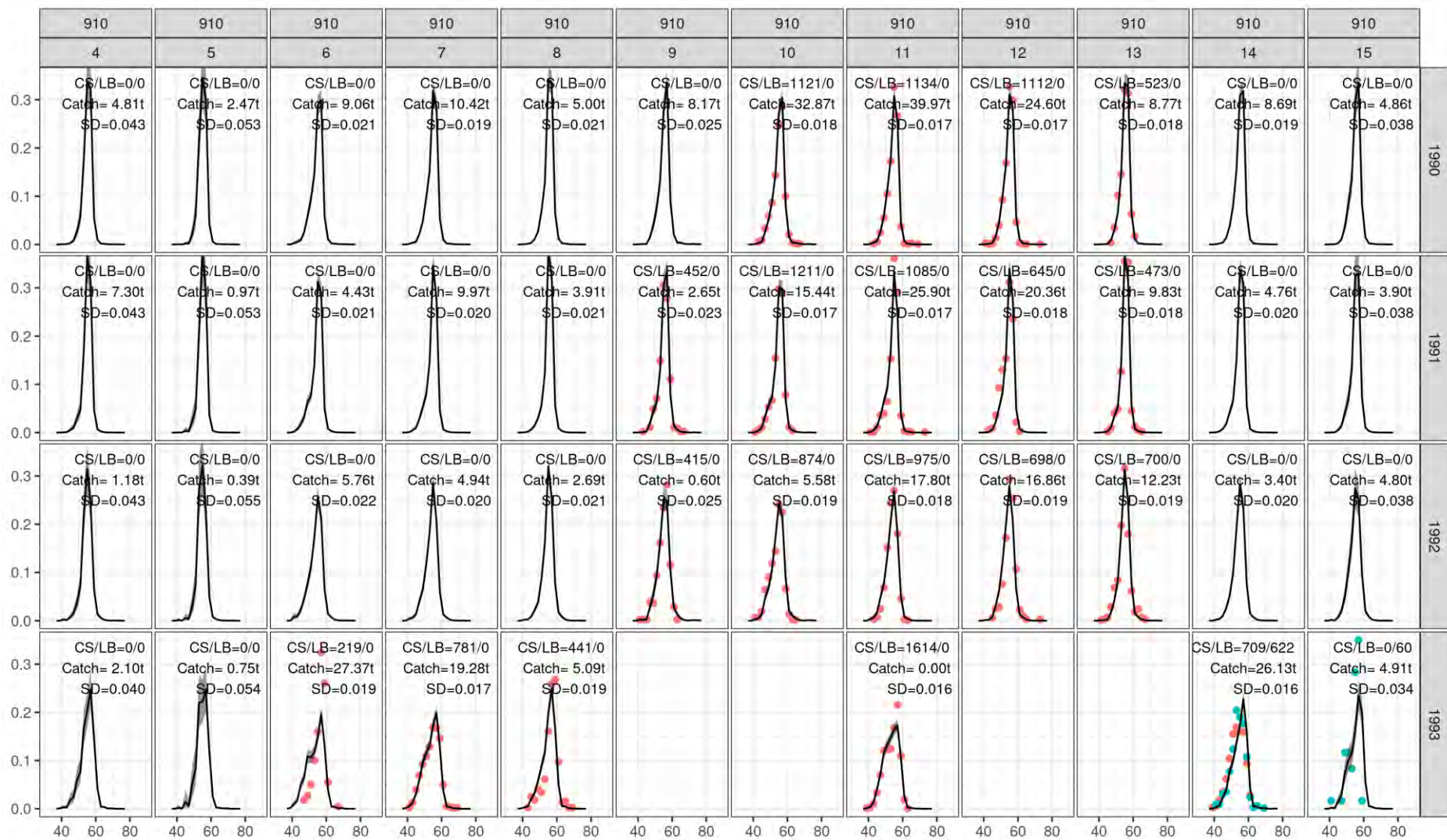


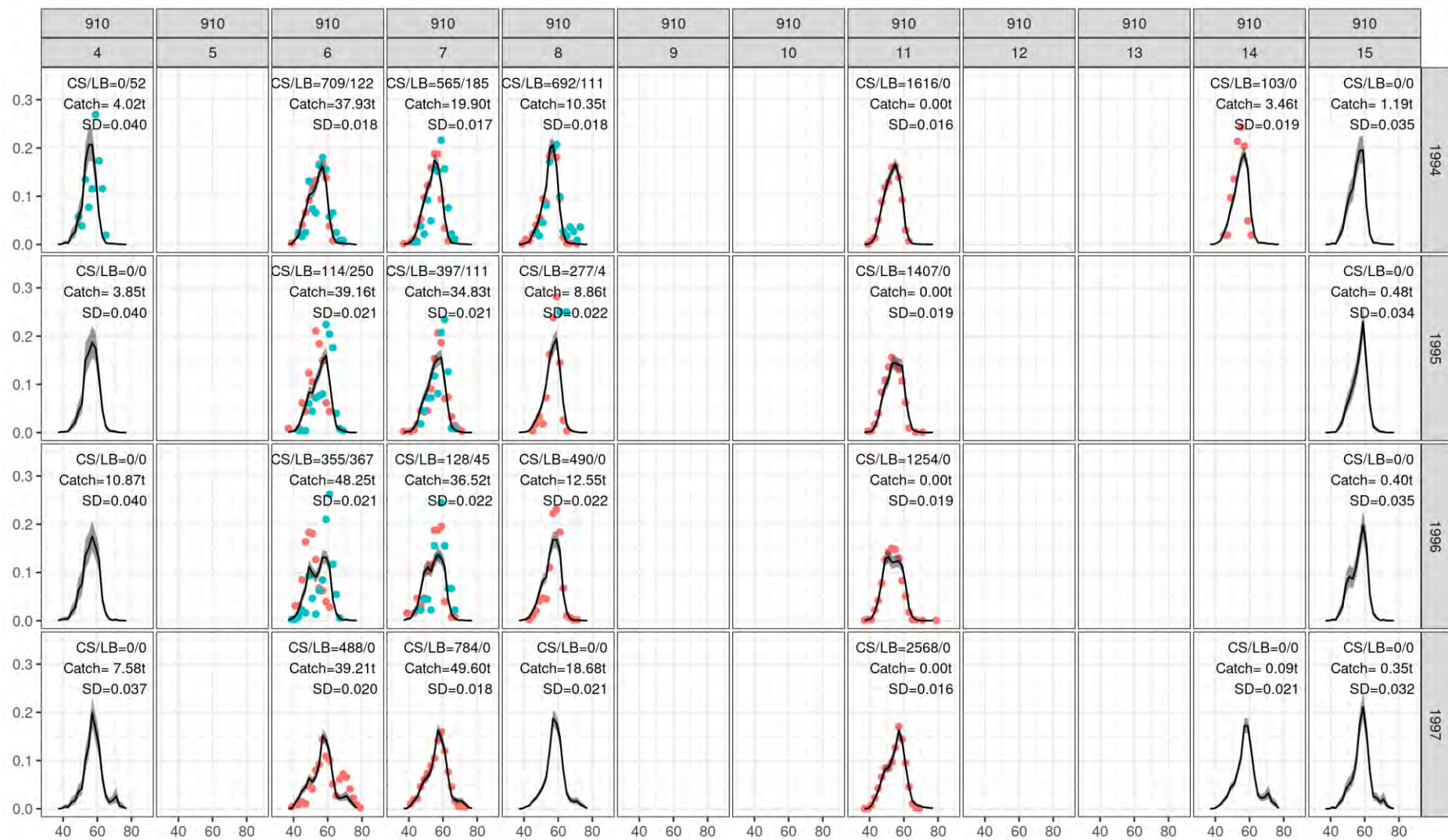


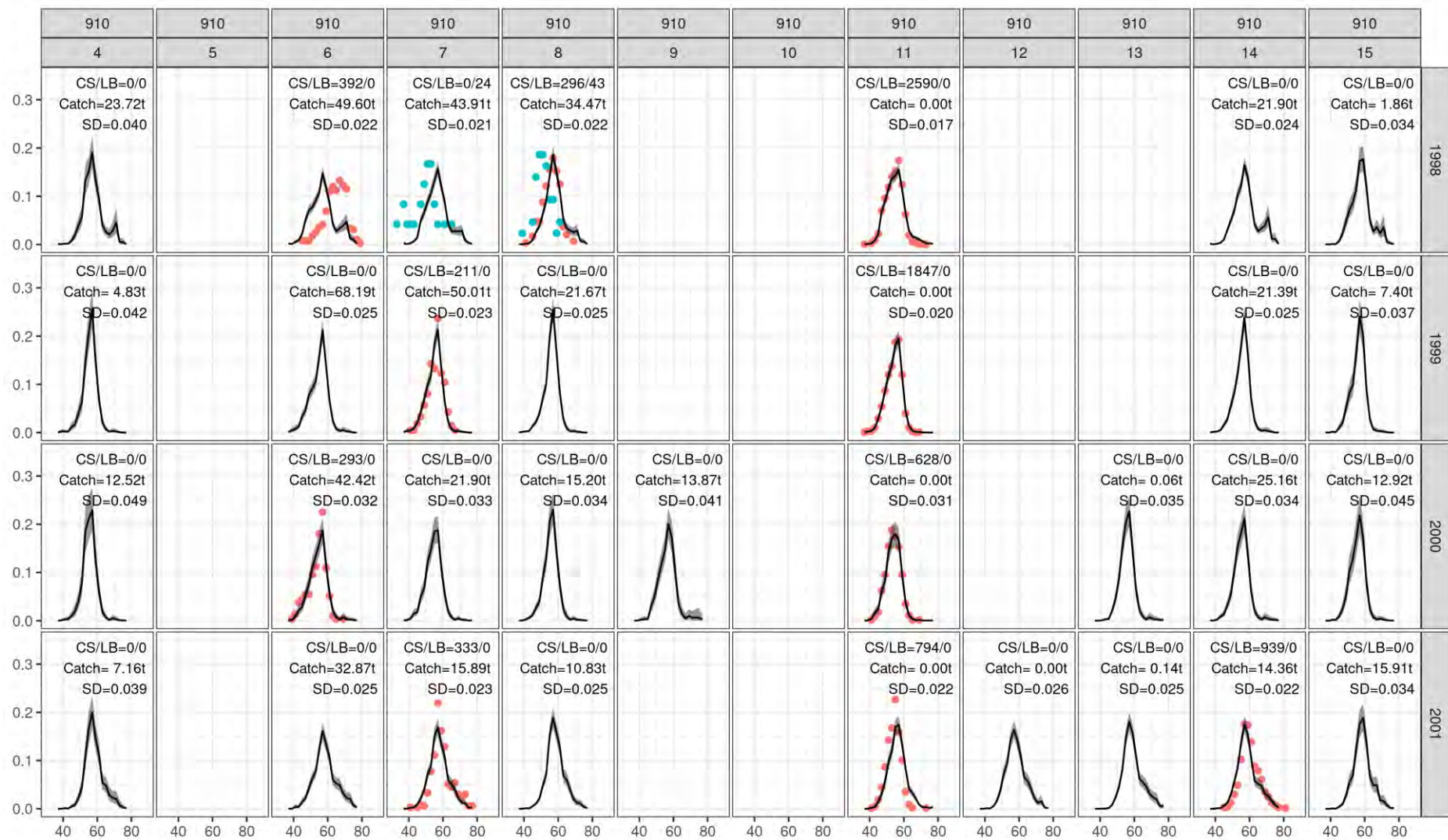


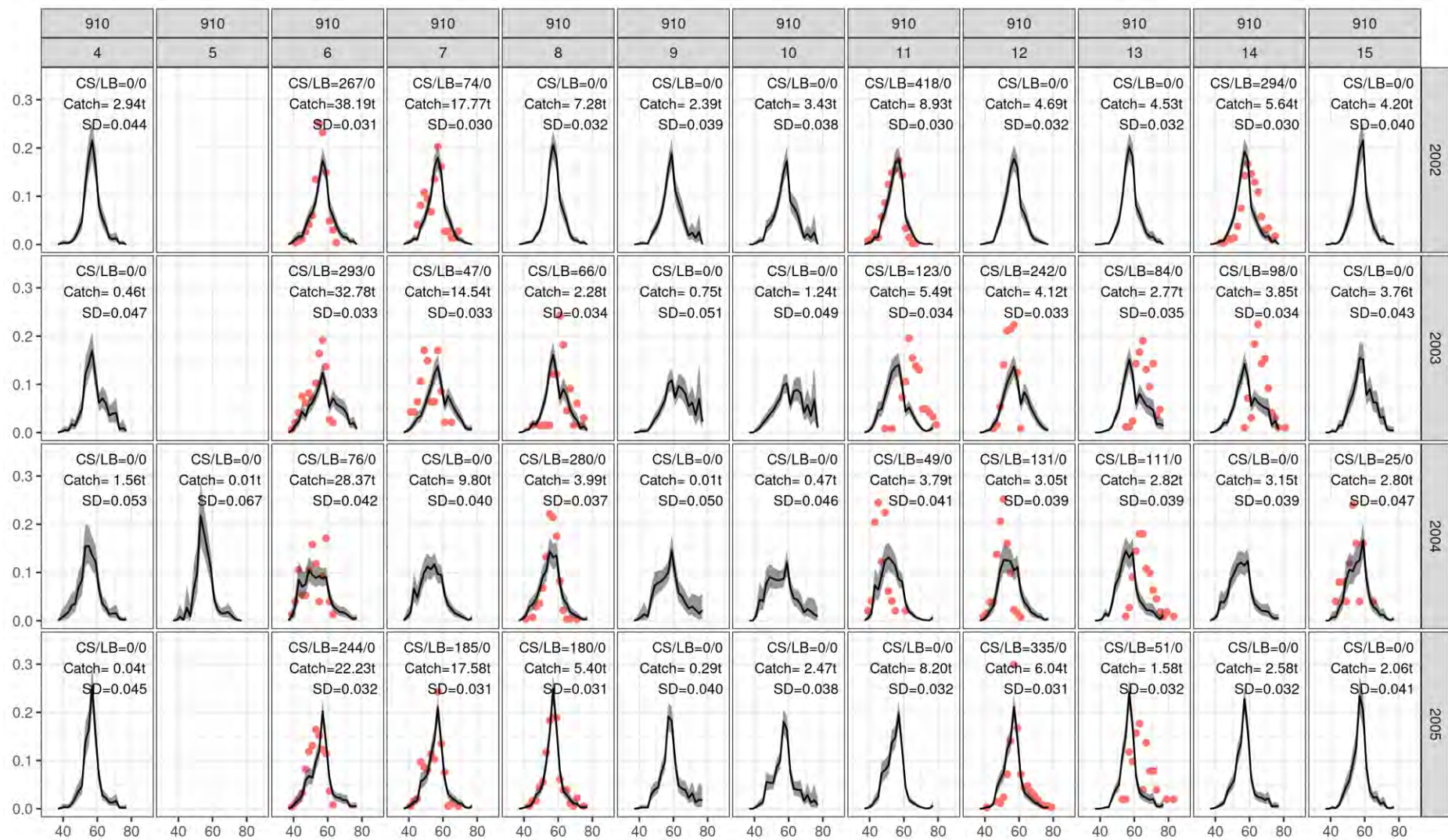


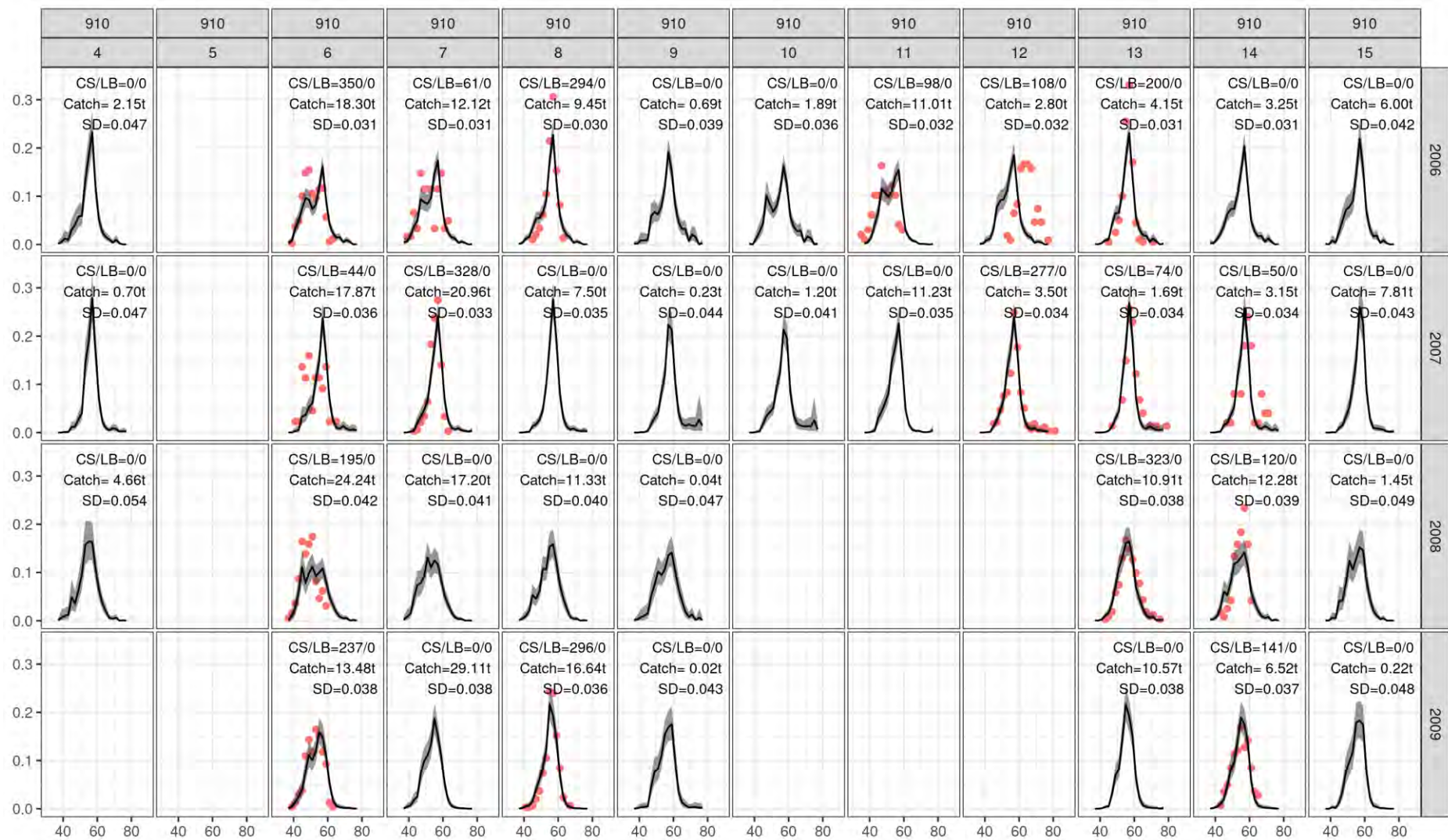


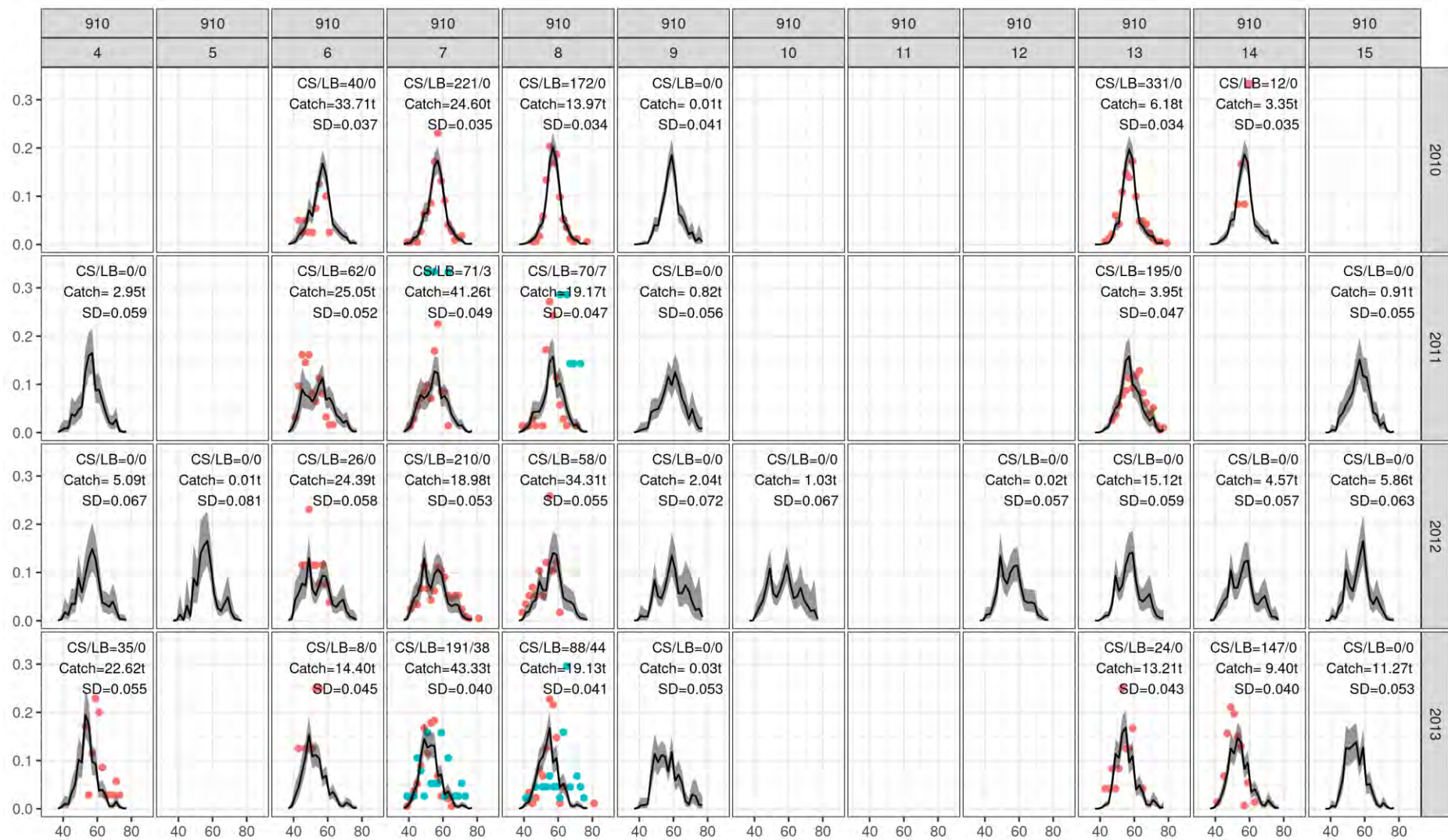


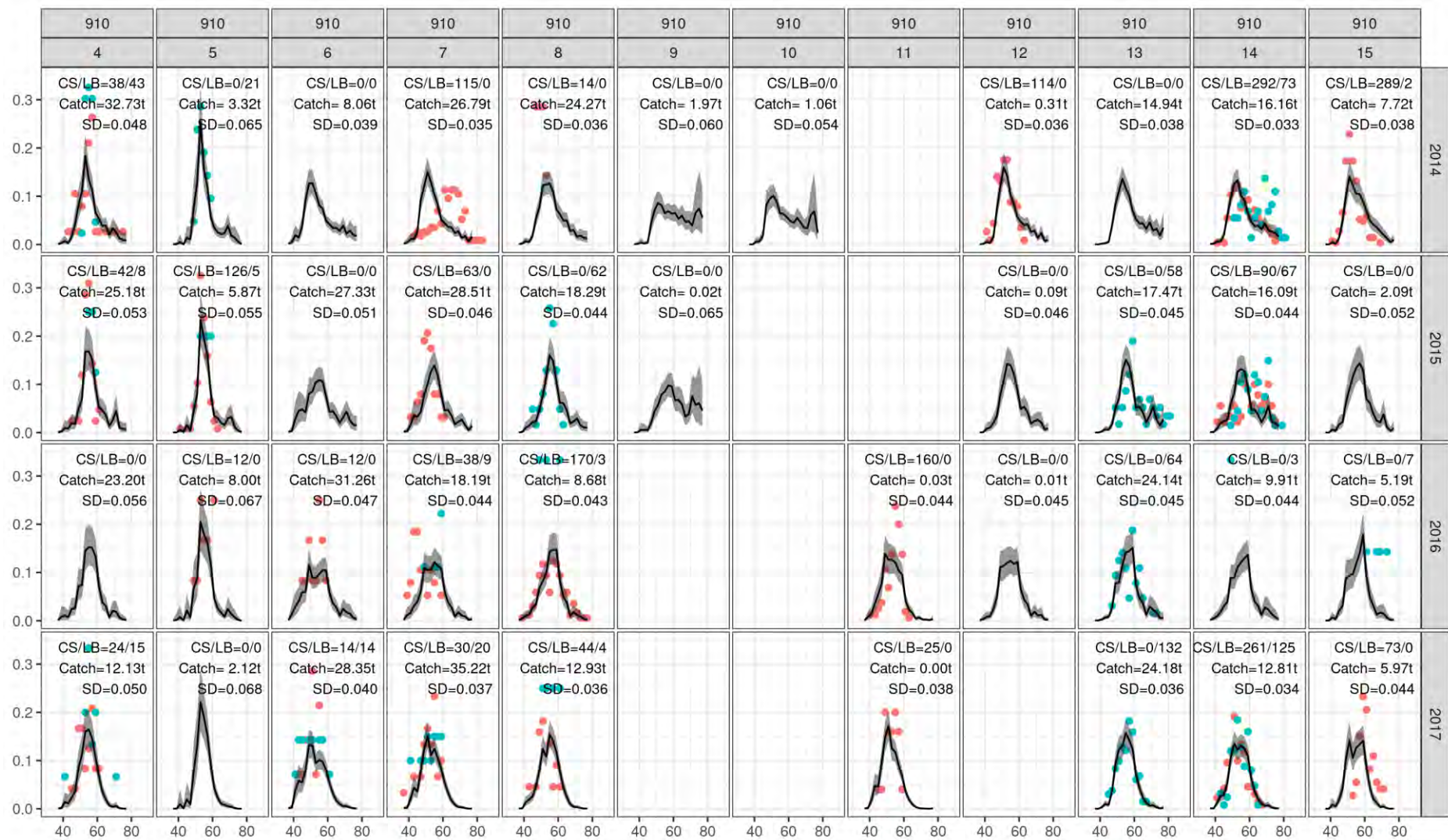


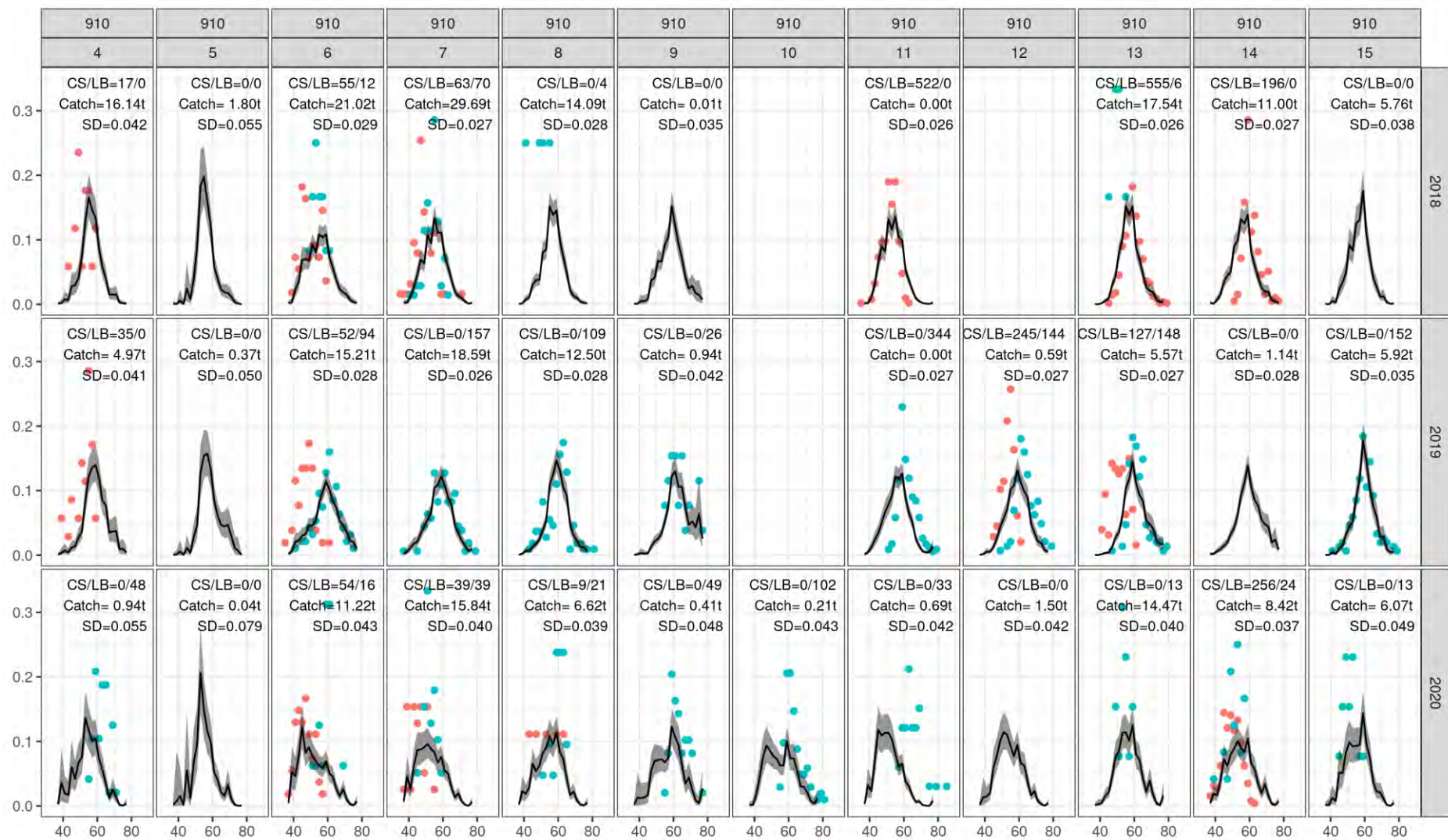


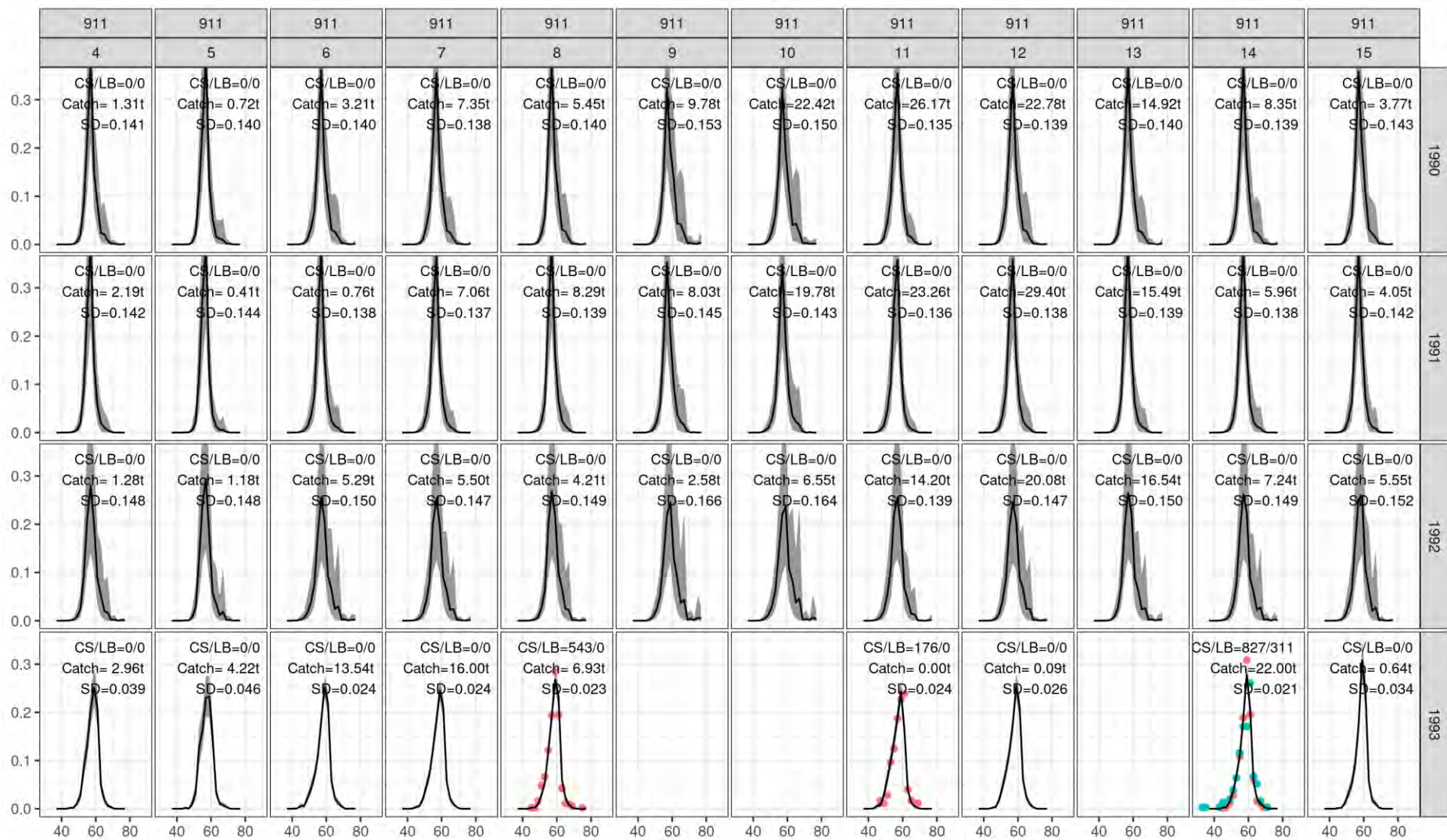


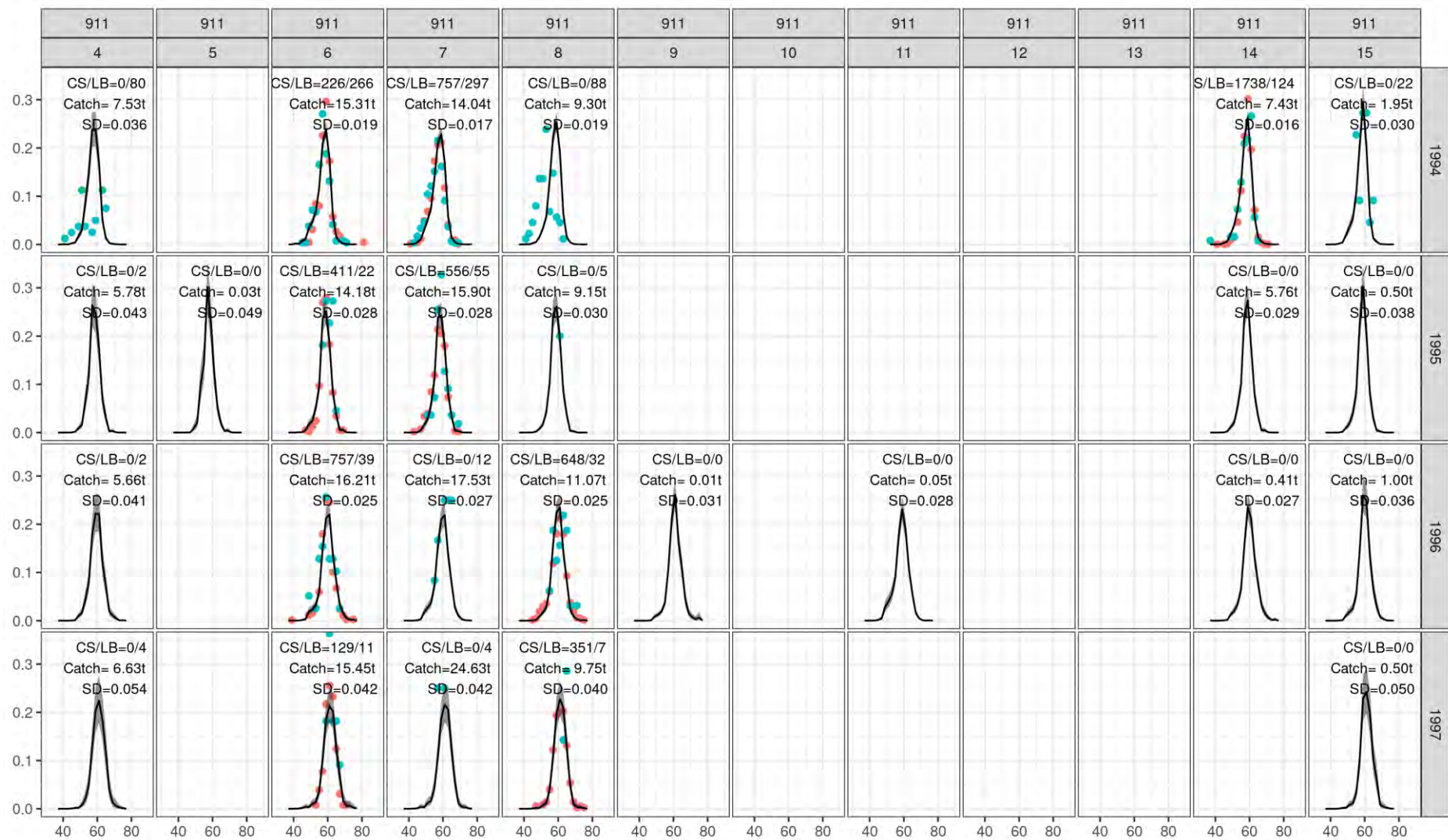


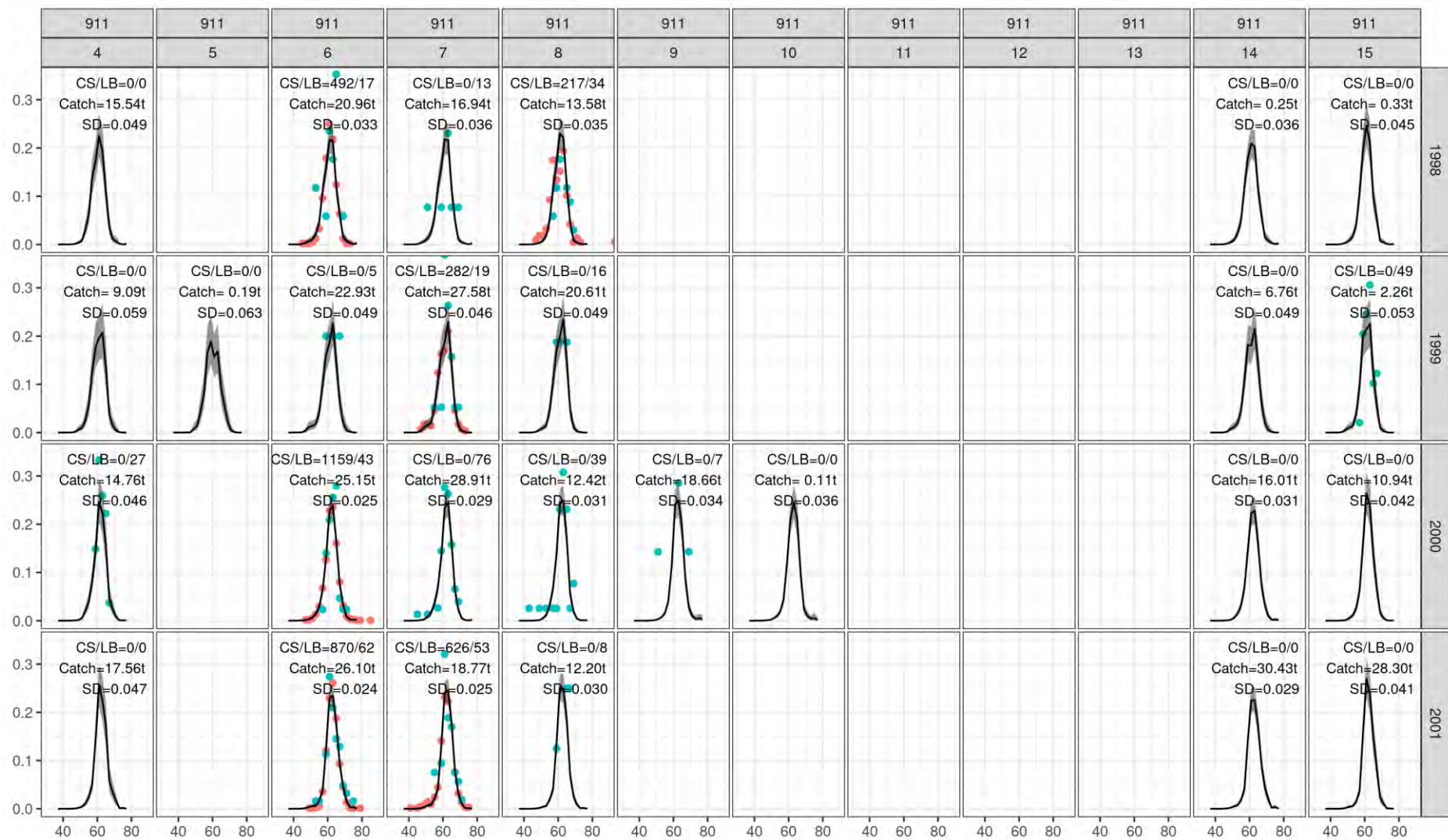


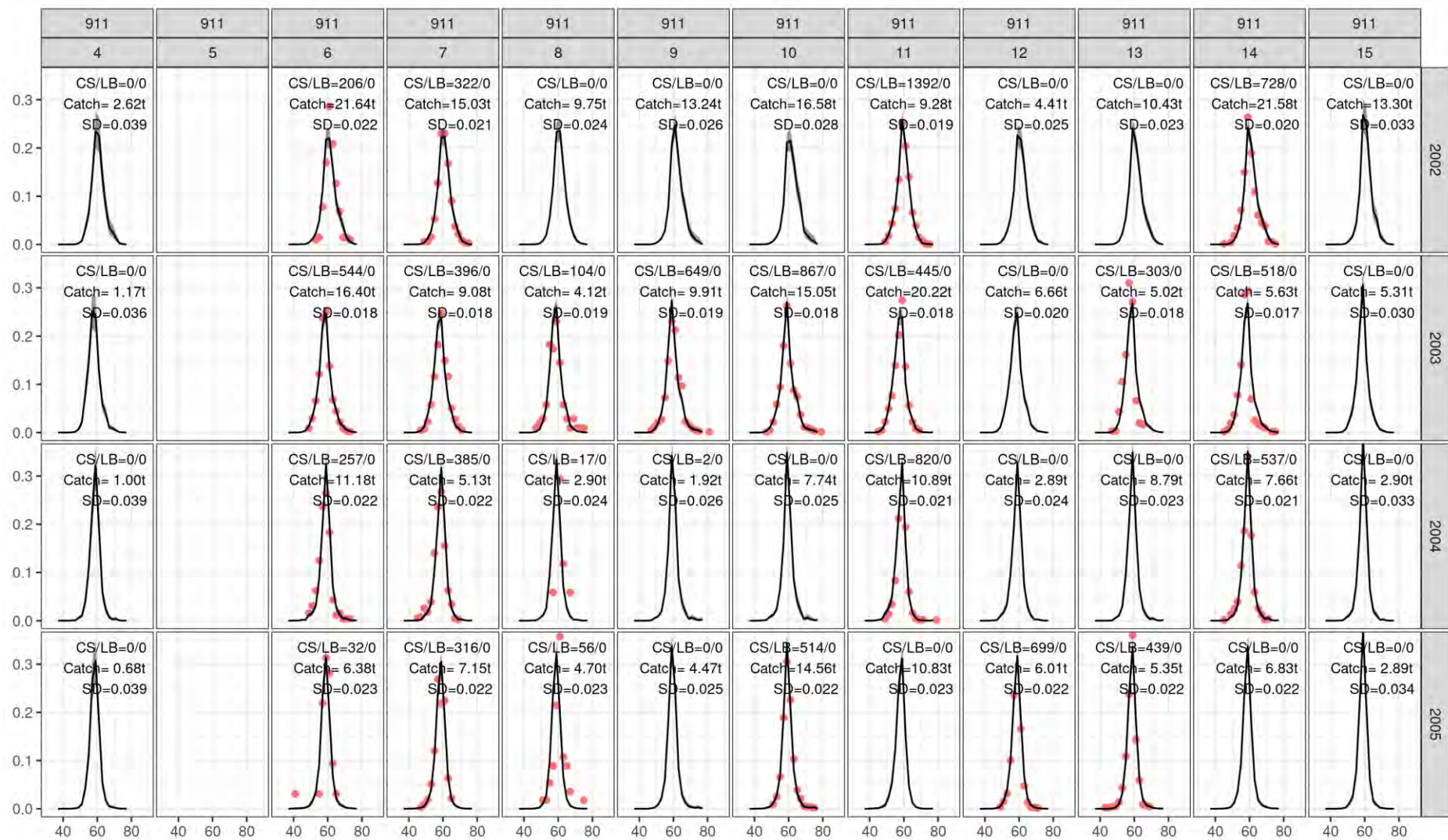


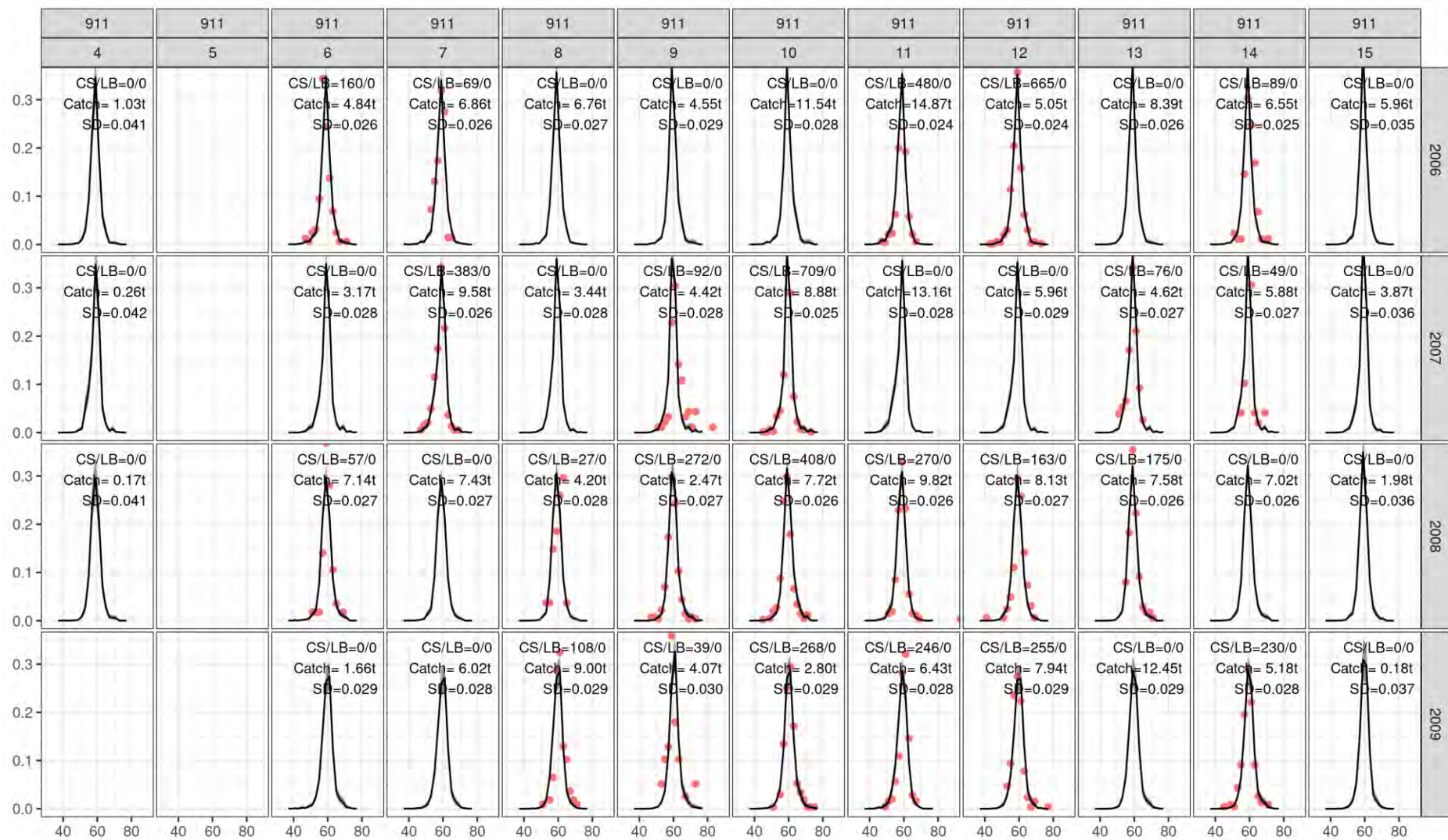


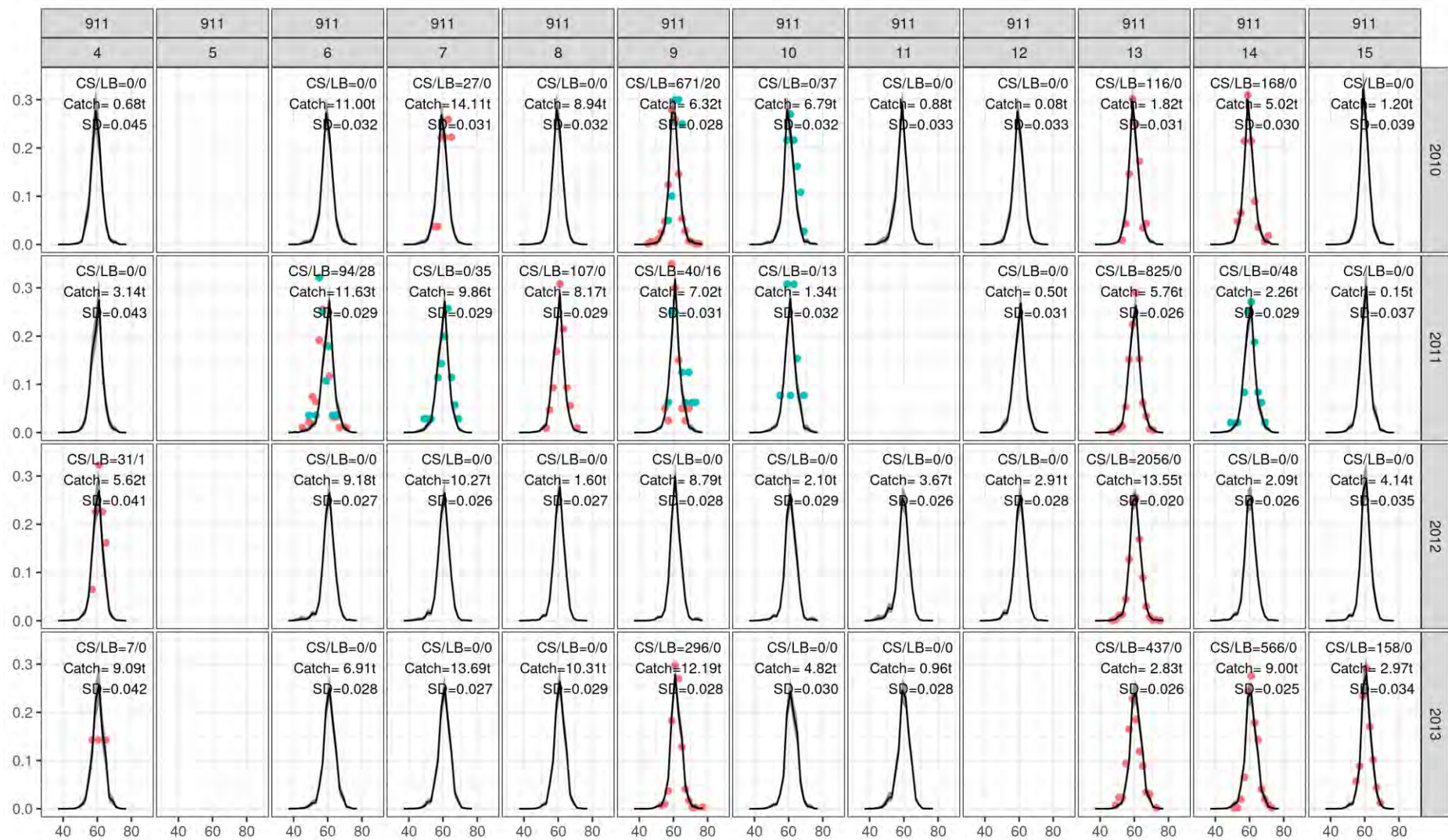


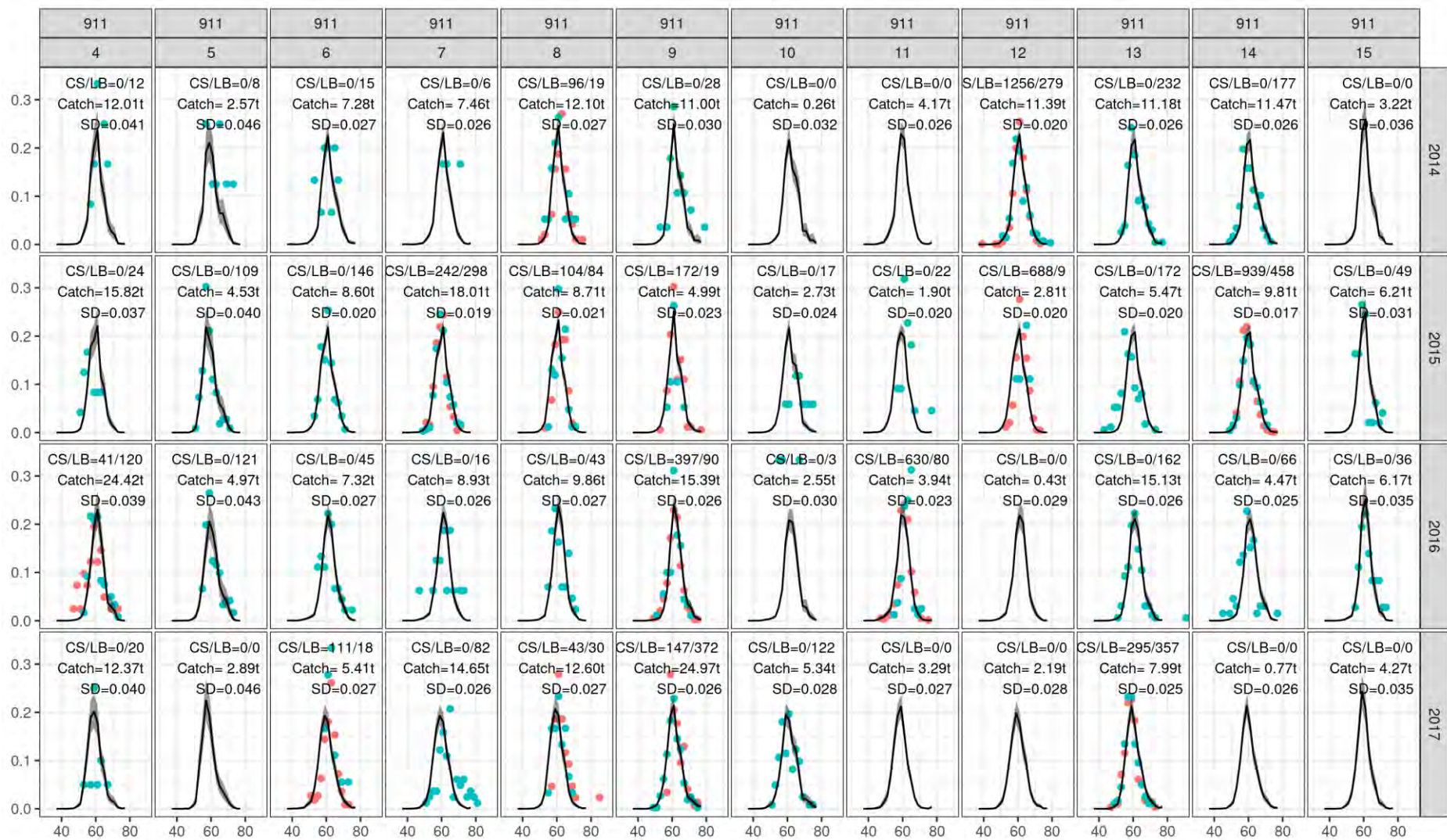


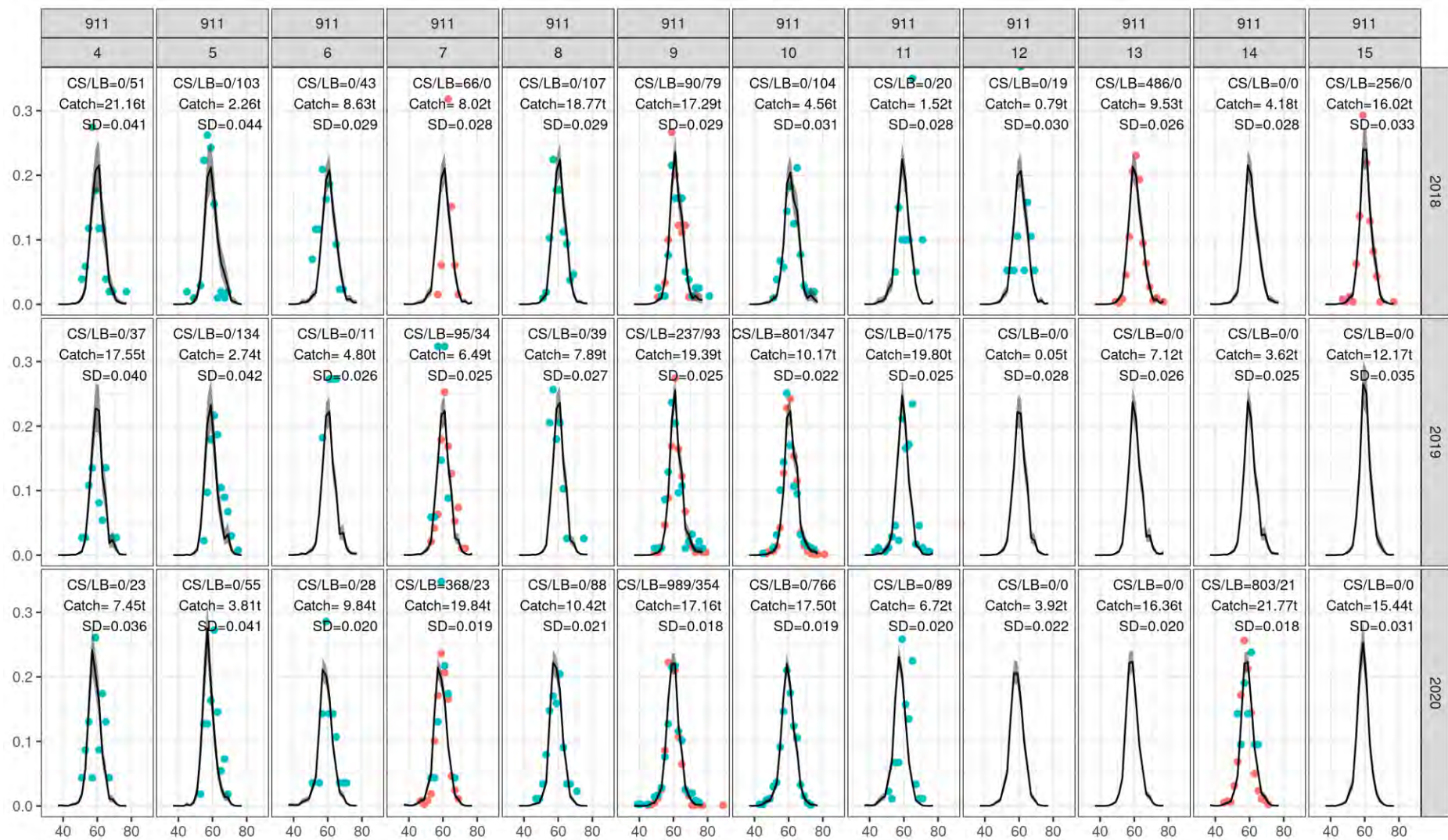










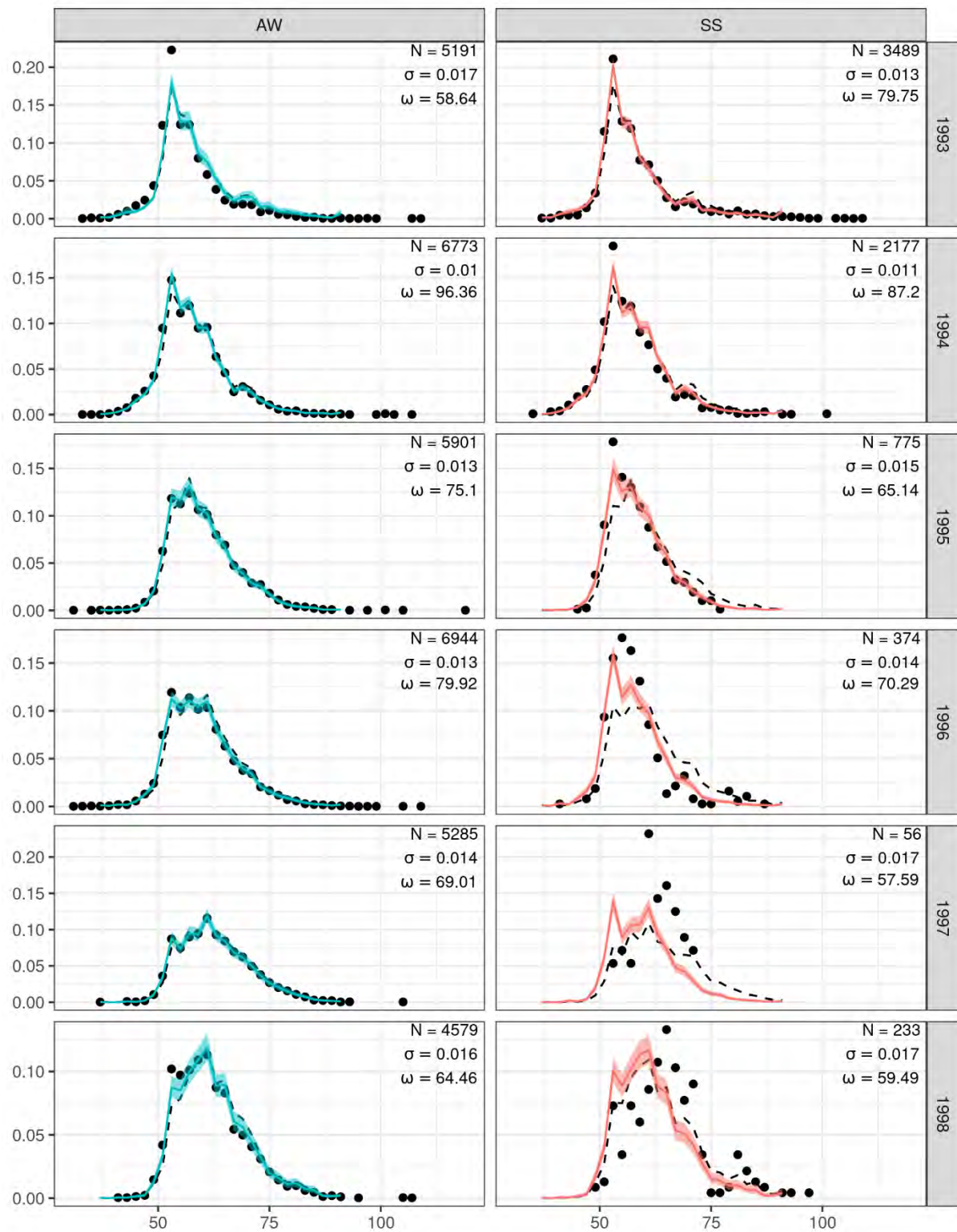


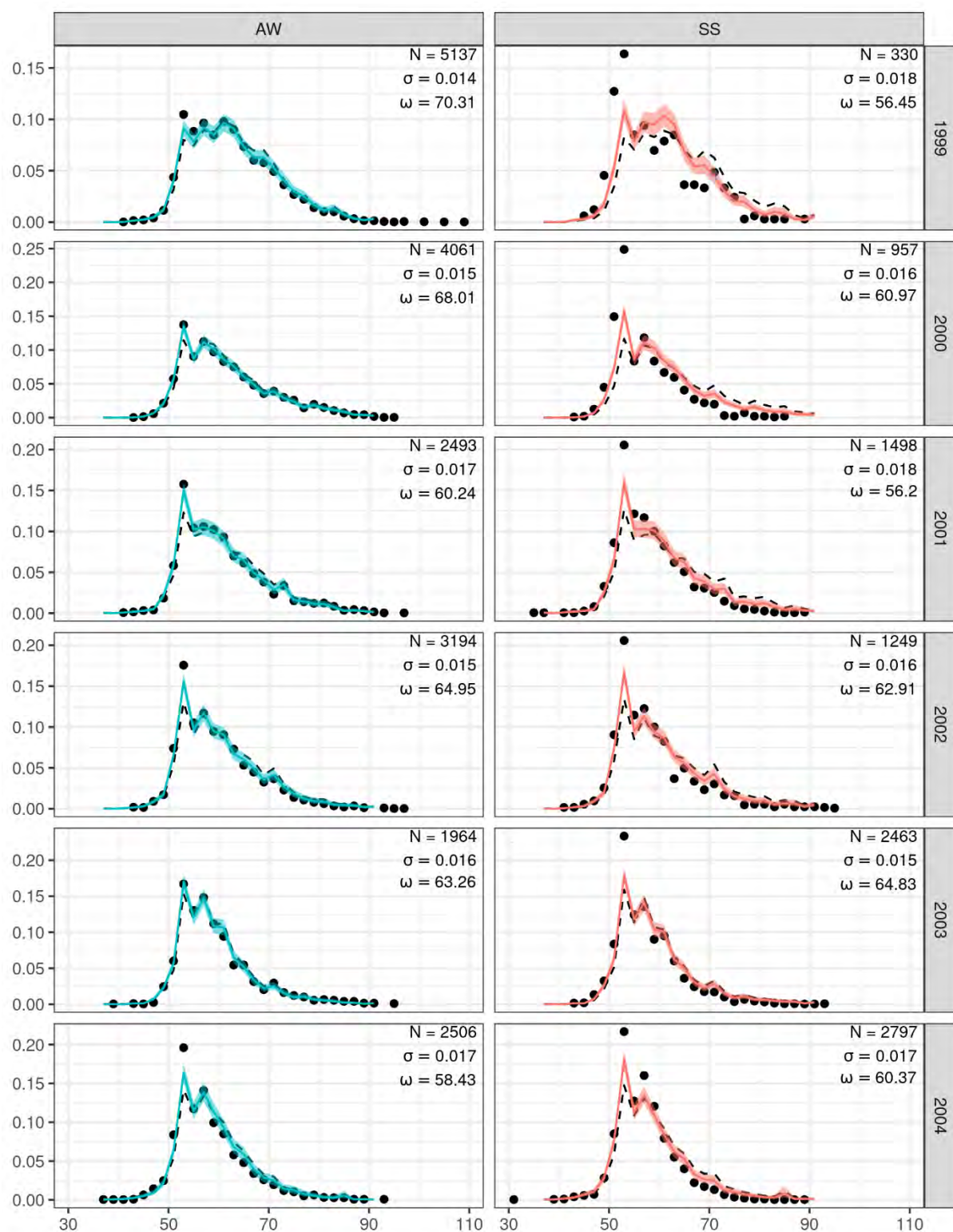
12. APPENDIX IV: SCALED LF DISTRIBUTIONS

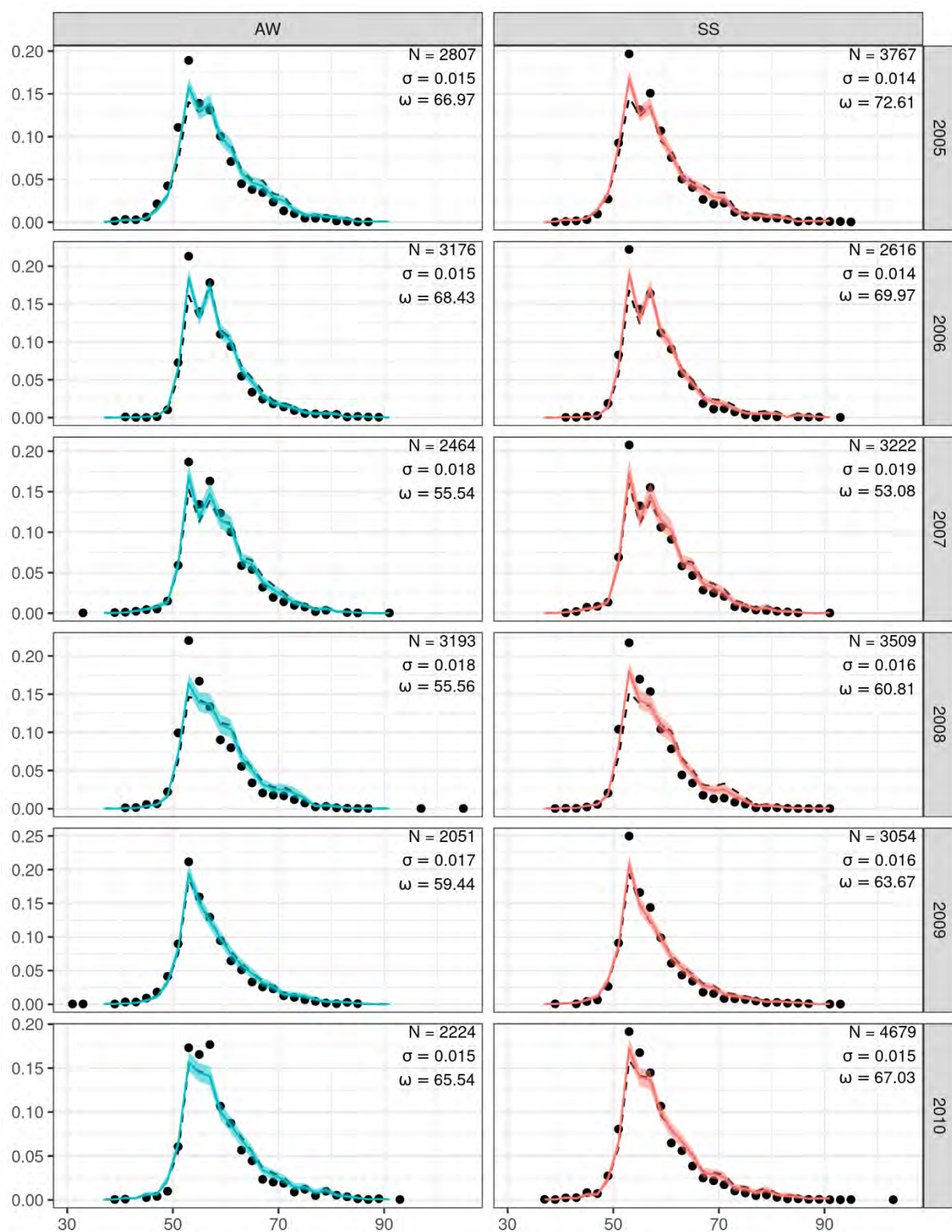
The figures in this appendix show the scaled LF distributions by fishing year, season, and length bin for each QMA, region, and sex category where the solid line is the mean and the shaded region is the 95% credible interval (Equation 13). The aggregated data (points, Equation 10) and unscaled predictions of the LF (dashed line) for the same strata are also shown. Each panel also includes text which provides:

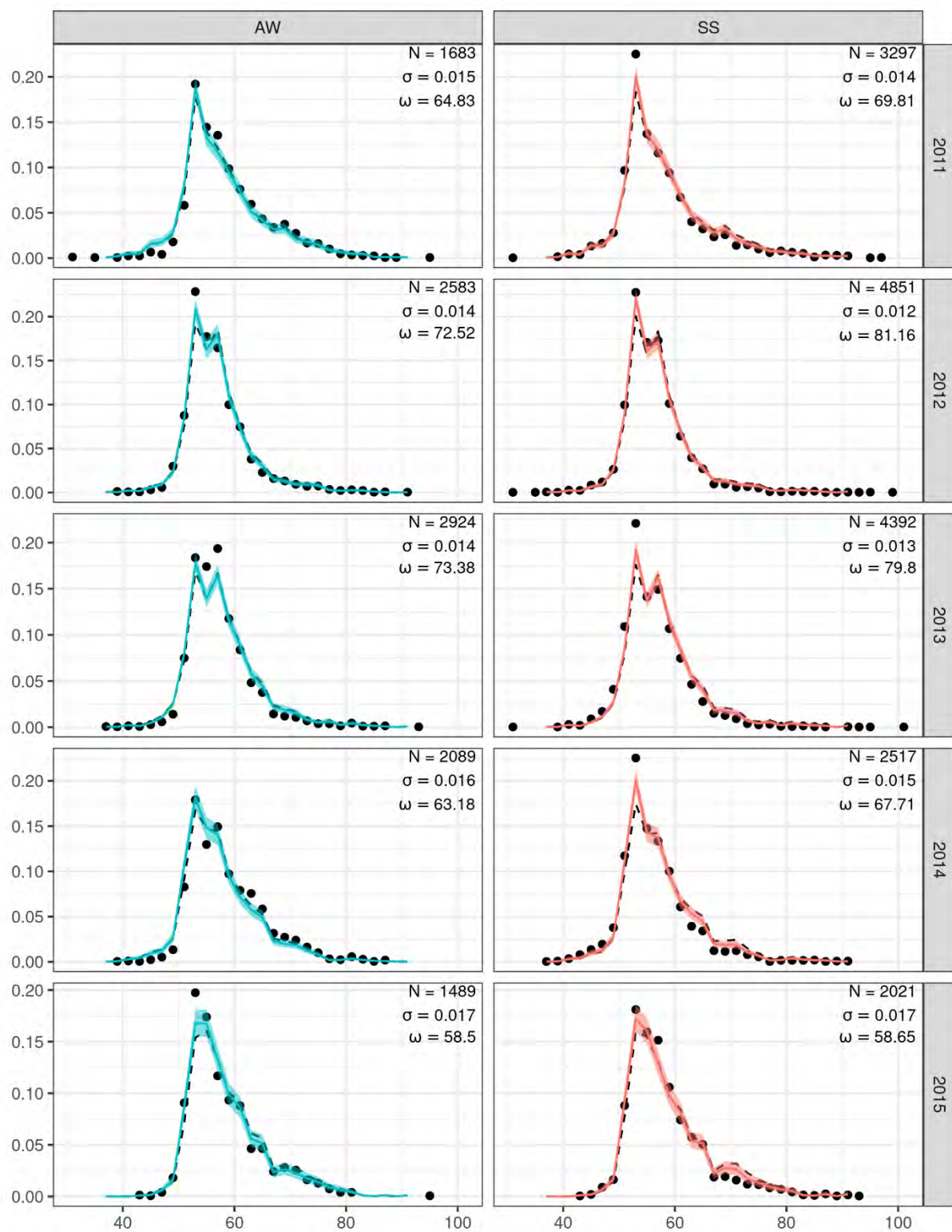
- the number of lobsters measured;
- the SD of the predicted LF for each stratum (Equation 13); and
- the data set weight for each stratum (Equation 14).

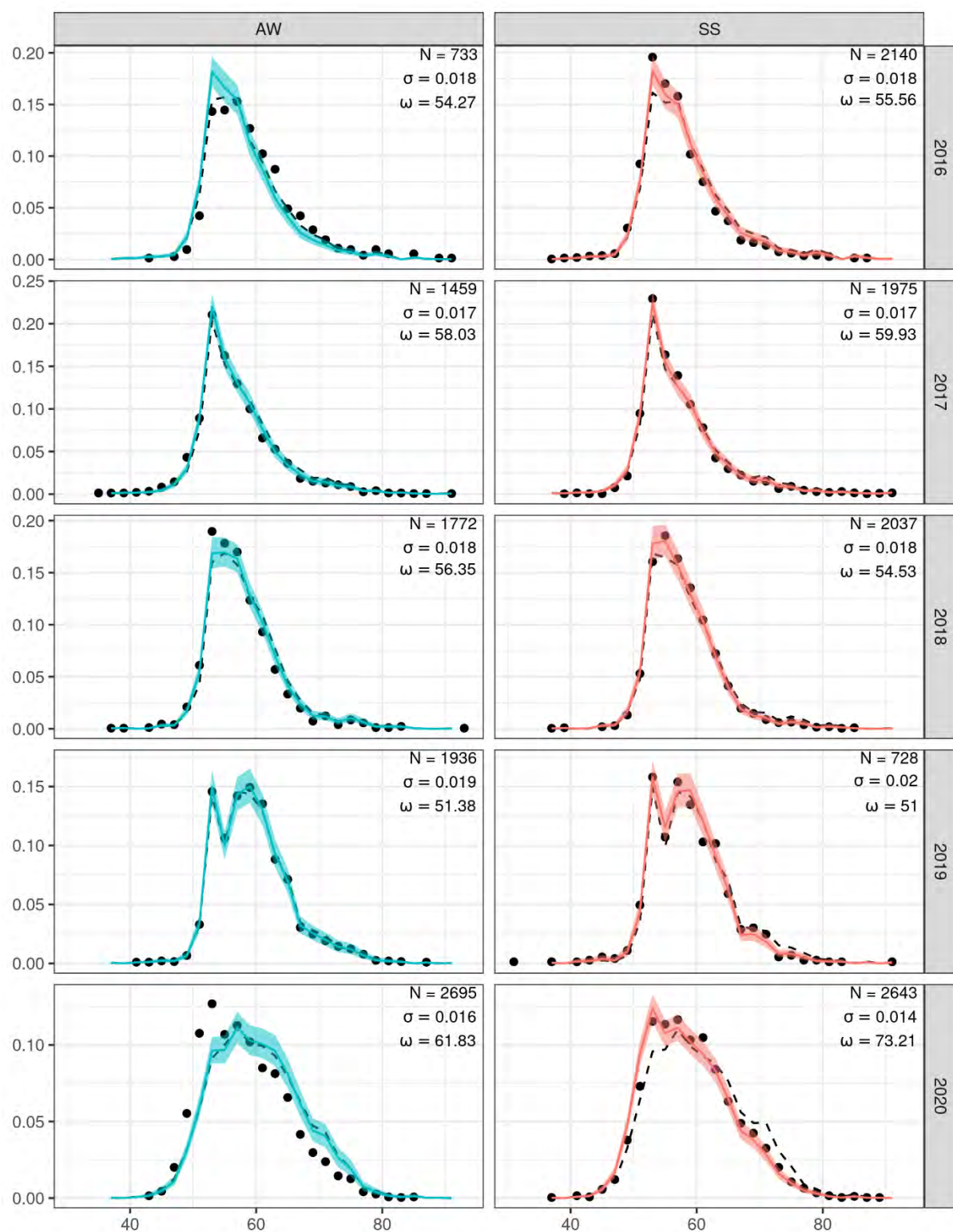
12.1 CRA 2 males



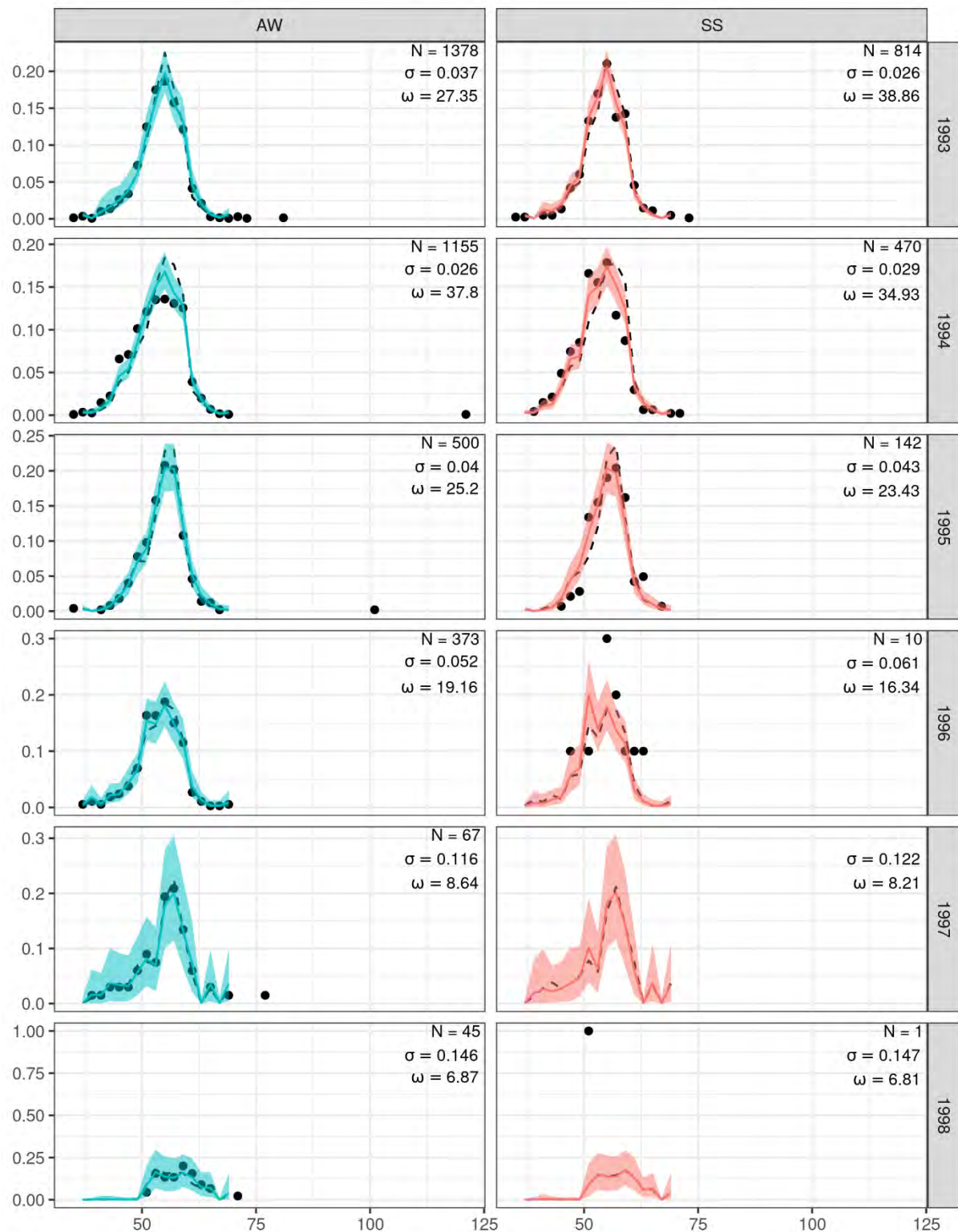


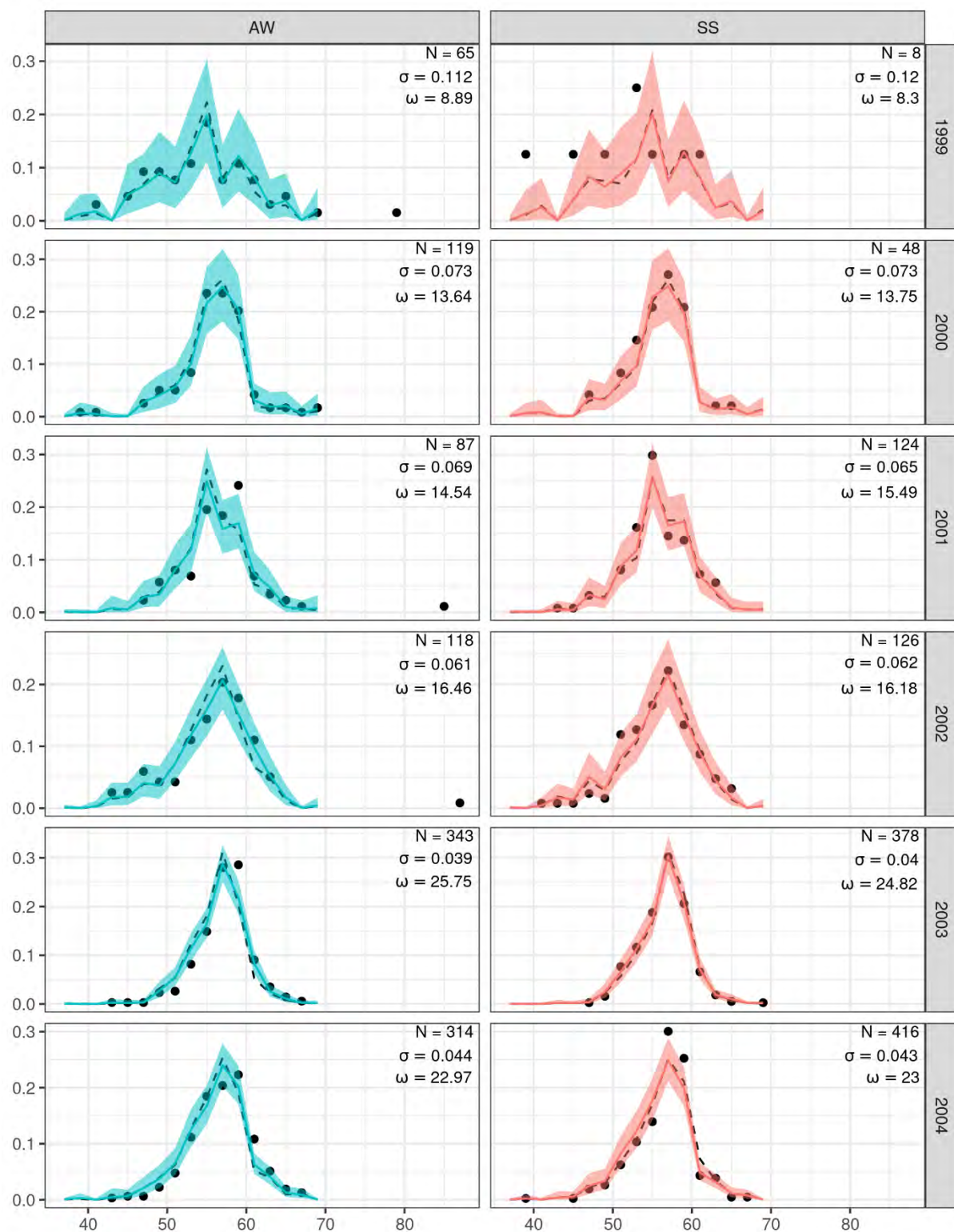


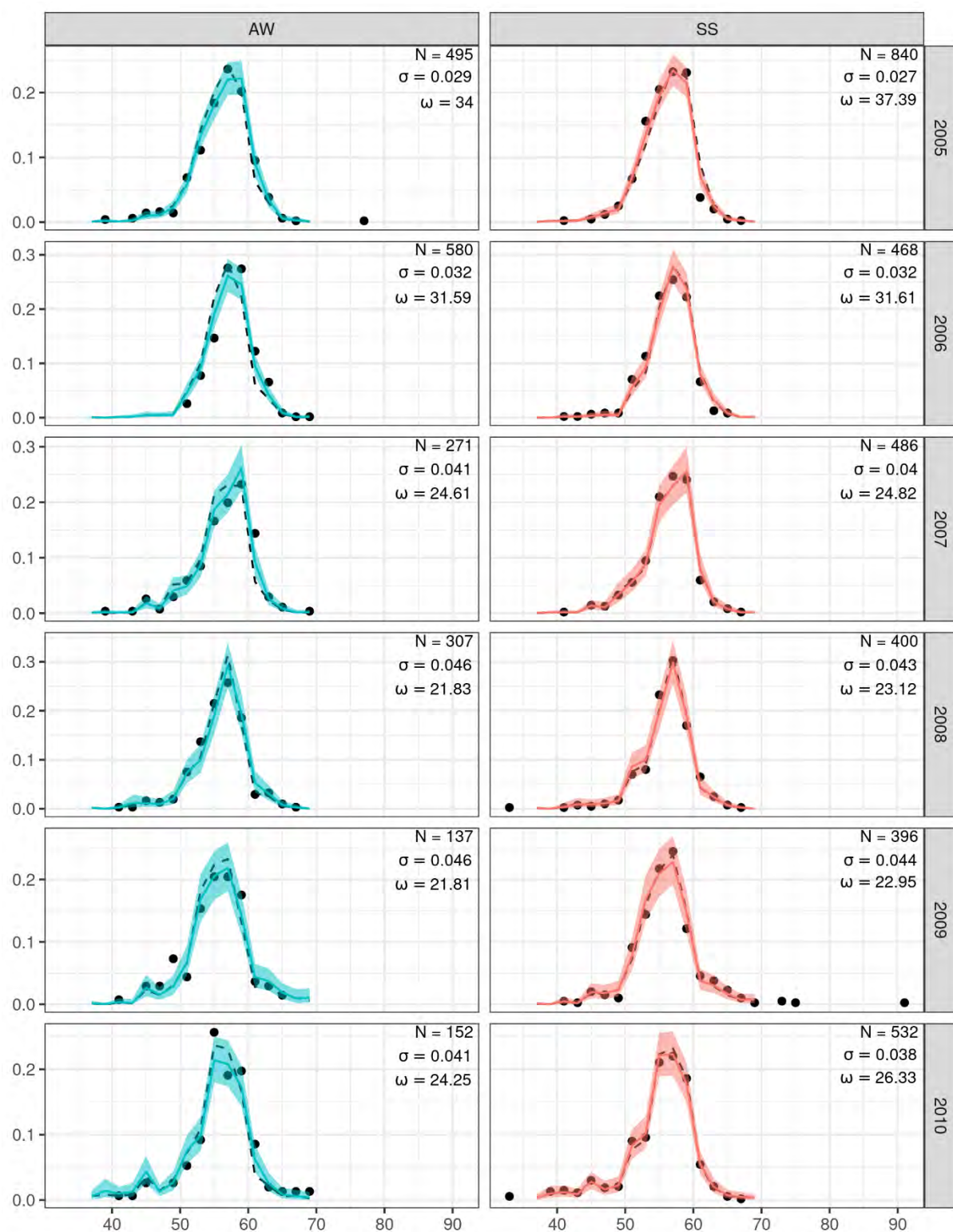


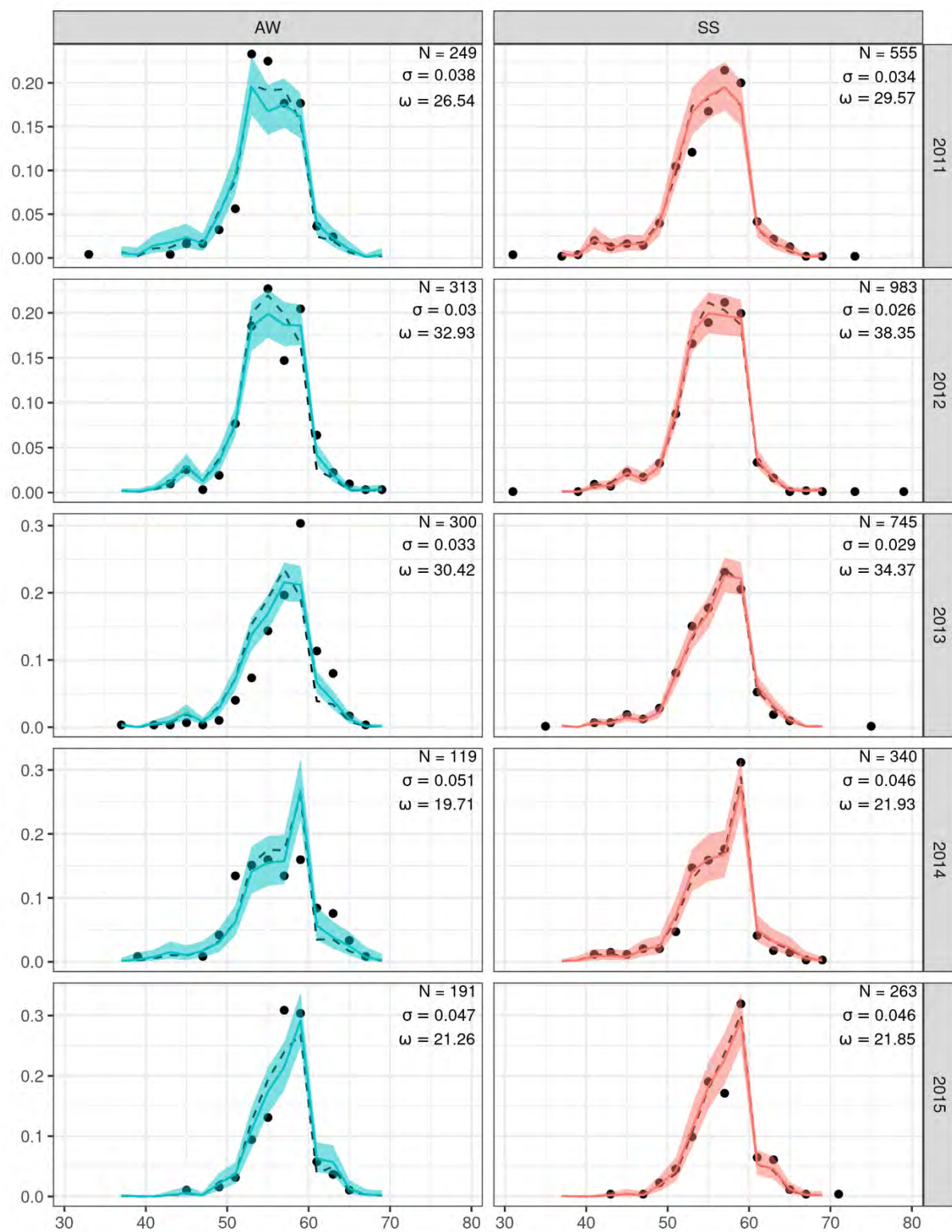


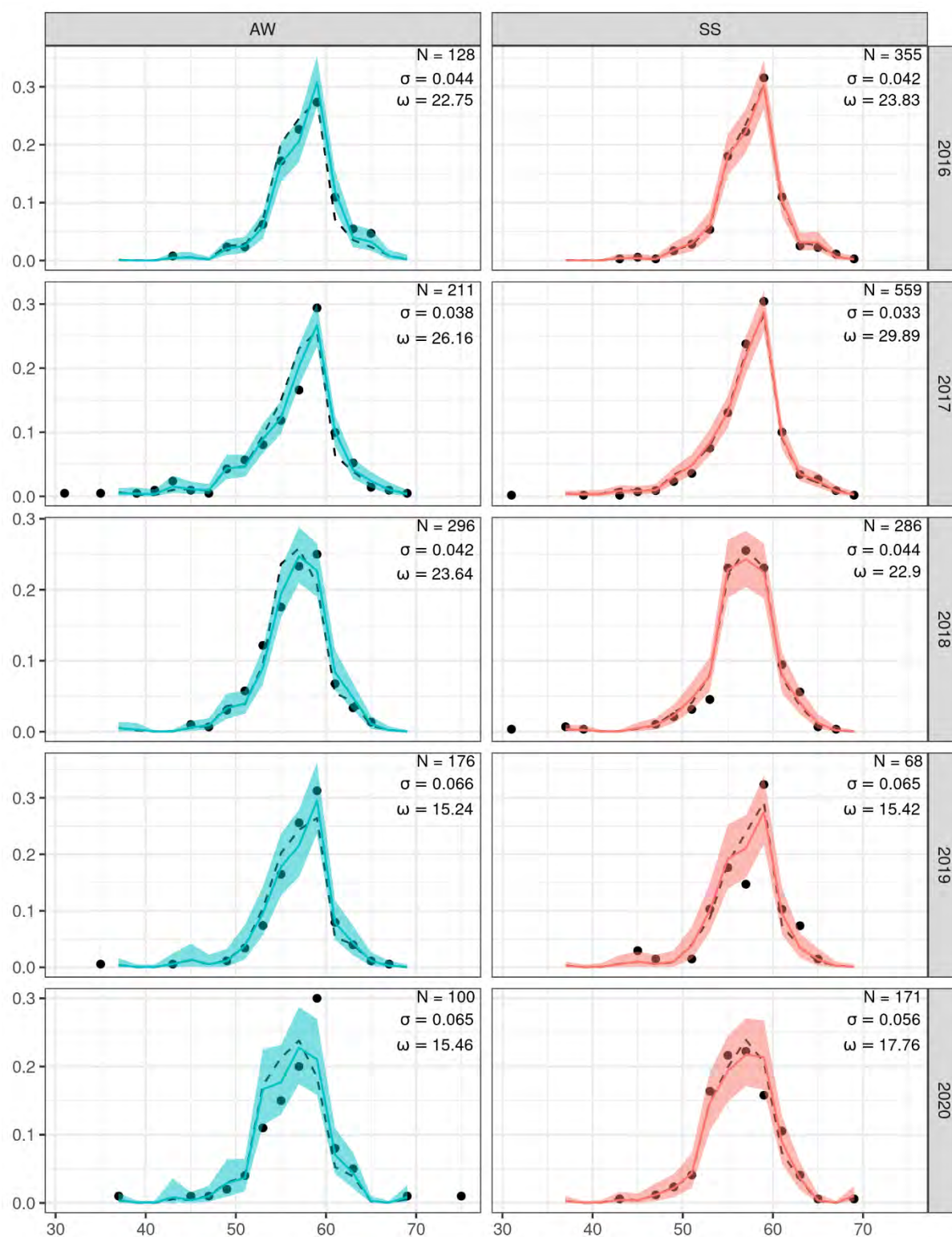
12.2 CRA 2 immature females



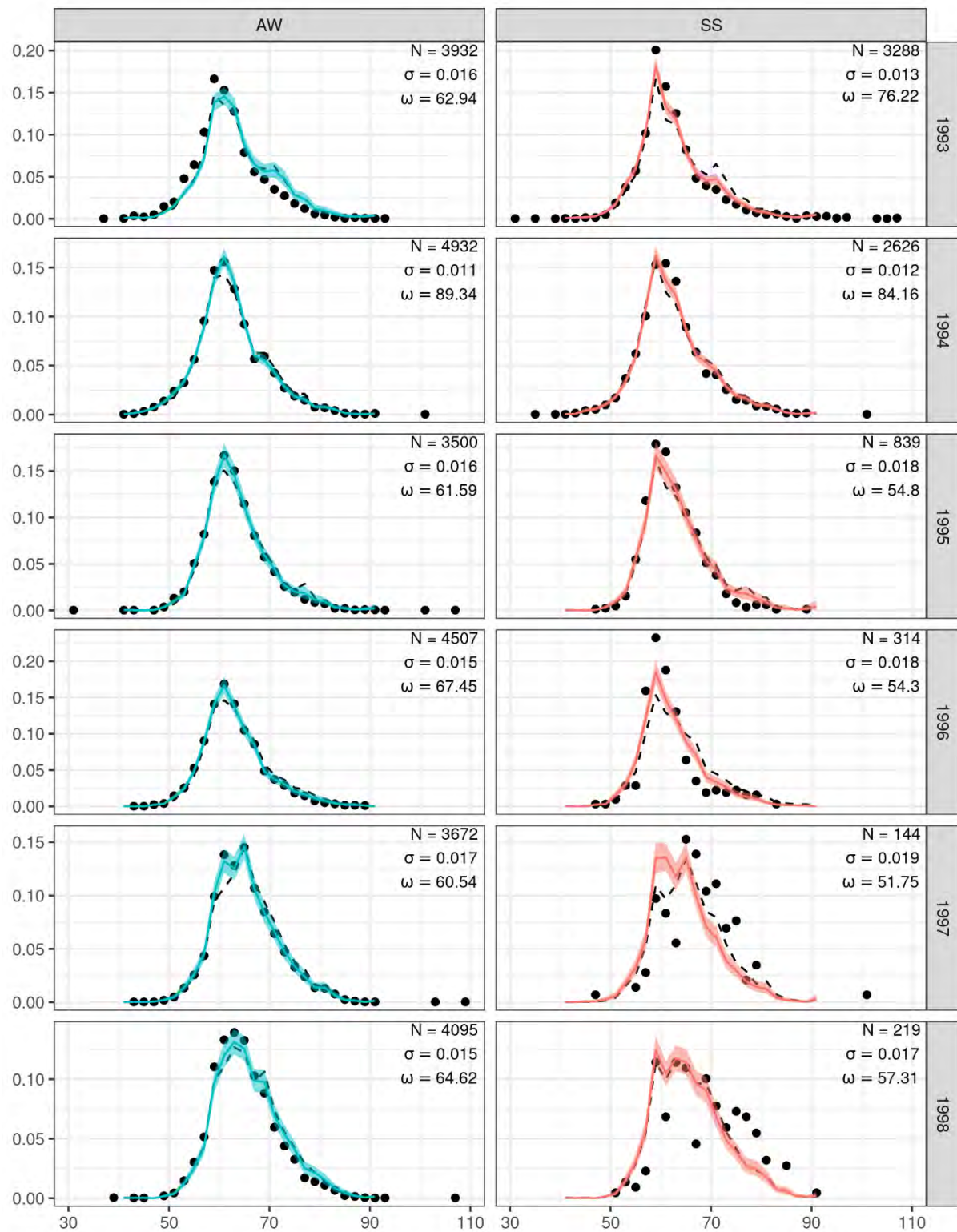


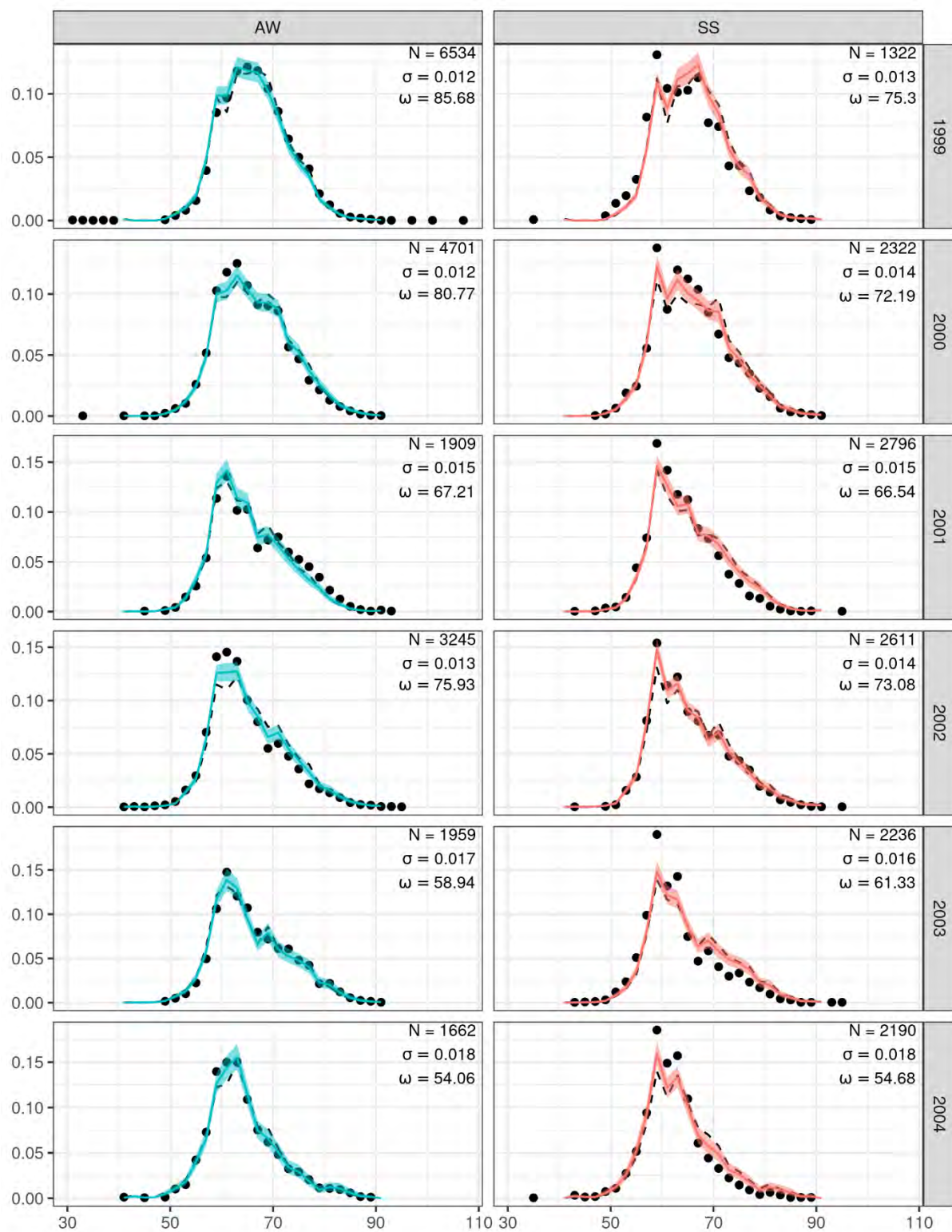


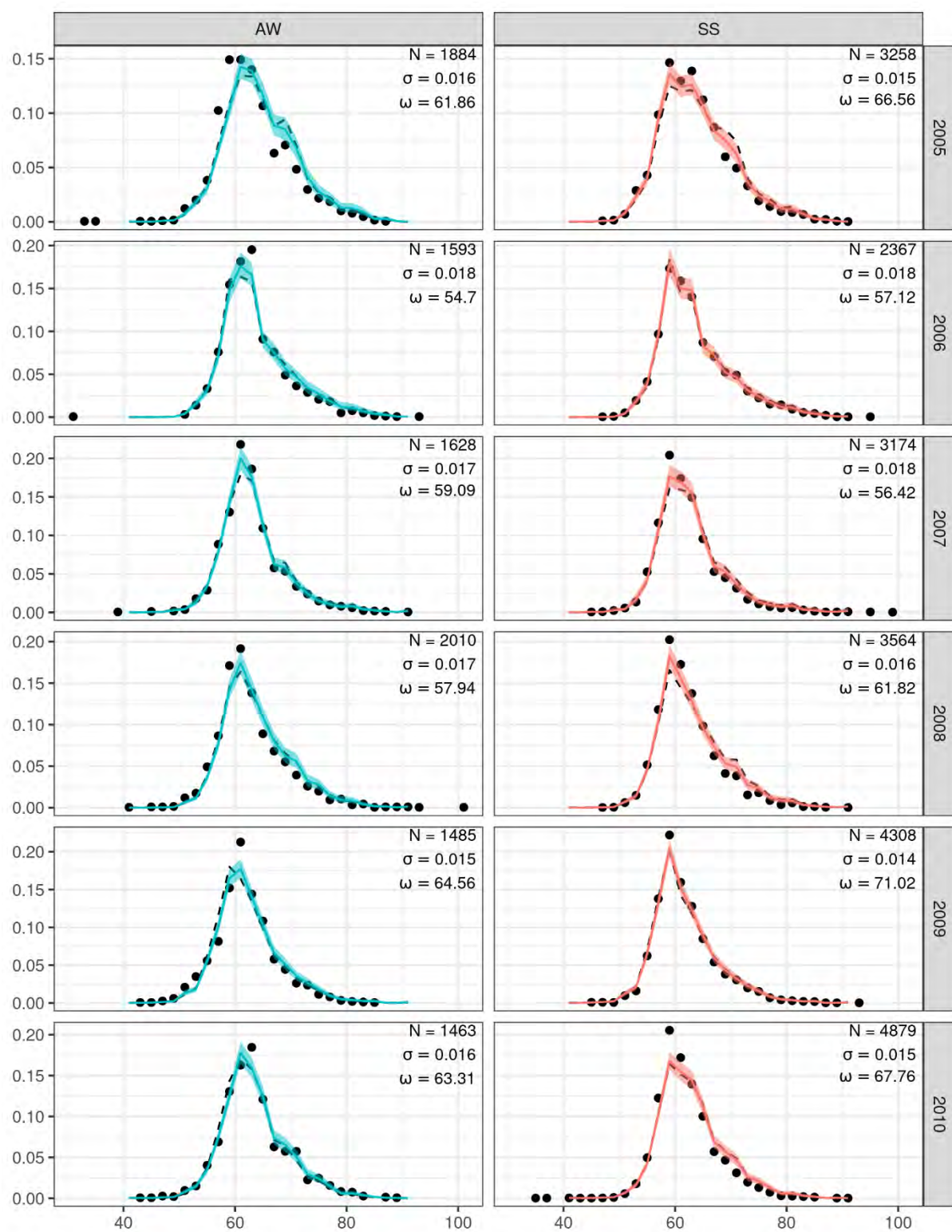


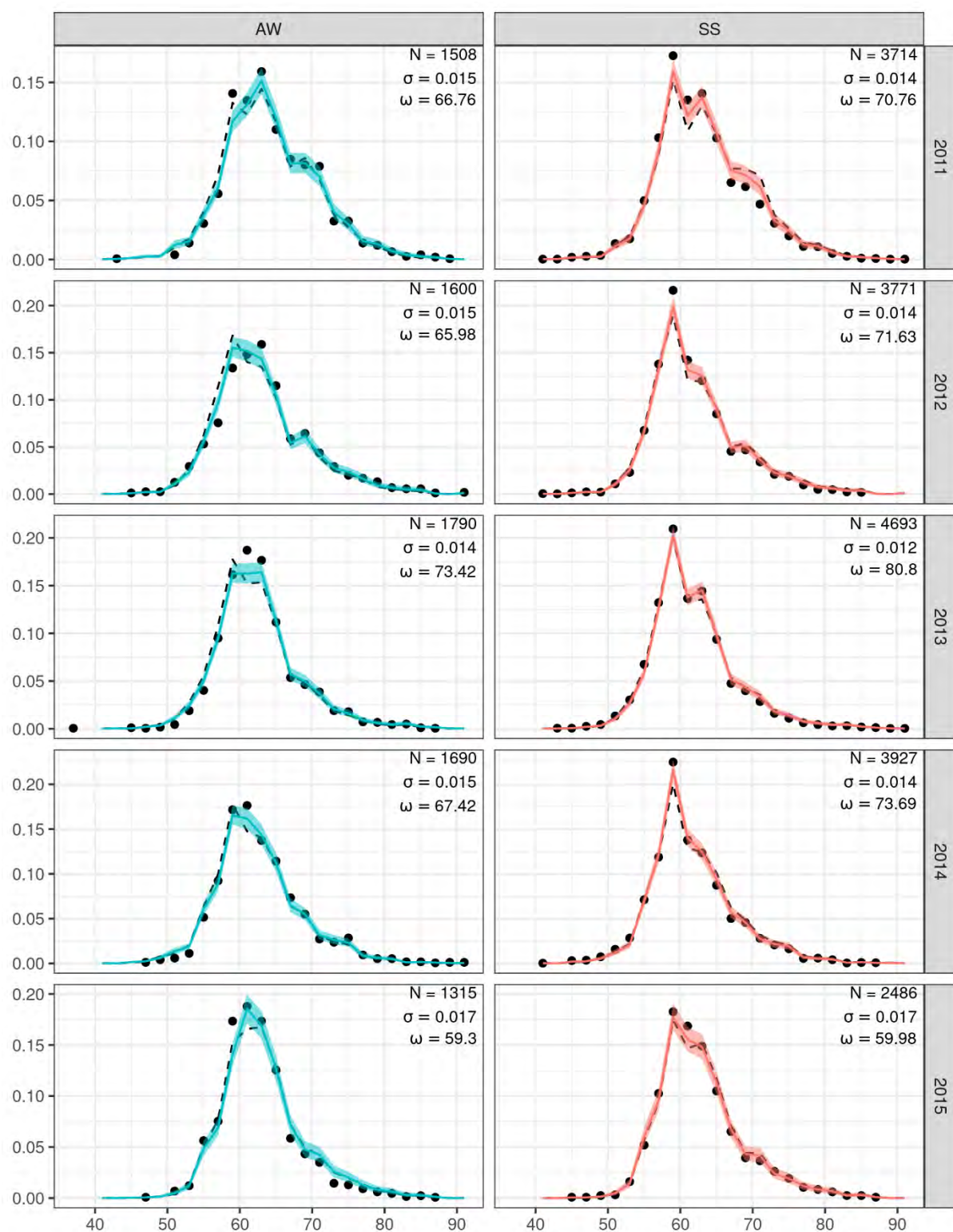


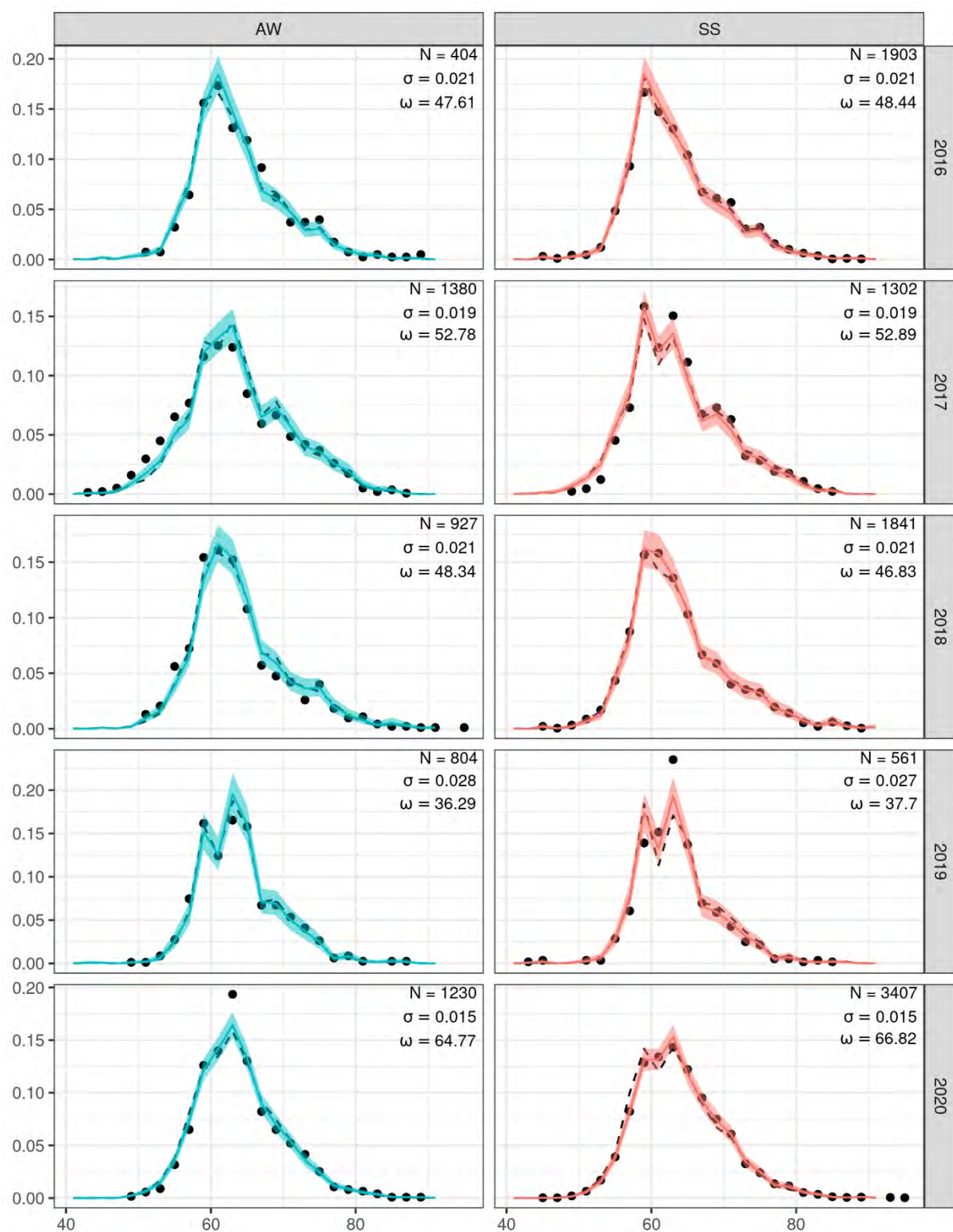
12.3 CRA 2 mature females



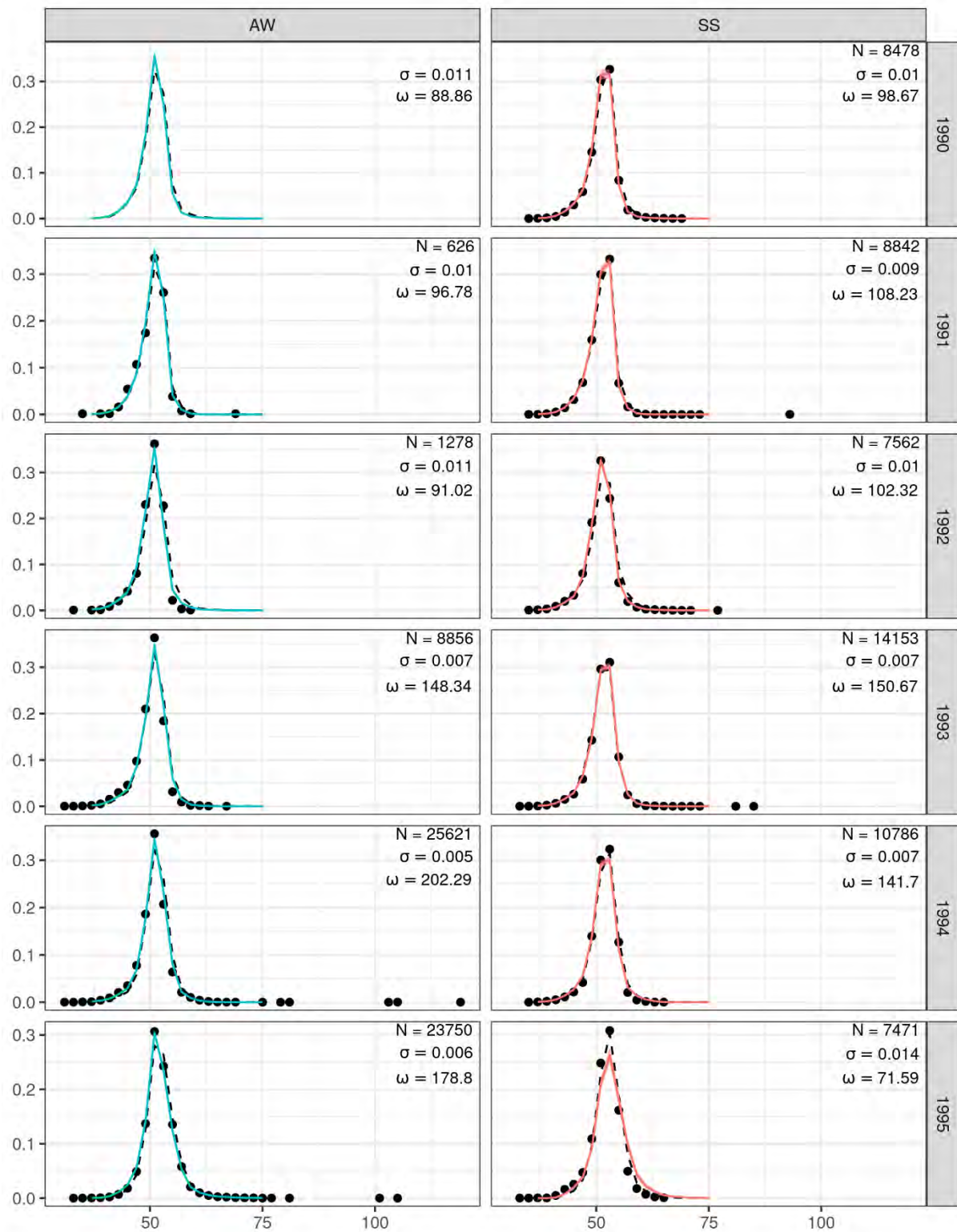


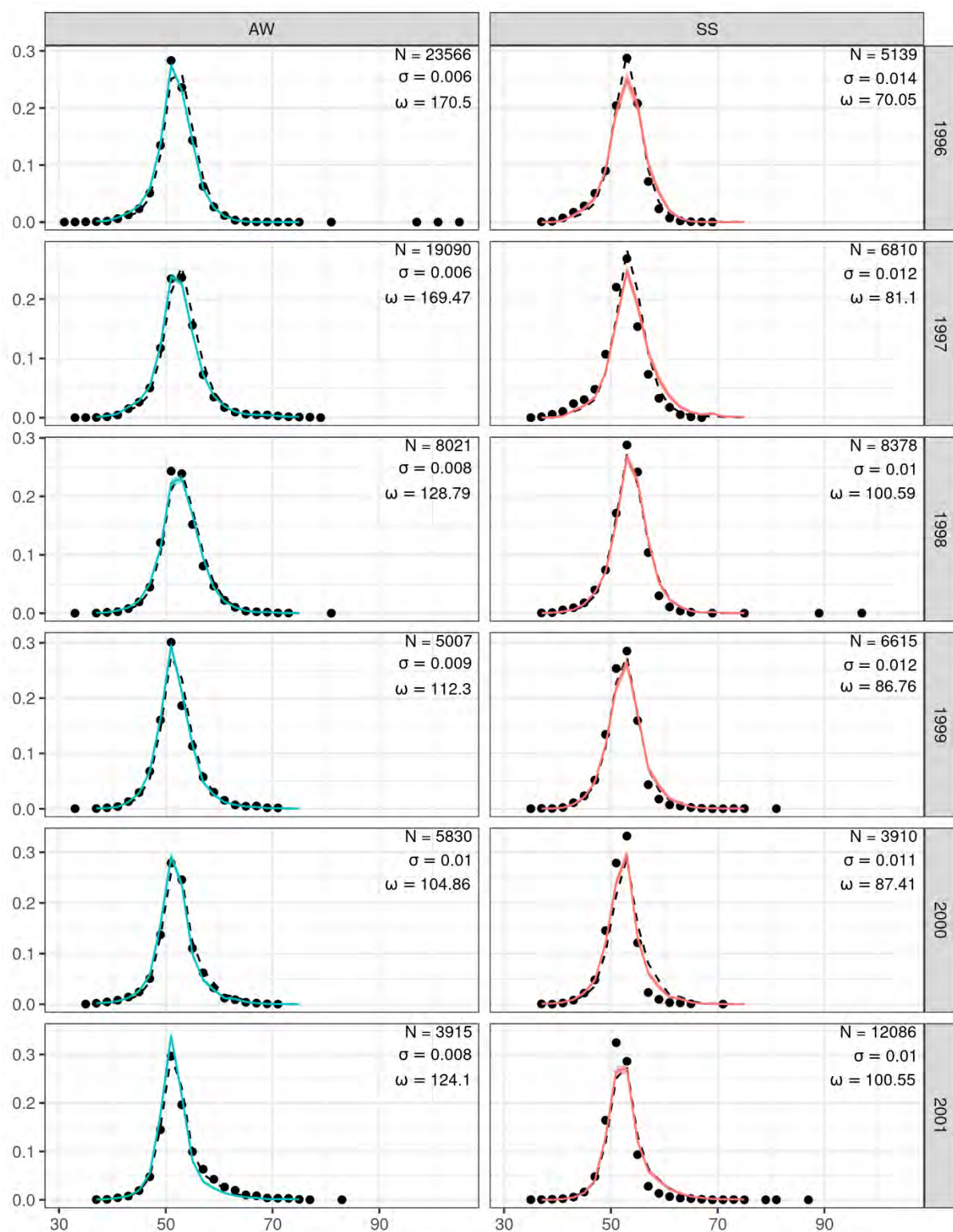


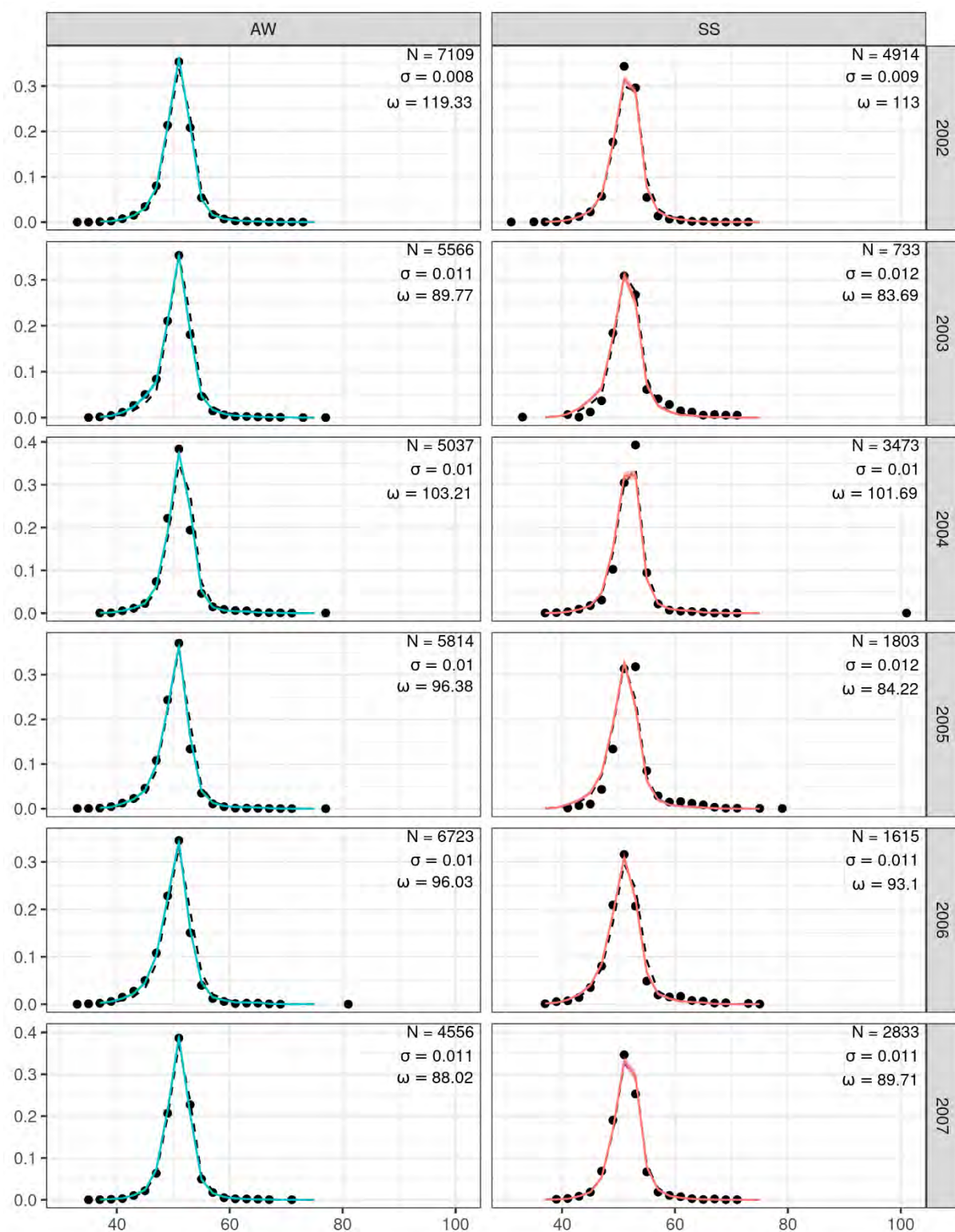


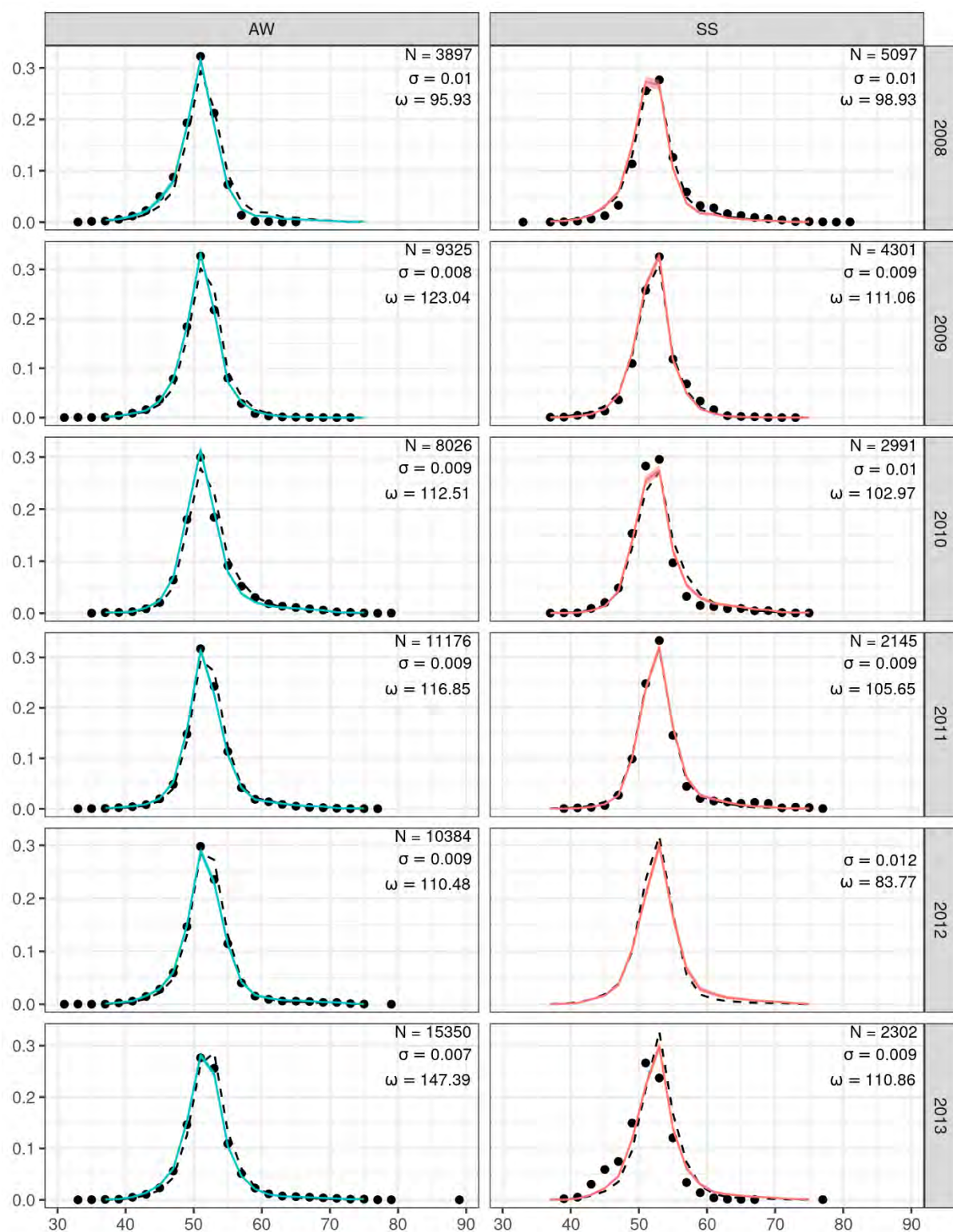


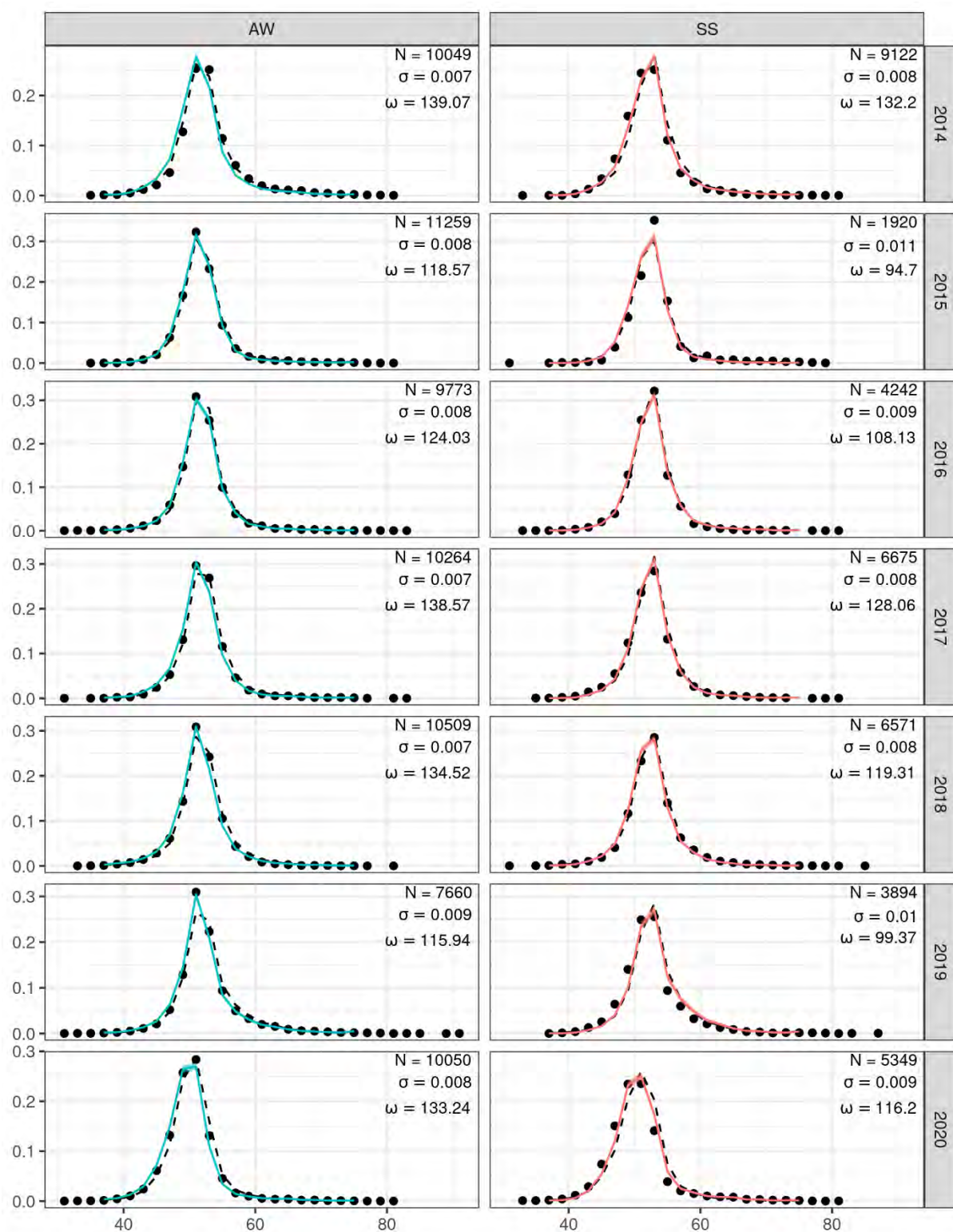
12.4 CRA 3 region 1 males



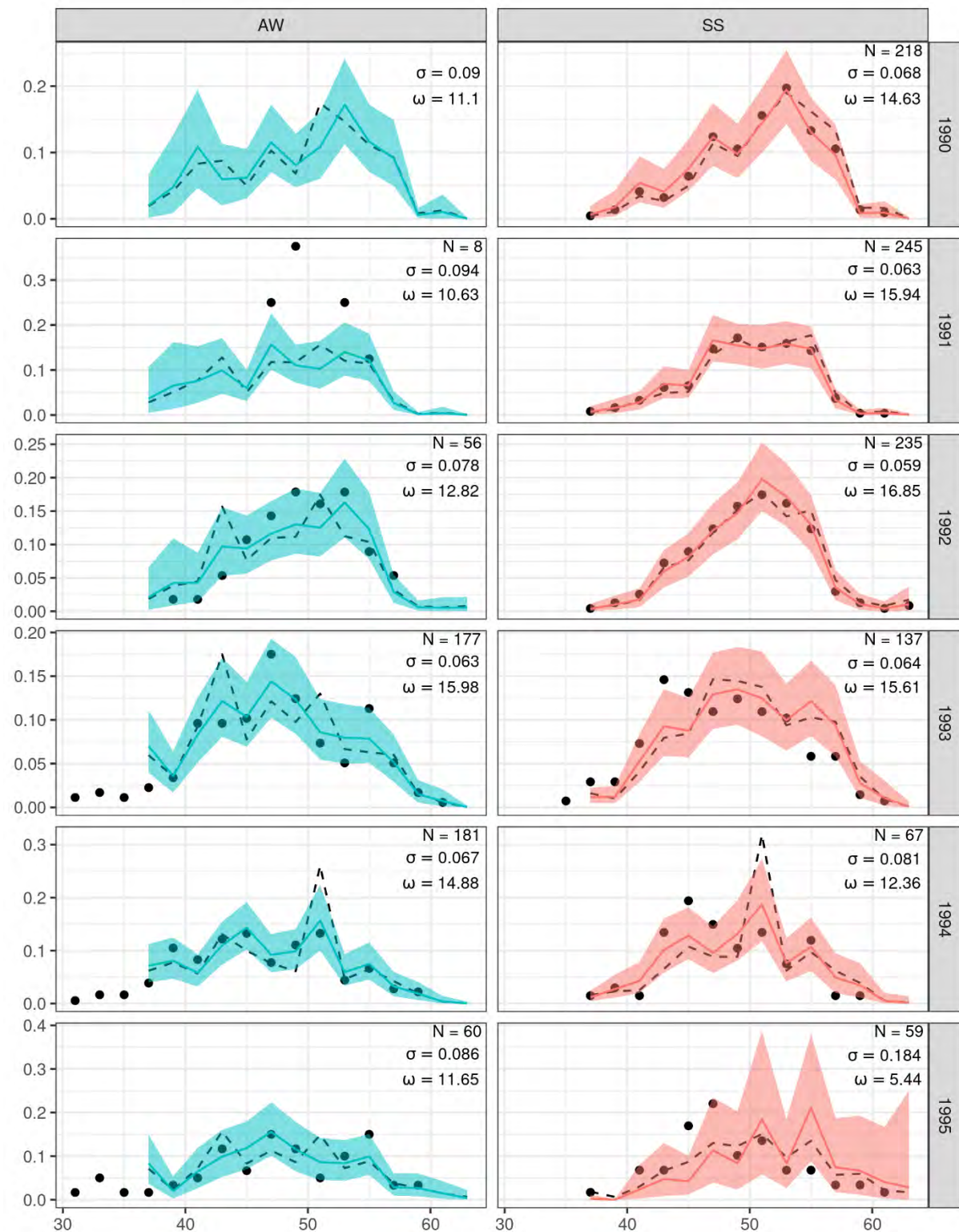


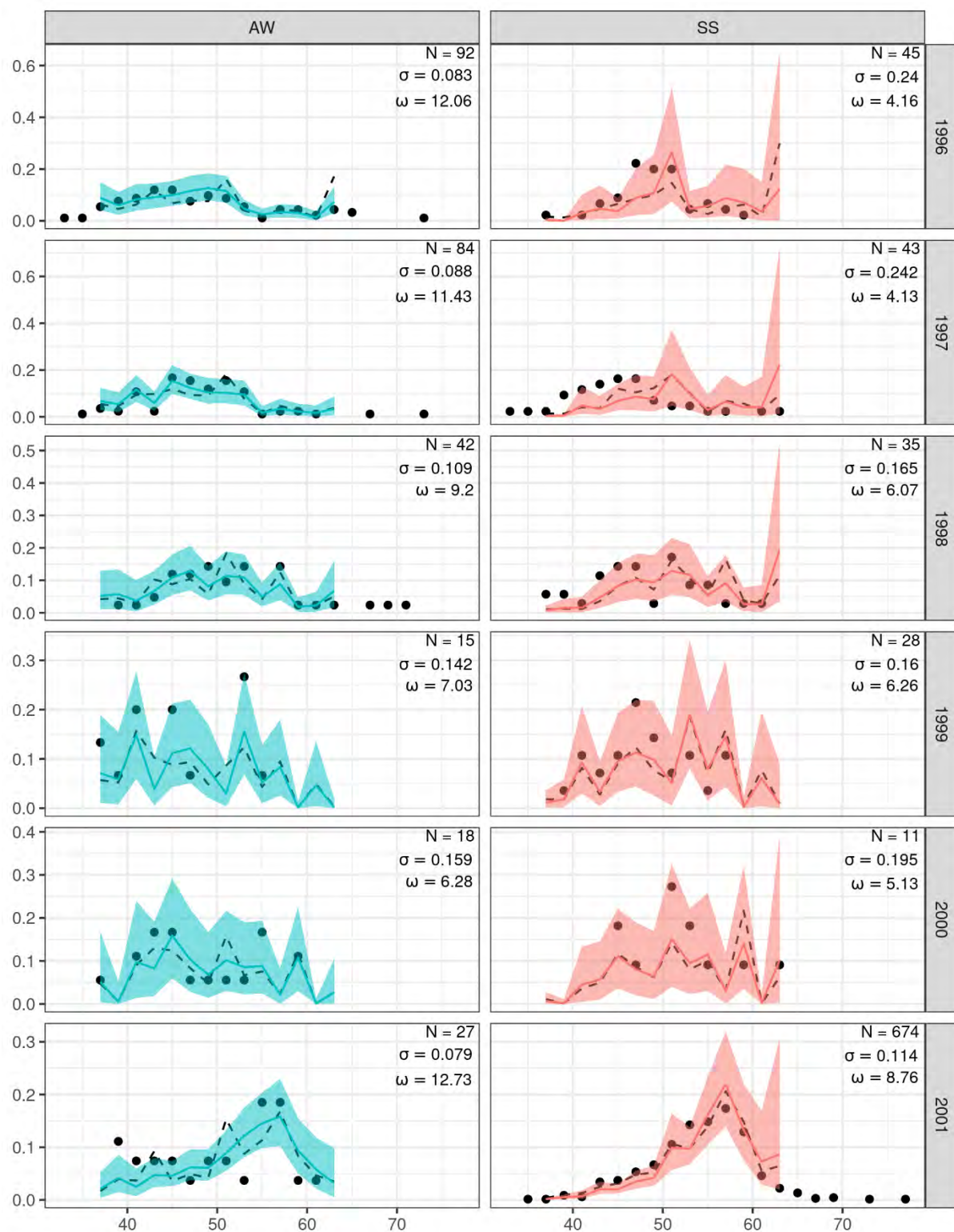


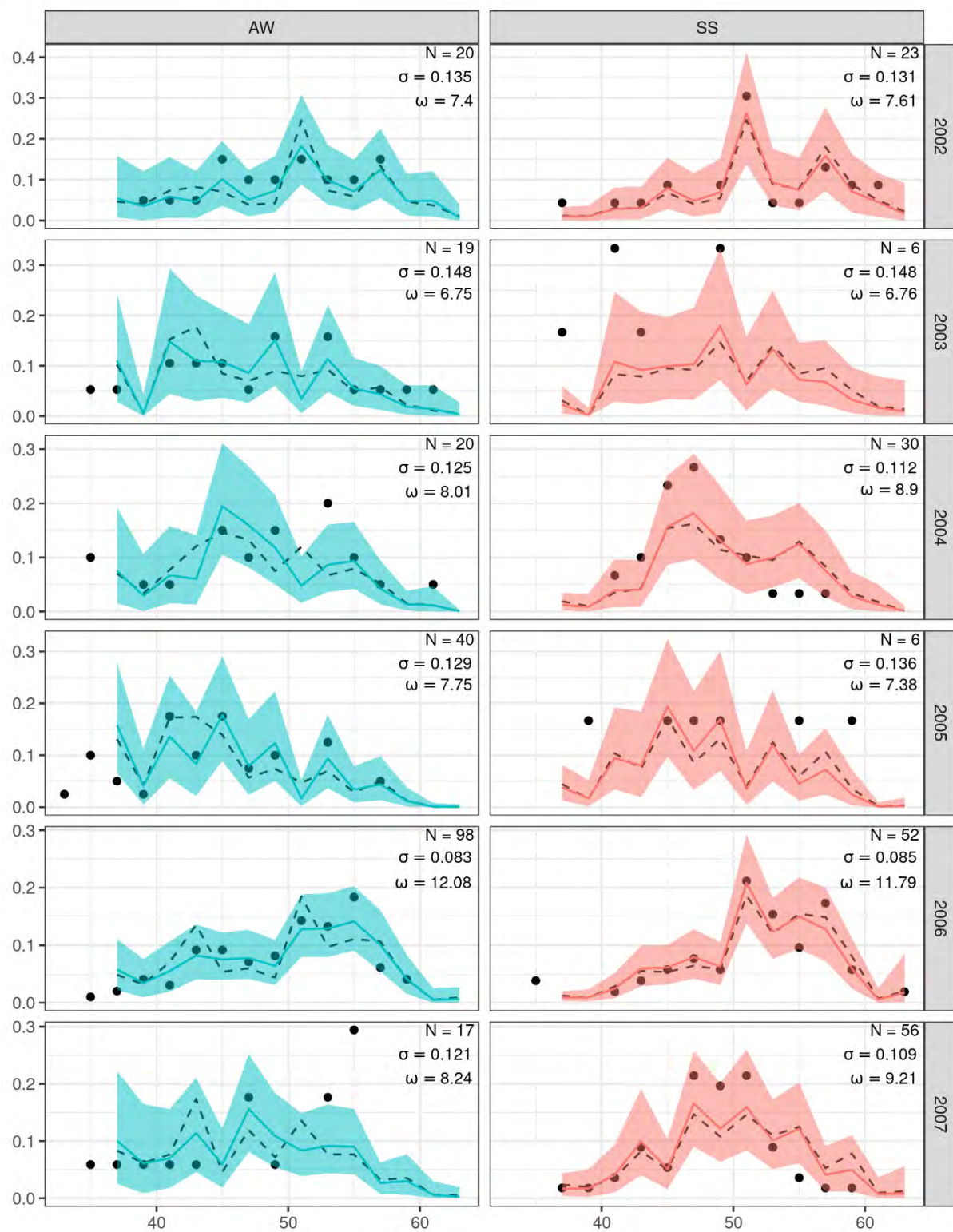


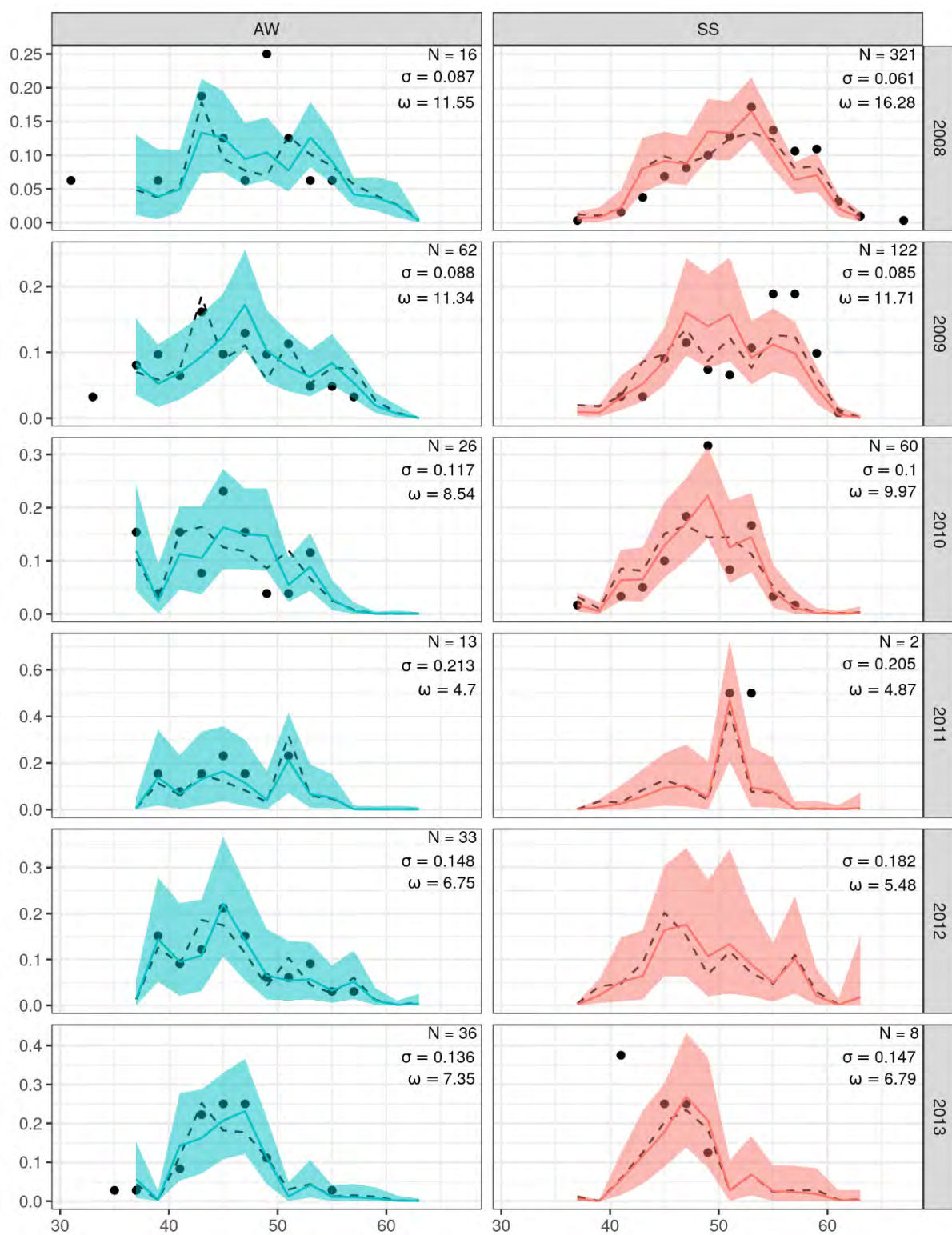


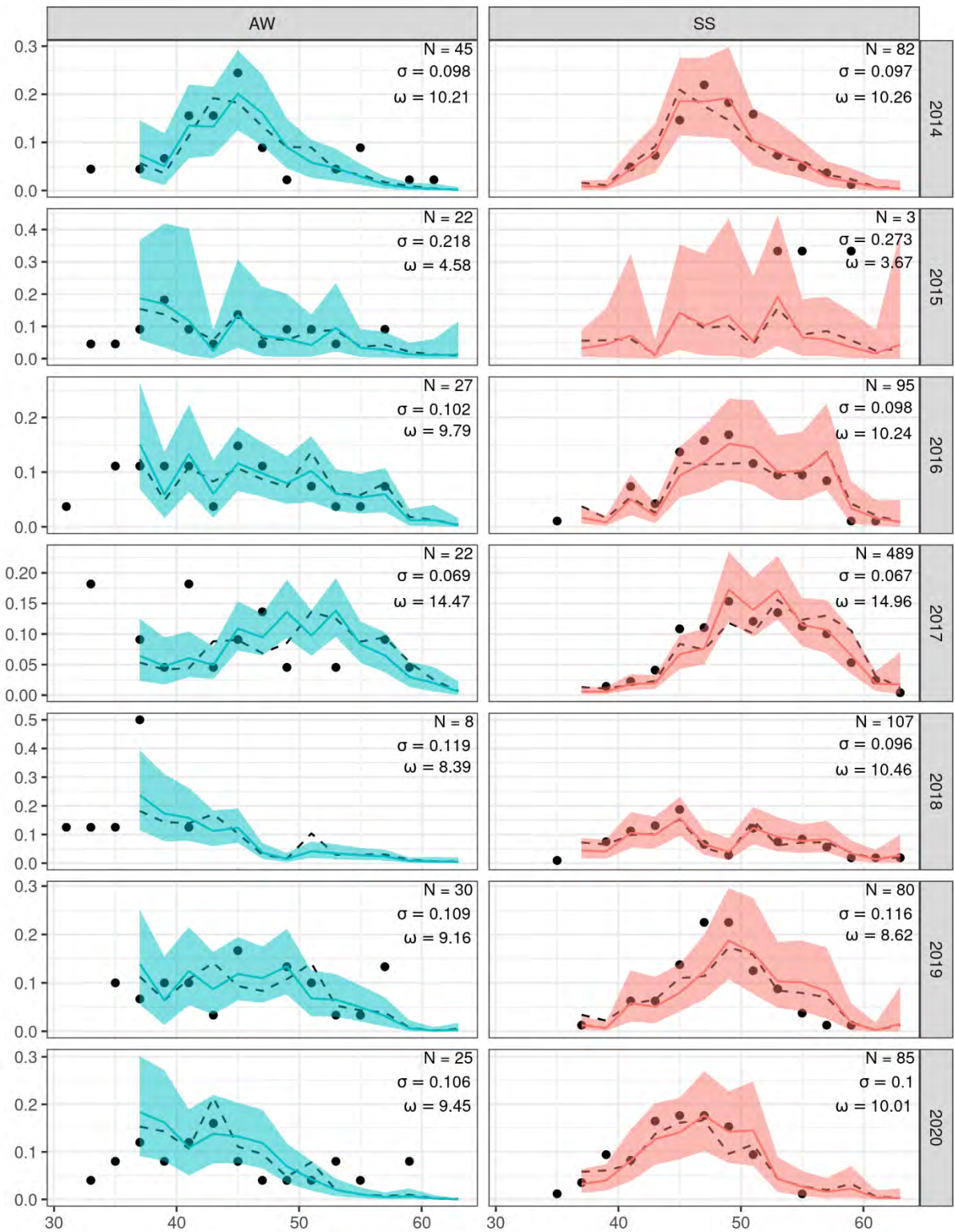
12.5 CRA 3 region 1 immature females



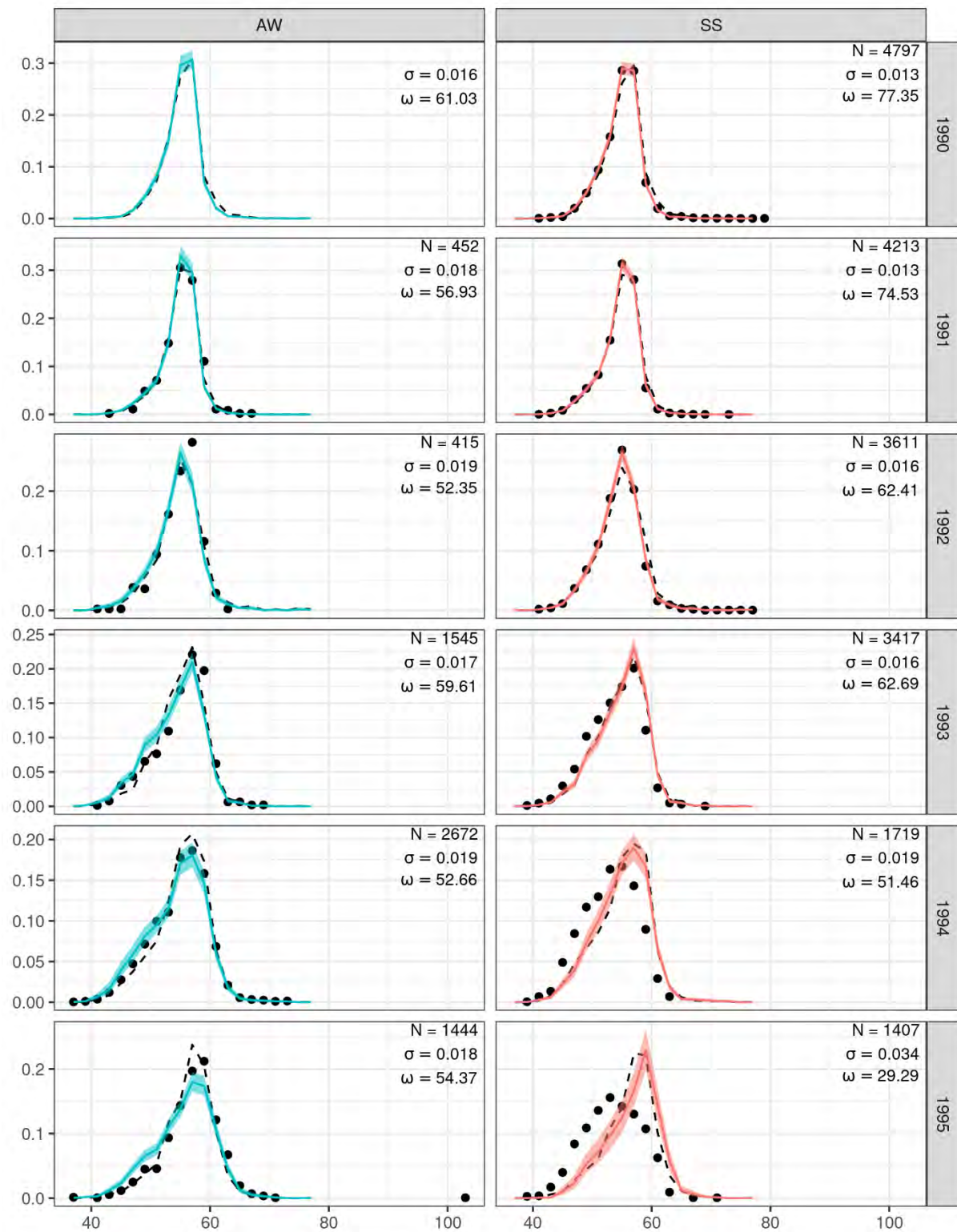


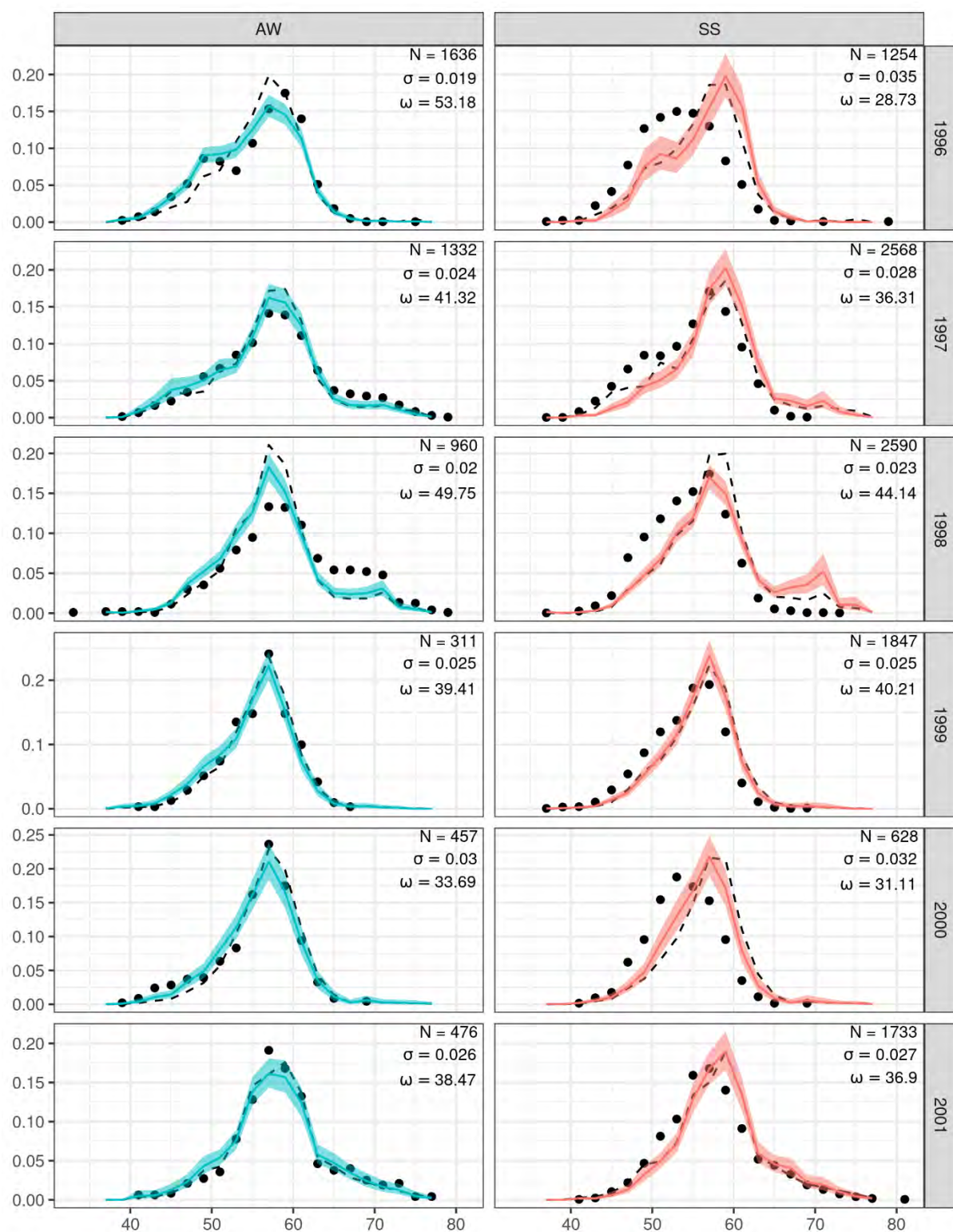


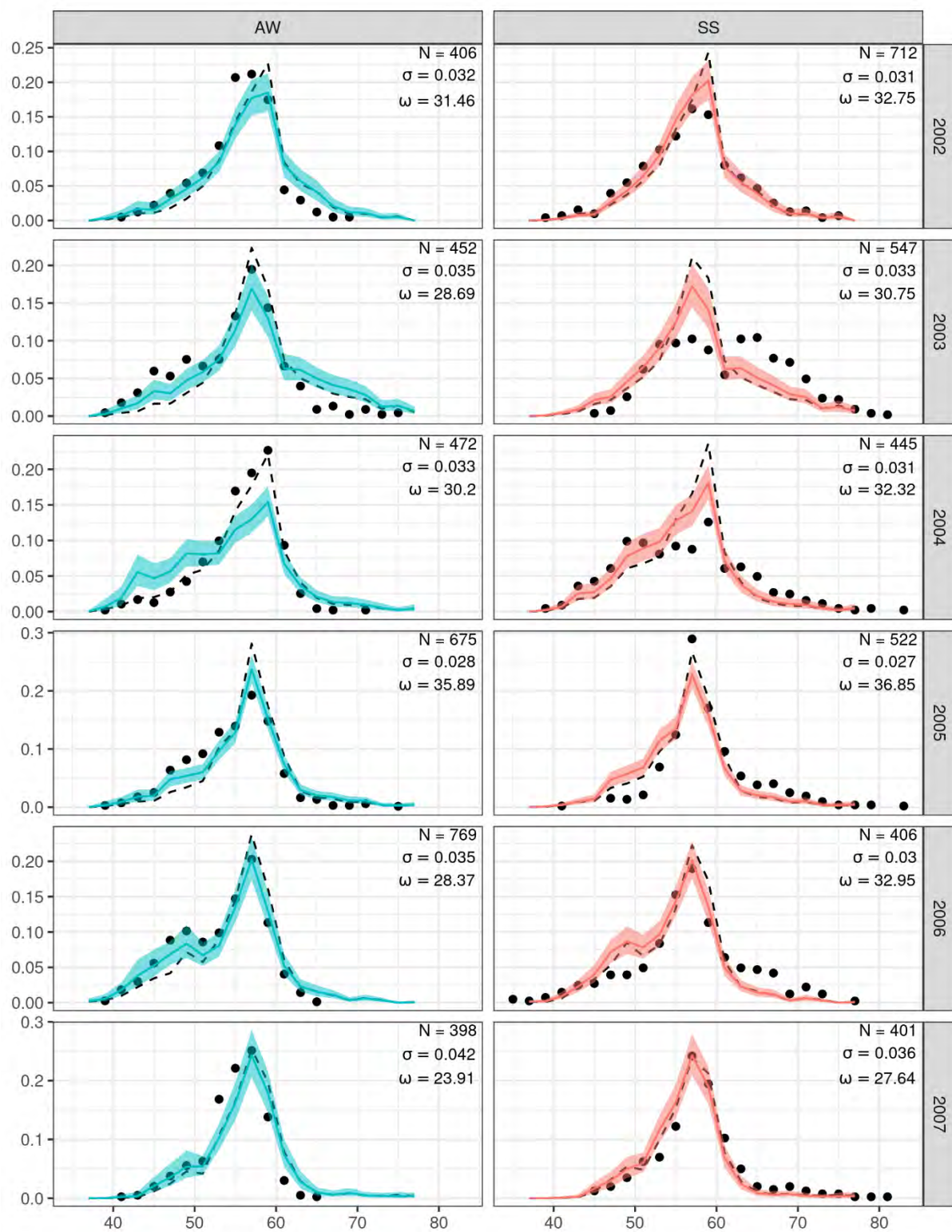


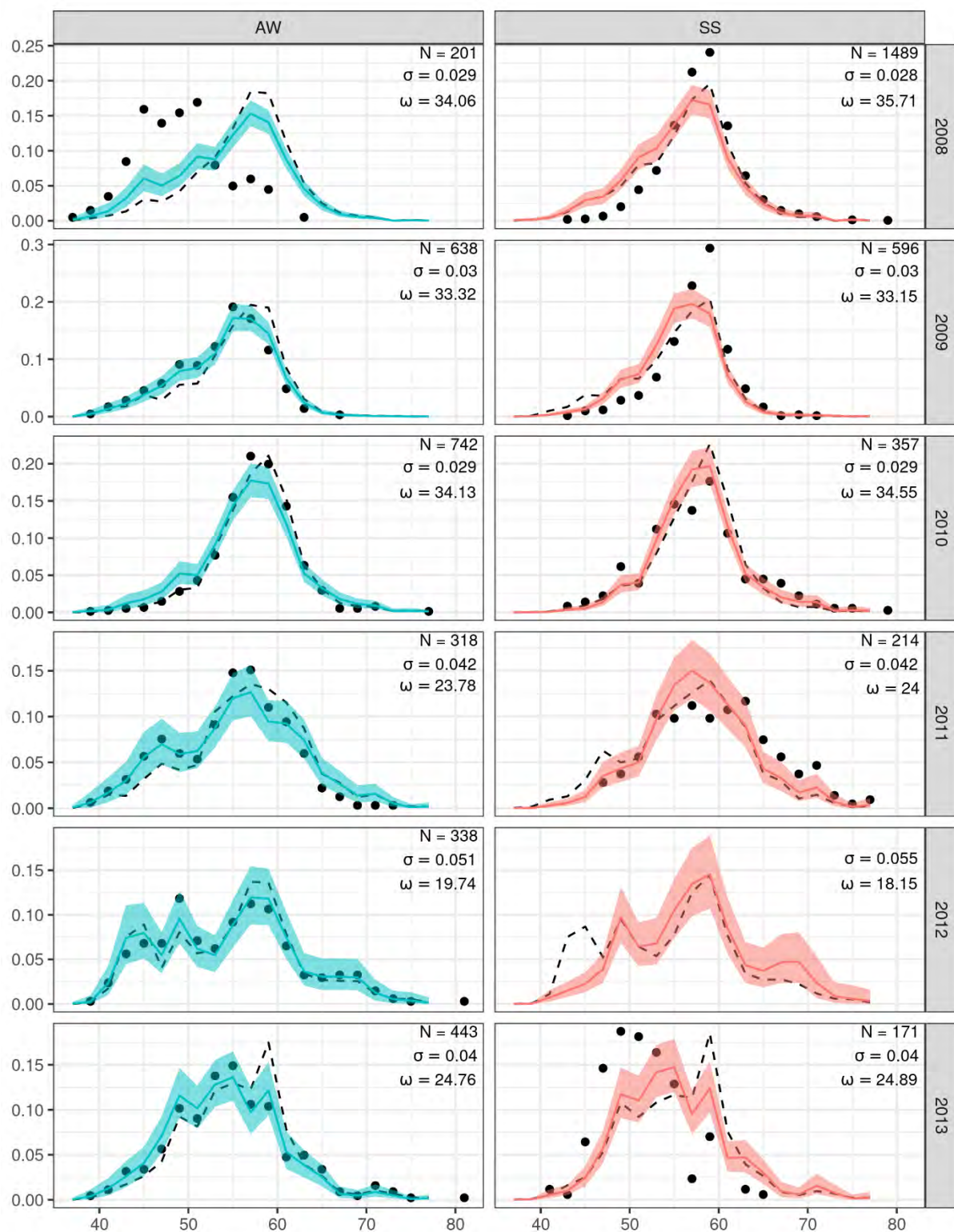


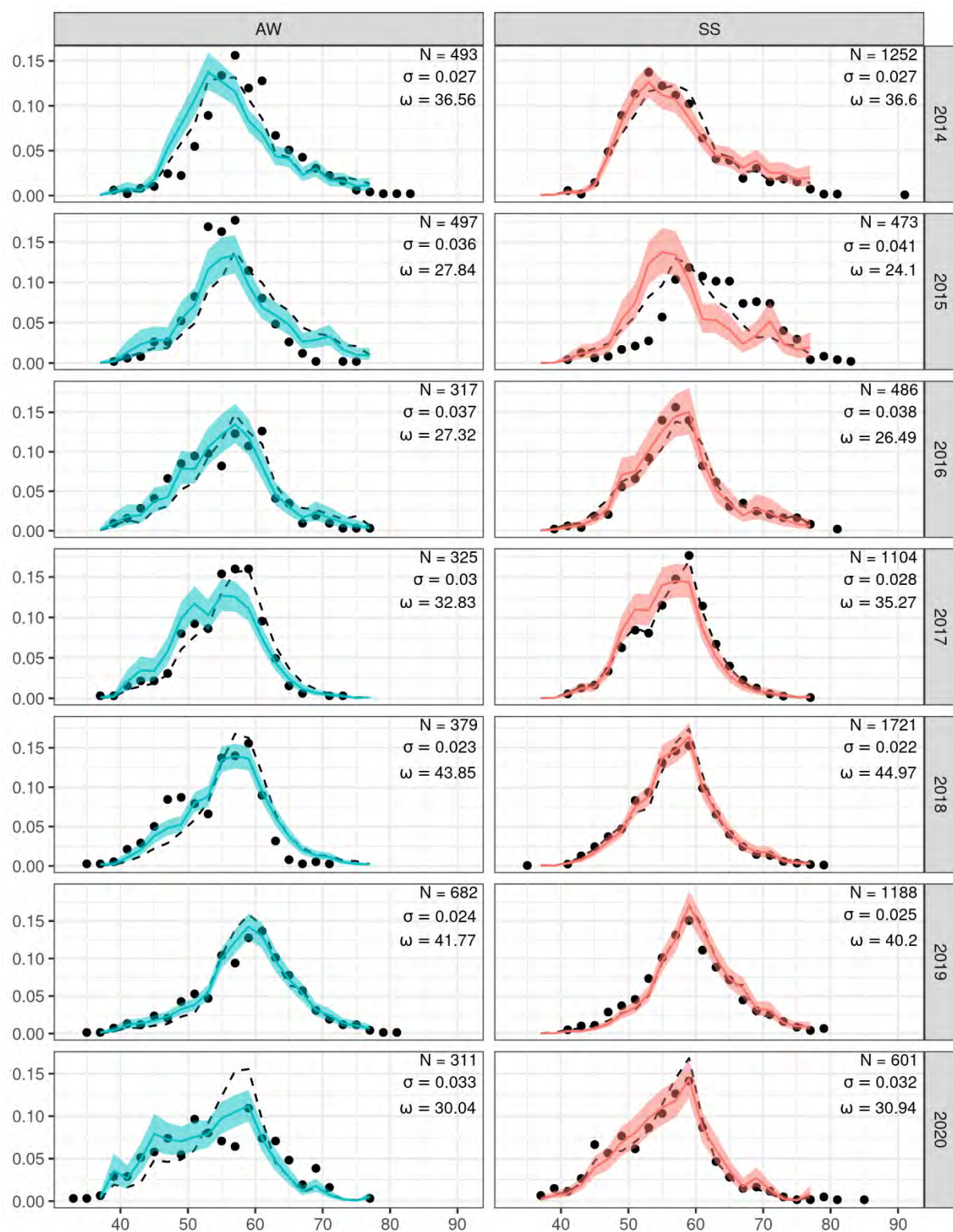
12.6 CRA 3 region 1 mature females



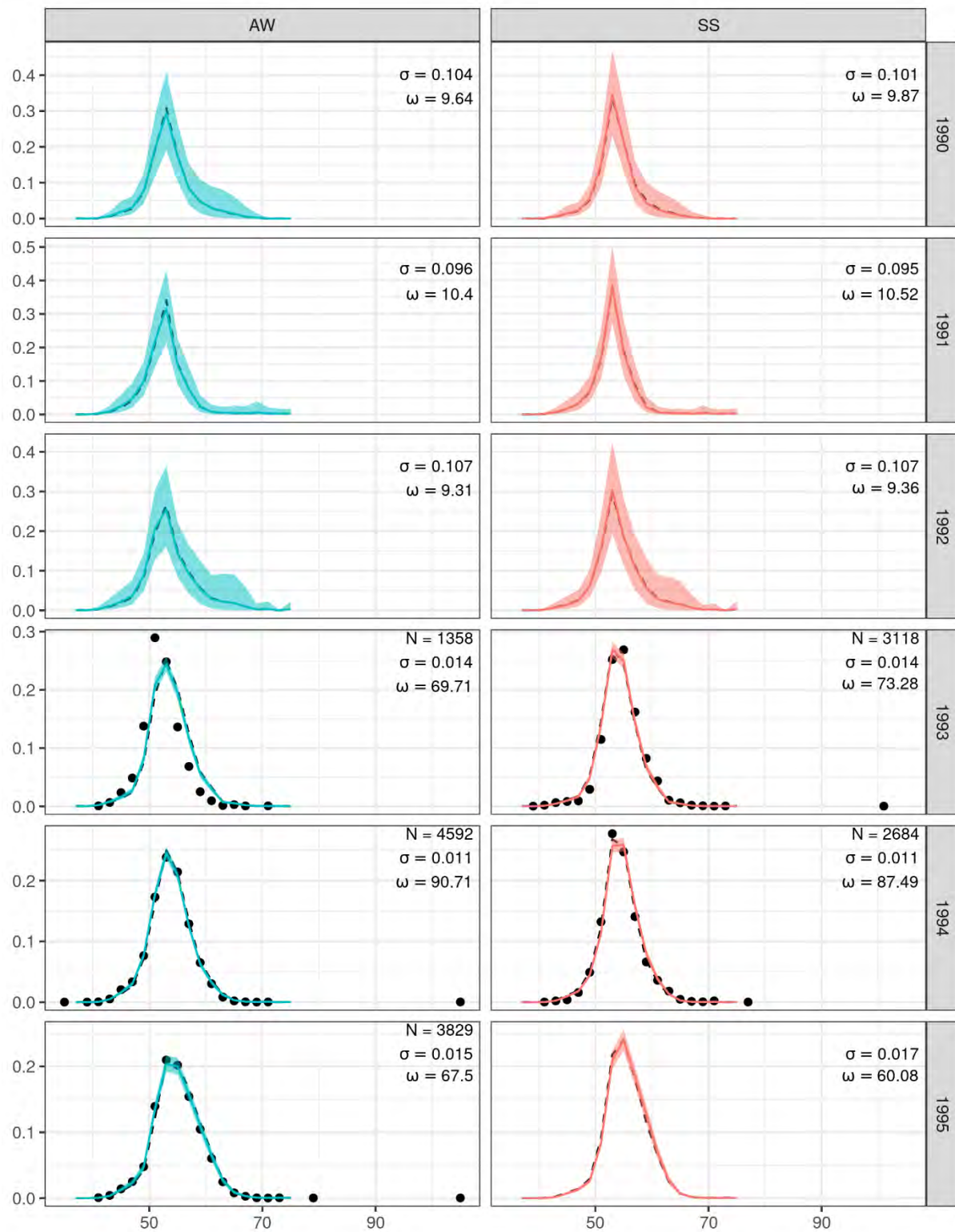


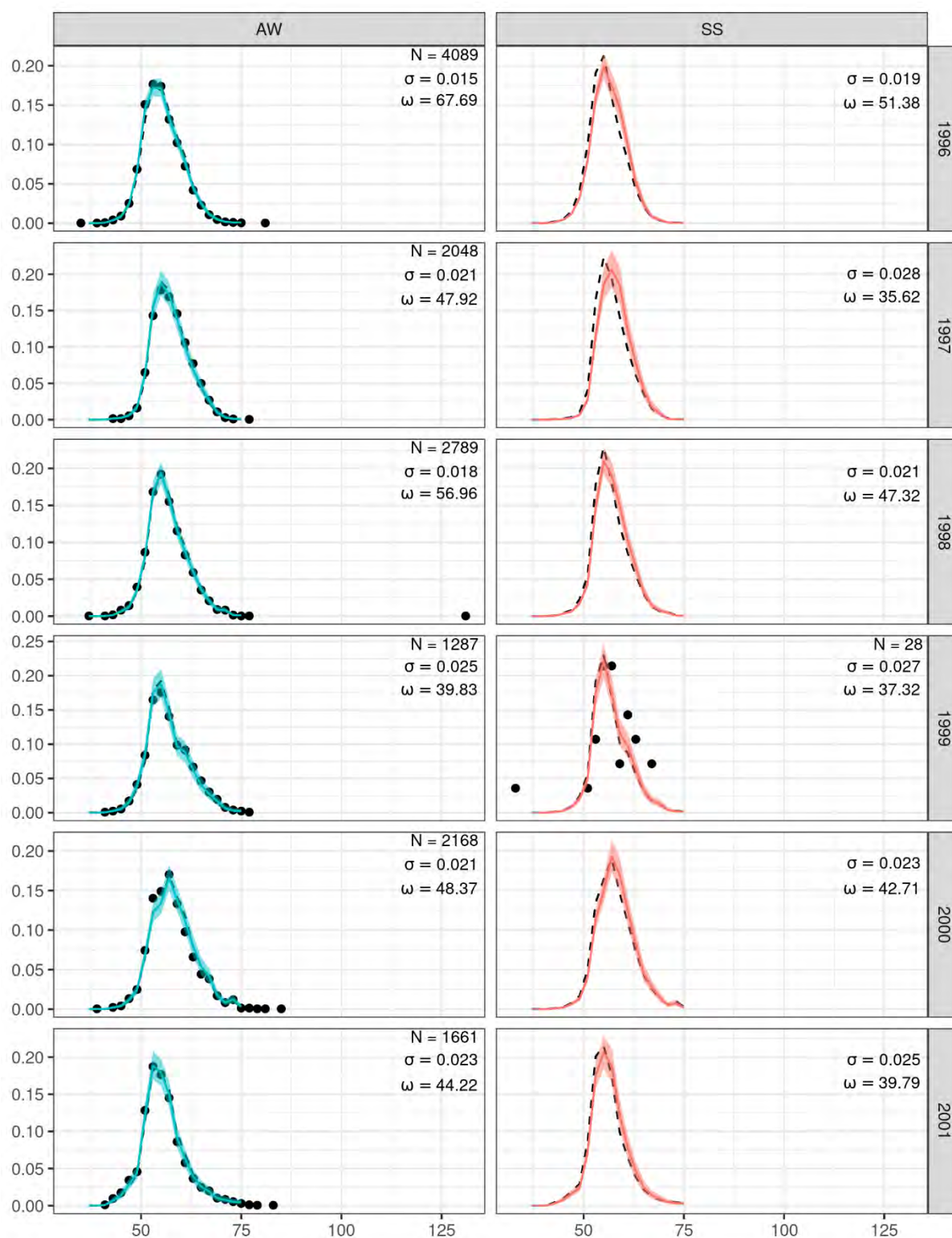


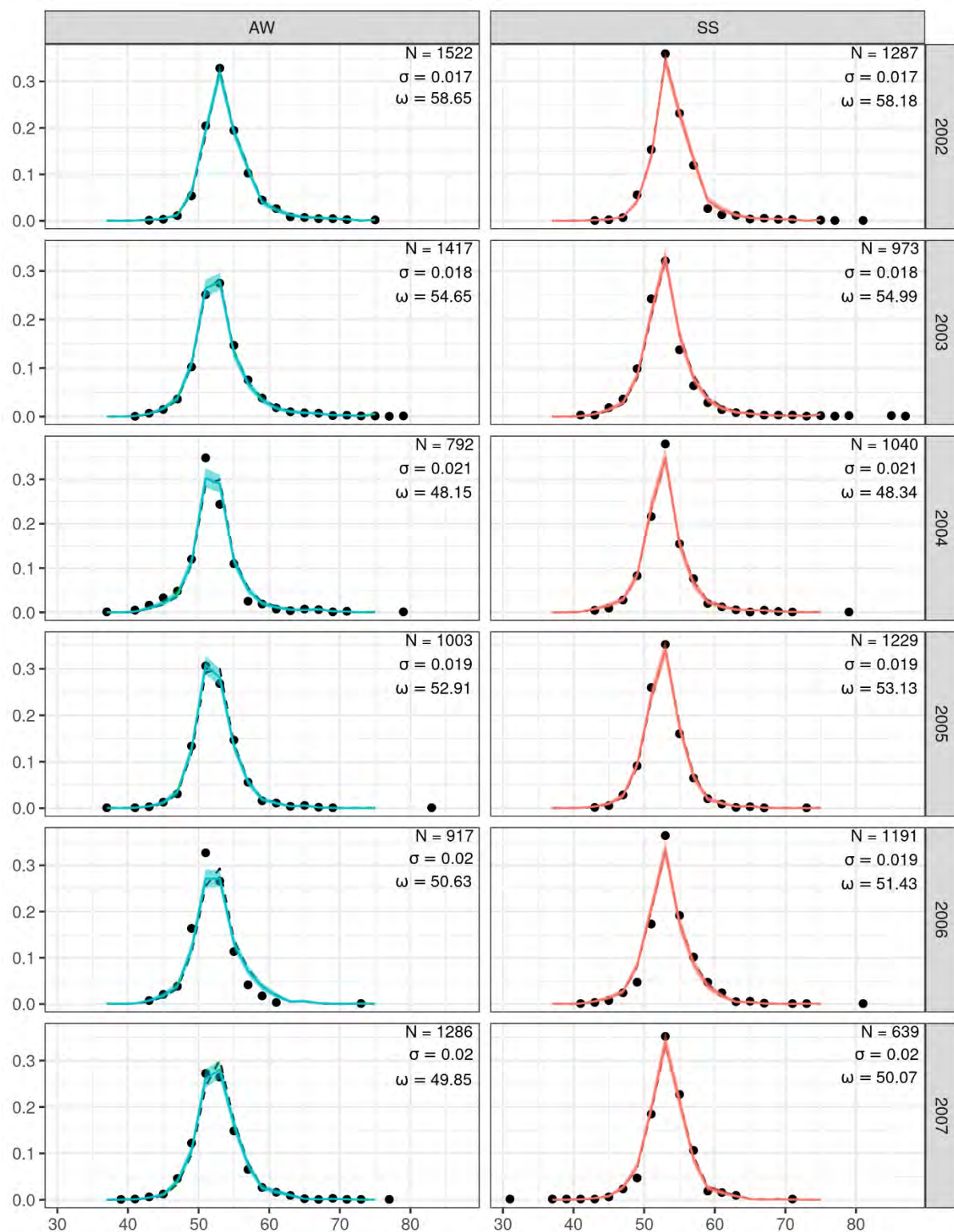


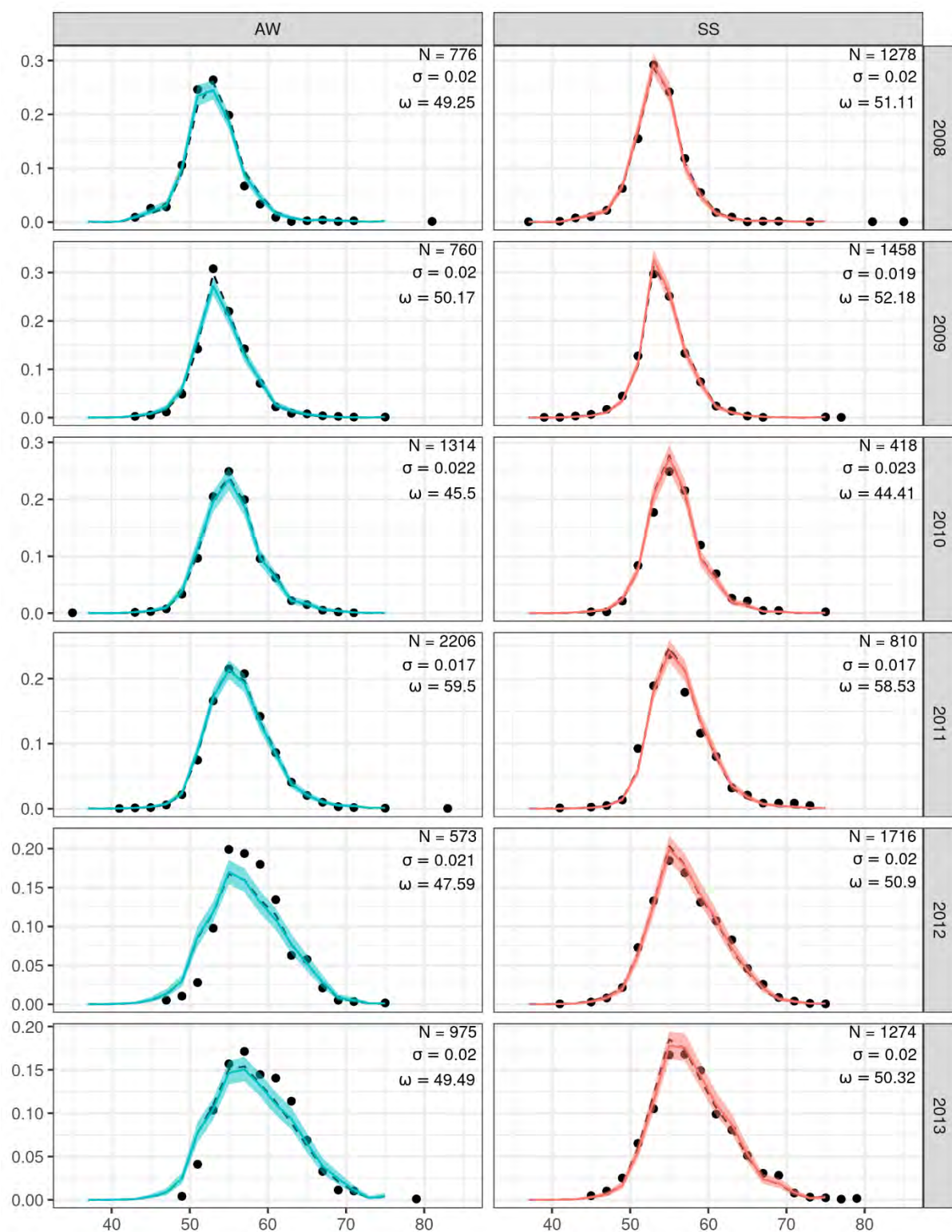


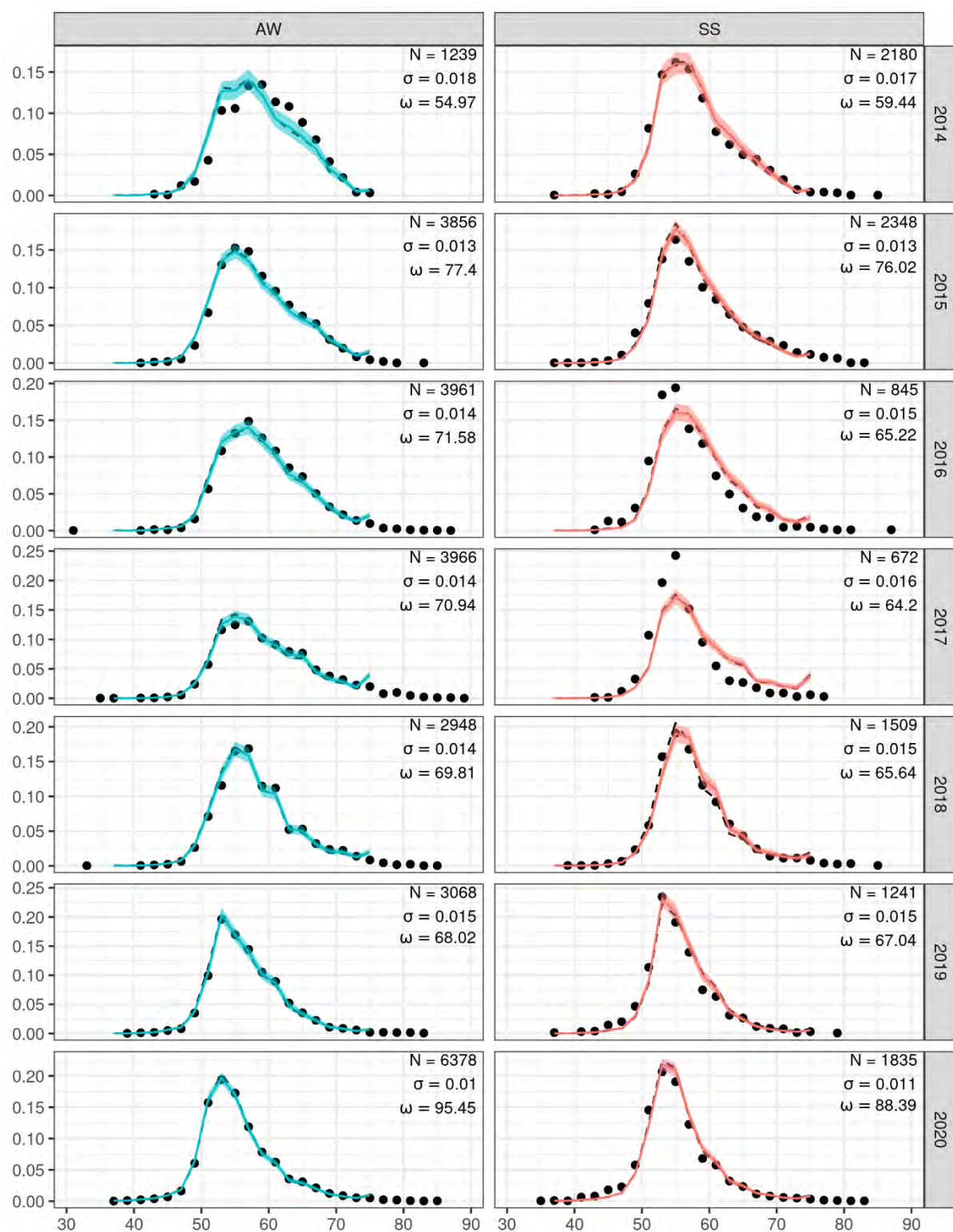
12.7 CRA 3 region 2 males



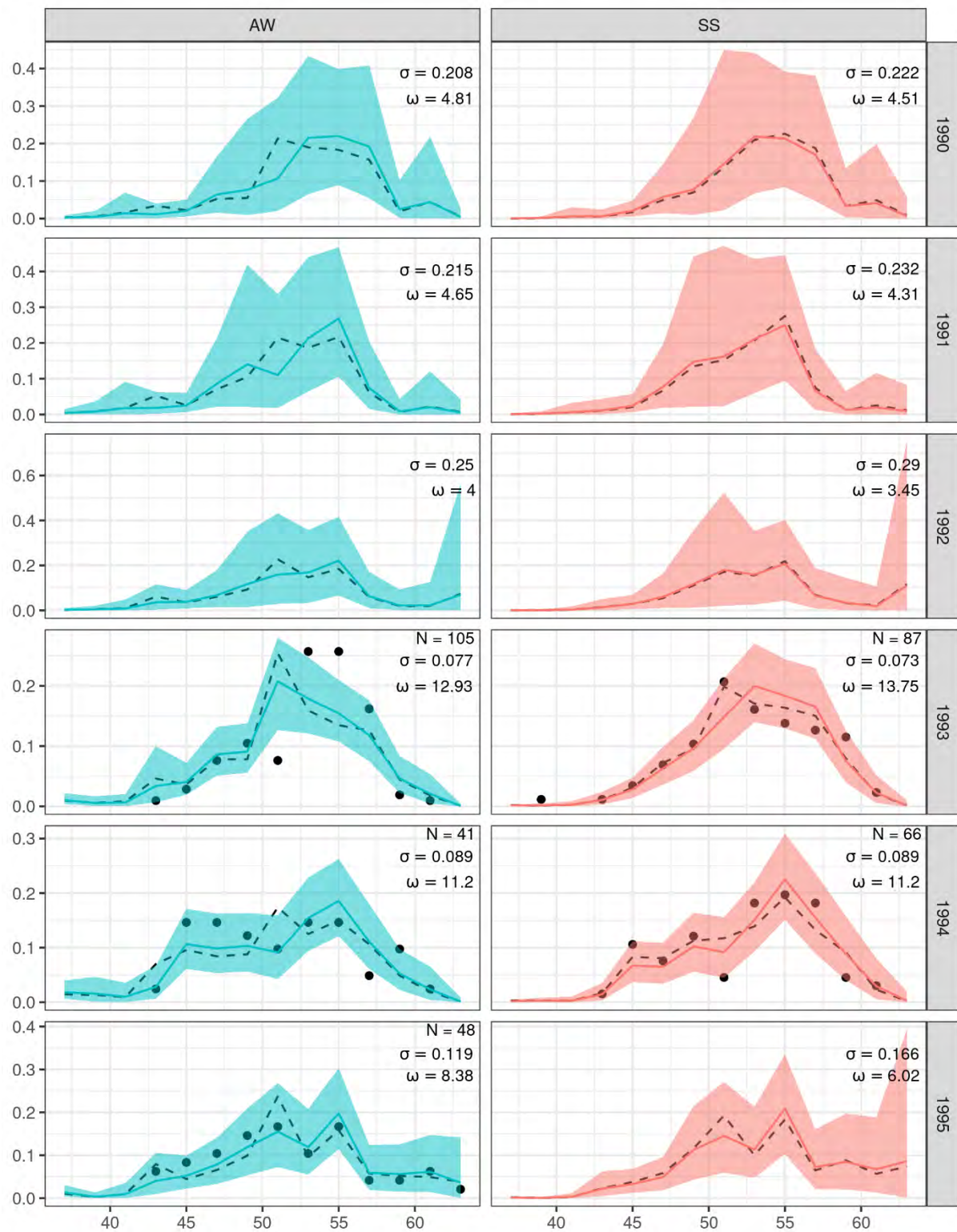


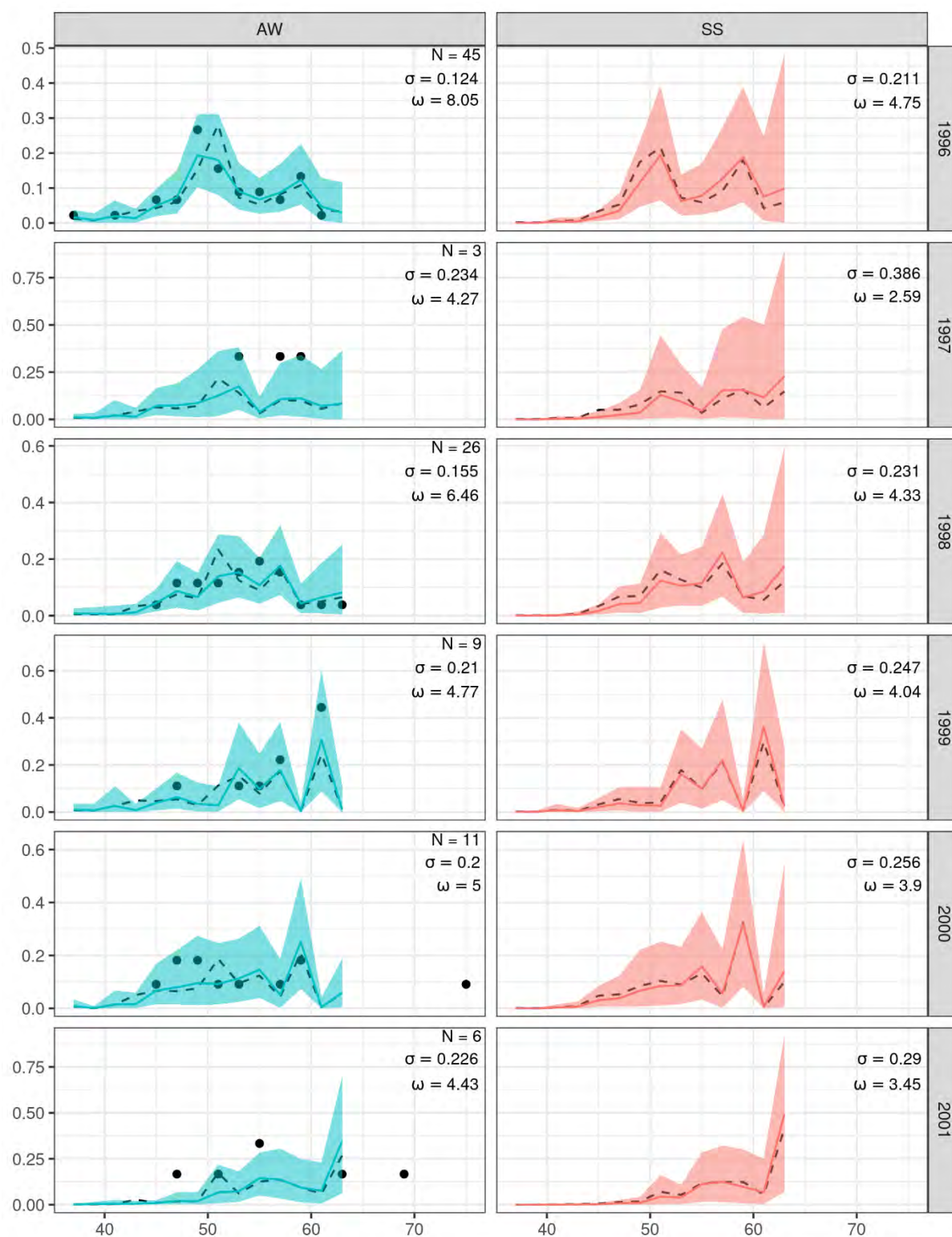


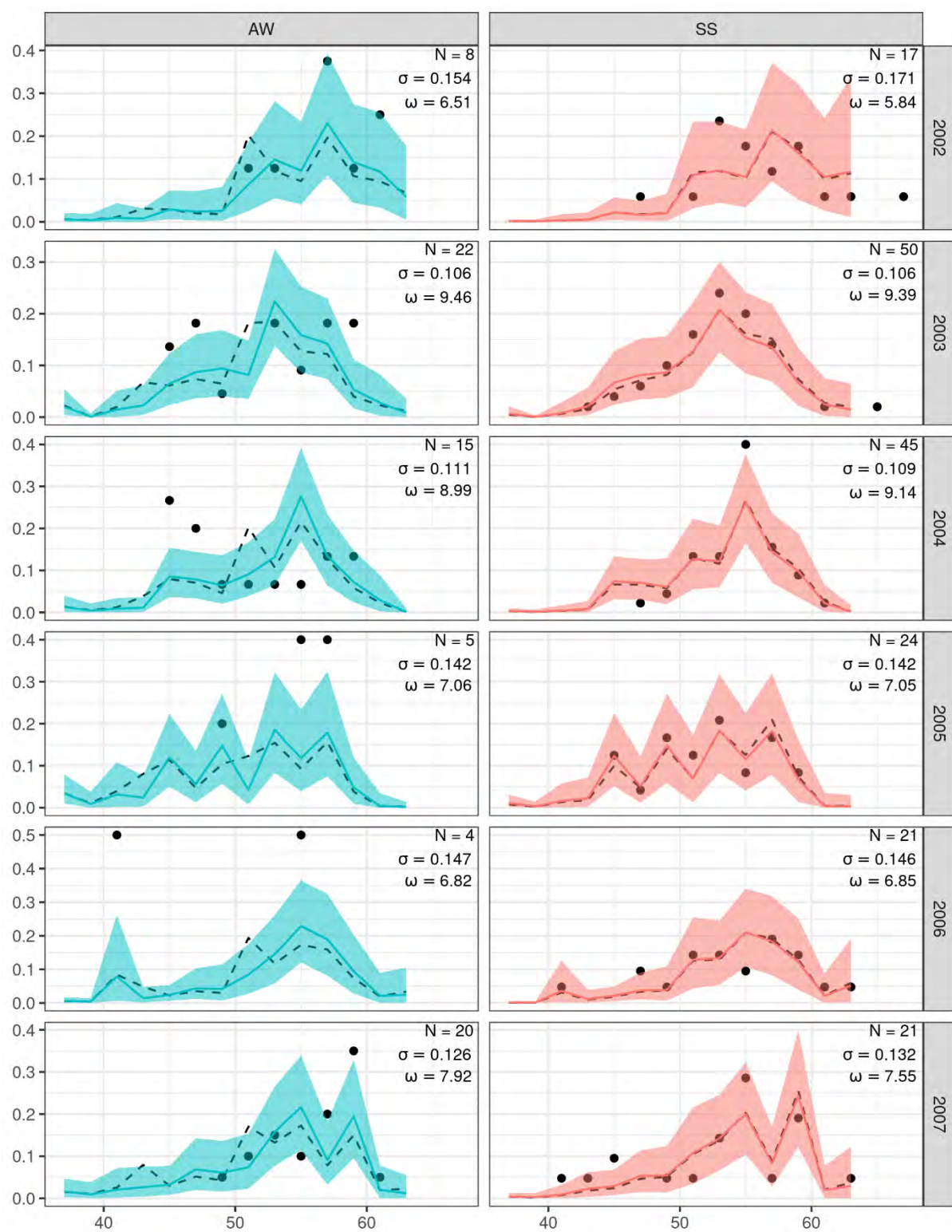


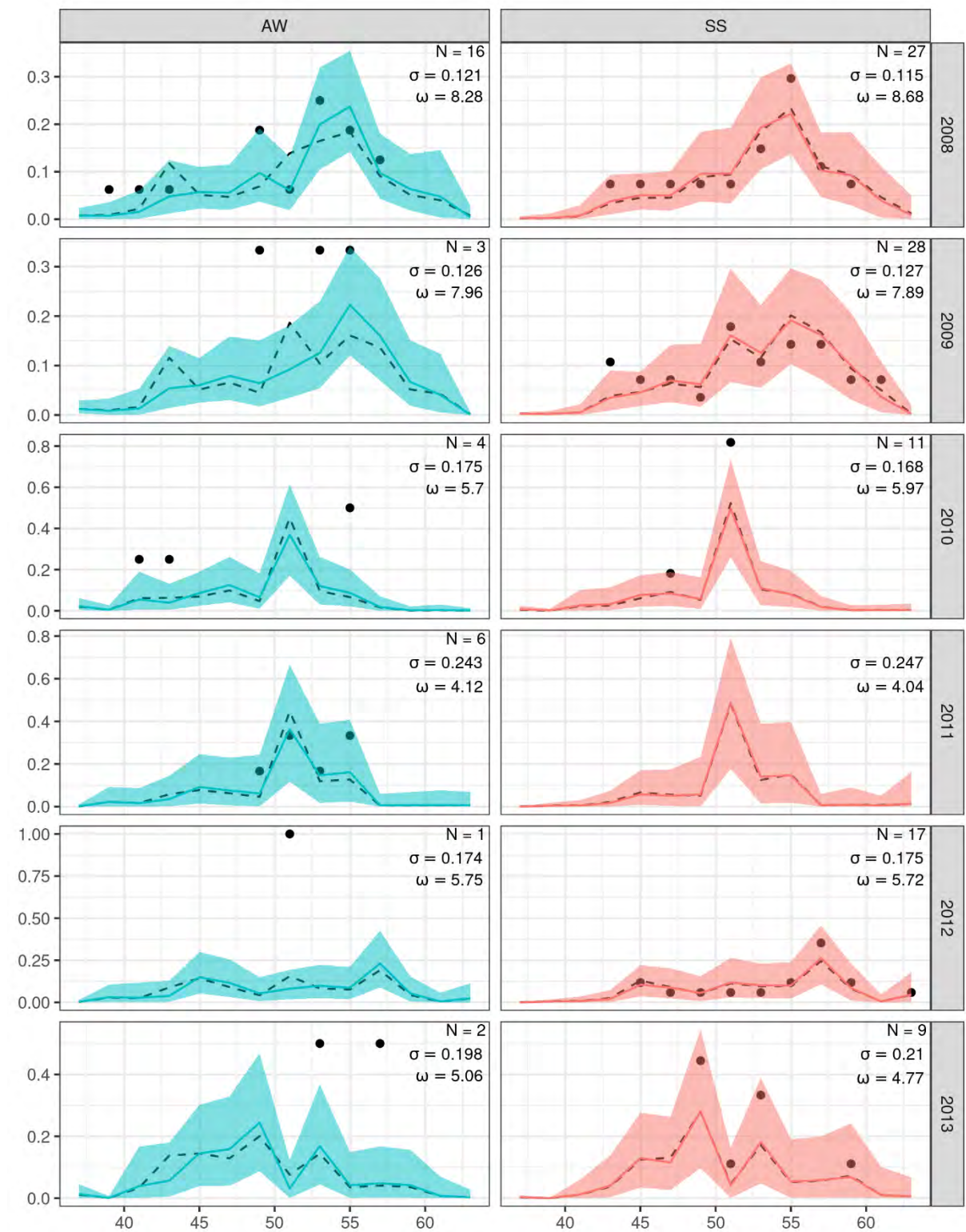


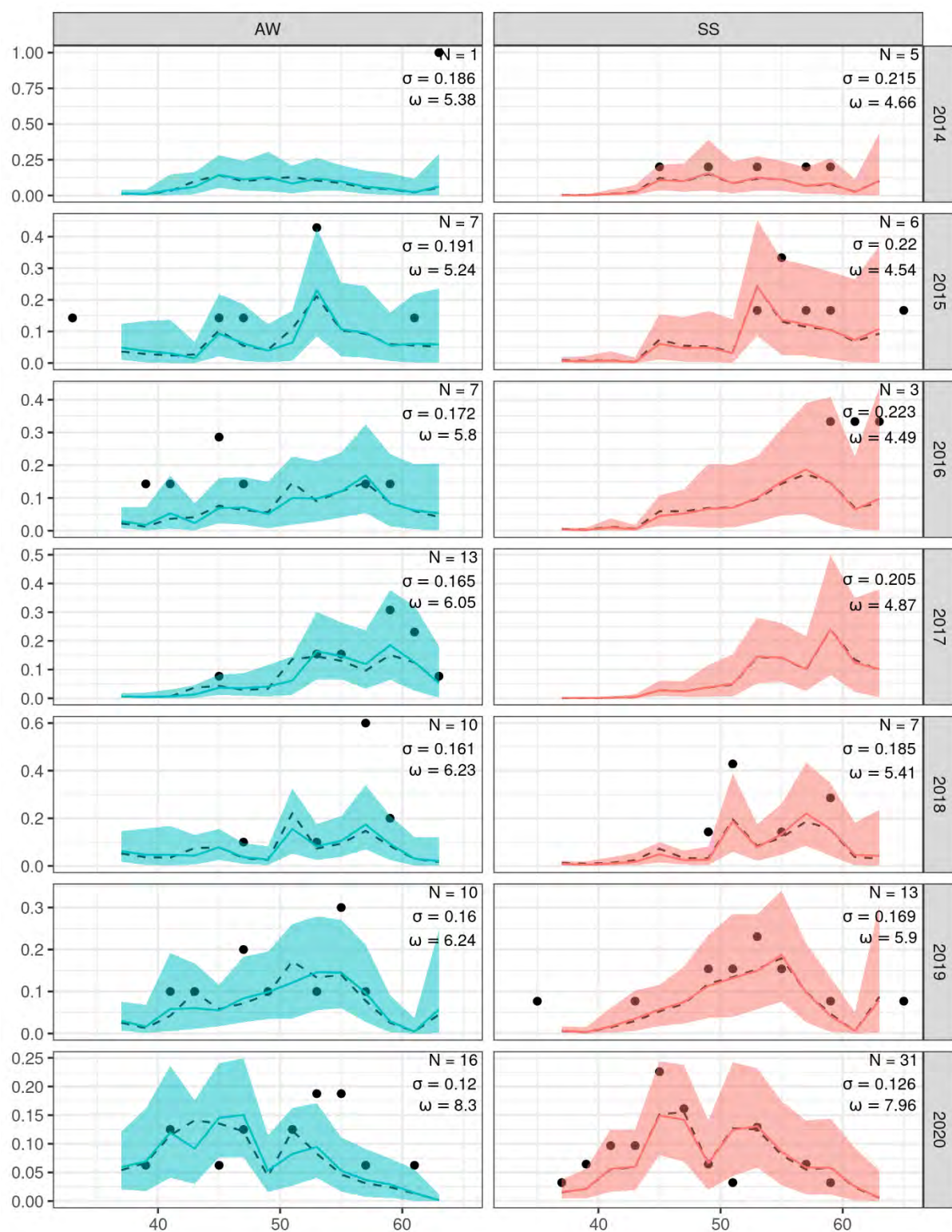
12.8 CRA 3 region 2 immature females











12.9 CRA 3 region 2 mature females

